

# Vortices and spirals at gap edges in 3D self-gravitating disk-planet simulations

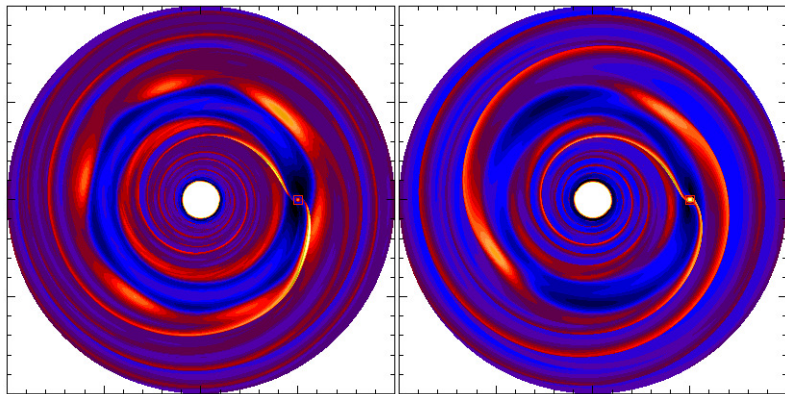
Min-Kai Lin

Canadian Institute for Theoretical Astrophysics

September 20 2012

# Unstable planetary gaps

- Giant planets in self-gravitating disks.



- Low mass disk → VORTICES (Koller et al., 2003; Li et al., 2005; Ou et al., 2007; Li et al., 2009; Yu & Li, 2009; Lin & Papaloizou, 2010, 2011a)
- High mass disk → SPIRALS (Meschiari & Laughlin, 2008; Lin & Papaloizou, 2011b, 2012)

# Outline

- Review
- Examples in 3D
- Discussion
- Upcoming

# Non-axisymmetric instabilities in structured 2D disks

- Potential vorticity (vortensity) extrema is necessary for instability:

$$\eta \equiv \frac{\kappa^2}{2\Omega\Sigma}$$

- Note: barotropic, non-magnetized
- Nearly-Keplerian disk:  $\kappa \sim \Omega$  so  $\eta \sim \kappa/\Sigma \propto Q$

# Non-axisymmetric instabilities in structured 2D disks

- Potential vorticity (vortensity) extrema is necessary for instability:

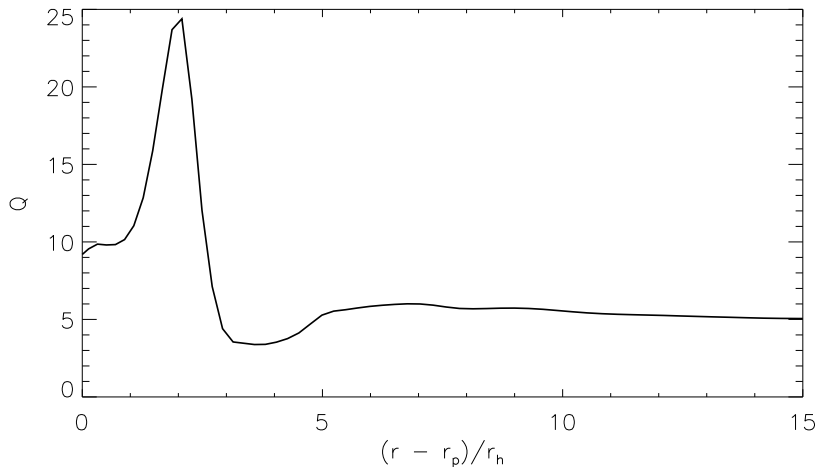
$$\eta \equiv \frac{\kappa^2}{2\Omega\Sigma}$$

- Note: barotropic, non-magnetized
- Nearly-Keplerian disk:  $\kappa \sim \Omega$  so  $\eta \sim \kappa/\Sigma \propto Q$

$$Q \equiv \frac{c_s \kappa}{\pi G \Sigma} = \frac{c_s}{\pi G} \left( \frac{2\Omega\eta}{\Sigma} \right)^{1/2}$$

- Planet gaps:  $Q$  and  $\eta$  look similar

## Planetary gaps in terms of $Q$



- Local  $\min(Q) \rightarrow$  vortices
- Local  $\max(Q)$  plus  $Q(r_{\text{out}}) \lesssim 2 \rightarrow$  spirals

# Checklist for 3D simulations

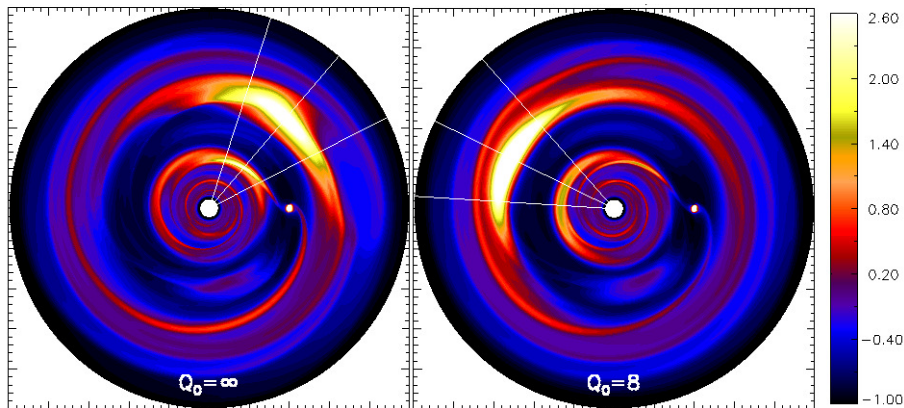
## 2D results:

- Vortex formation in low  $M_d$ , fast merging
- More vortices with increasing  $M_d$ , resisted/delayed merging
- Spirals with large  $M_d$

## 3D setup:

- Inviscid 3D disk in spherical polars
- Locally isothermal (now with energy equation)
- Self-gravity parametrized by  $Q_0 = Q(r_{\text{out}})$  or  $M_d$
- Giant planet: 1 or 2-Jupiter mass

$Q_0 = \infty$  verses  $Q_0 = 8$

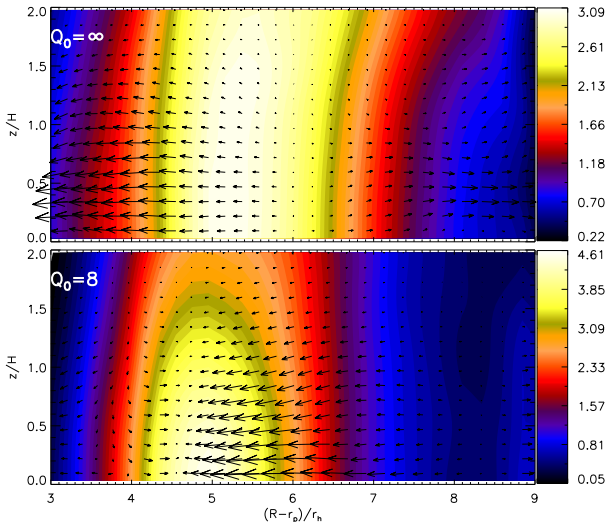


- Vortex formation, checked.



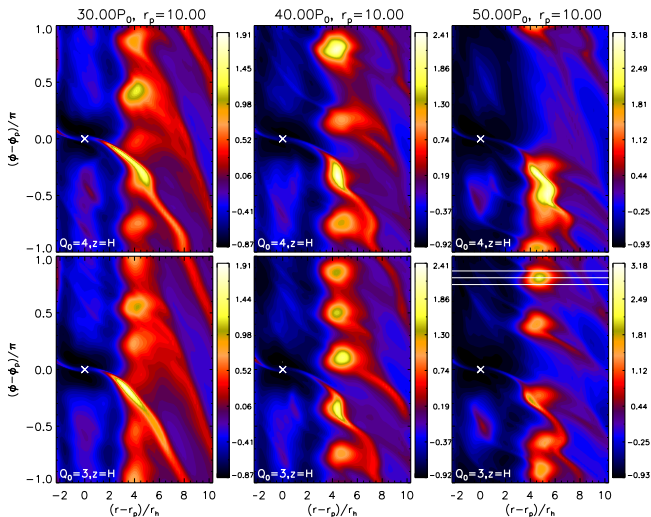
$Q_0 = \infty$  verses  $Q_0 = 8$

Vortex vertical structure (relative density perturbation)



- Initially  $Q \sim 10$  in this region  $\rightarrow$  but  $\Delta\rho/\rho$  is stratified

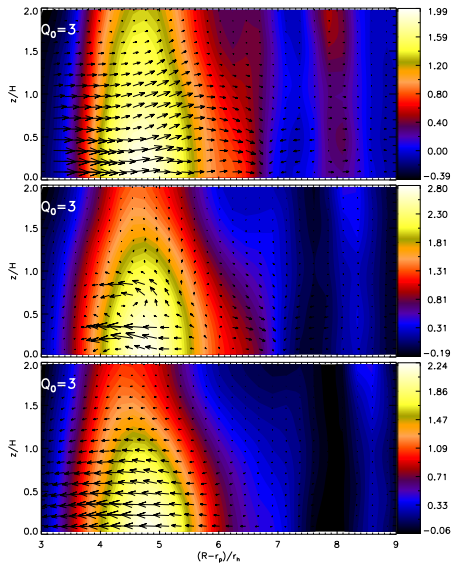
$$Q_0 = 4 \text{ and } Q_0 = 3$$



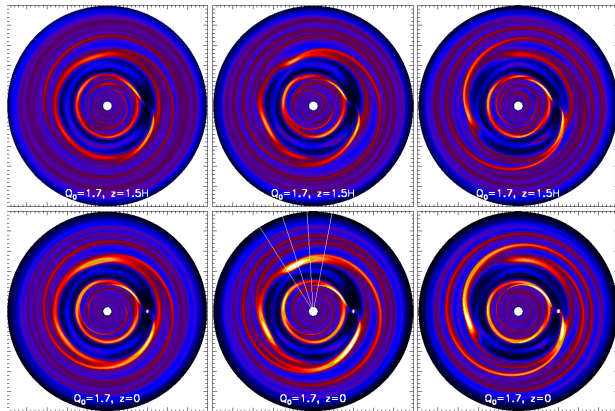
- More vortices and resisted merging, checked.
- Caution: boundary potential needs  $m >$  vortices

$$Q_0 = 3$$

- Unperturbed  $Q \sim 4$
- No merging yet
- Most stratified at vortex core
- Unable to identify 'typical' vertical flow pattern (cf. anti-cyclonic horizontal flow)

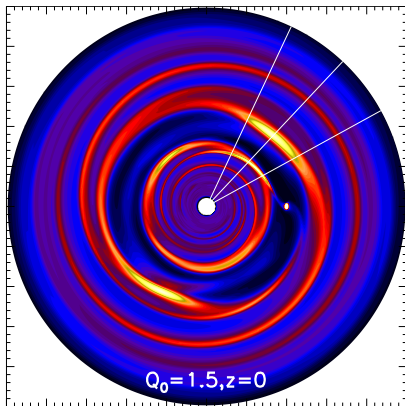


$$Q_0 = 1.7 \text{ and } Q_0 = 1.5$$



- Spirals in massive disks, checked.
- Sharp  $\max(Q)$  circumvents need for low  $Q$  locally ( $Q_{\text{edge}} \sim 10$ ), but need exterior disk to feel the edge disturbance
- They provide *positive co-orbital torques* (outward migration possible)

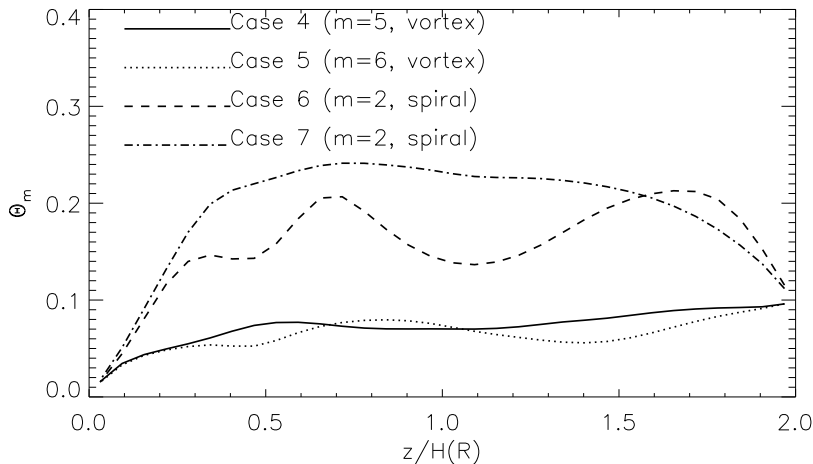
$$Q_0 = 1.7 \text{ and } Q_0 = 1.5$$



- Spirals in massive disks, checked.
- Sharp  $\max(Q)$  circumvents need for low  $Q$  locally ( $Q_{\text{edge}} \sim 10$ ), but need exterior disk to feel the edge disturbance
- They provide *positive co-orbital torques* (outward migration possible)

# Three-dimensionality

- Empirical measure:  $\sqrt{v_z^2/(v_R^2 + v_z^2)}$



- Approximately 2D disturbance
- Vertical Mach number  $\sim$  few per cent

# Discussion

- Confirmed that most 2D results persist in 3D, long term 3D evolution unknown (H. Li's talk)
- Strongest 3D effect: vertical self-gravity

Speculations:

- Vortex stability ('flattened' under its own weight)
- Reduction of vertical boundary effects: avoid complicated physics at upper and lower active layers  
(failed to find linear vortex modes with, e.g.  $\delta v_{\parallel} = 0$  or  $\delta \rho = 0$  at upper disk boundary)

## Back to linear stability

- Linear 3D adiabatic disturbances governed by PDE eigenvalue problem

$$U(W) = 0,$$

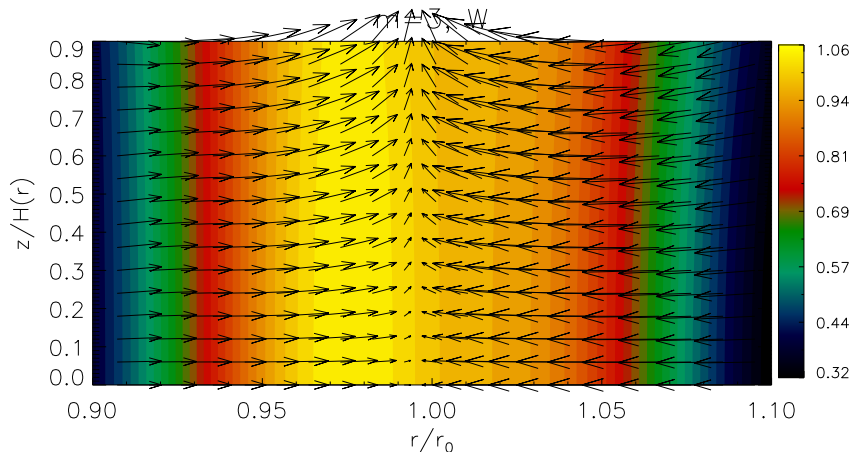
where  $W \equiv \delta p / \rho$ . (freedom to implement vertical b.c.)

- Equilibrium:  $\rho \propto \rho^\Gamma$  and  $\gamma \geq \Gamma \geq 1$ .
- Solve for nonhomentropic 3D thin disks with a density bump  $\rightarrow$  Rossby wave instability.
- Thick tori version (harder) done by Frank & Robertson (1988) and Kojima et al. (1989)  $\rightarrow$  clues.
- Difficult in general, but brute force works OK for RWI.



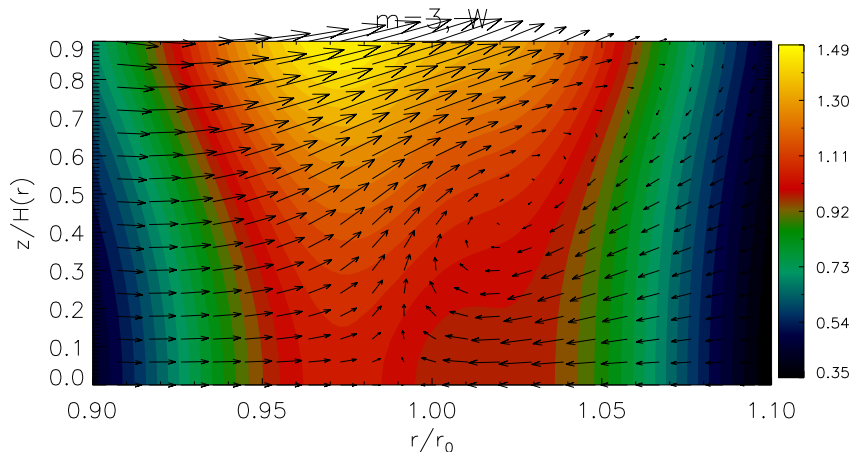
# Homentropic verses nonhomentropic

- $\gamma/\Gamma = 1$  (polytrope)



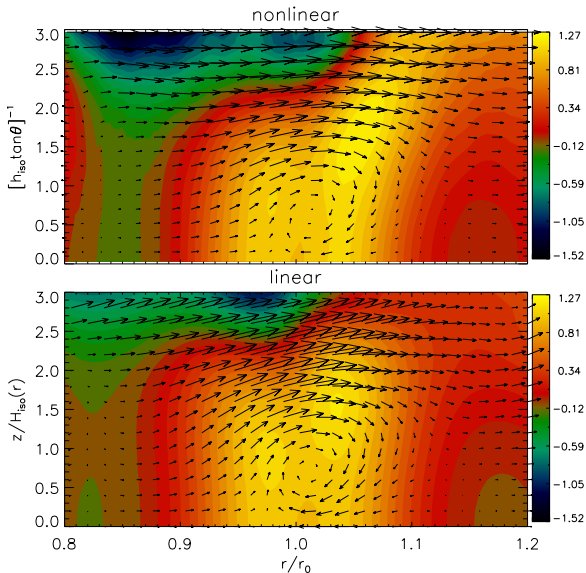
# Homentropic verses nonhomentropic

- $\gamma/\Gamma = 1.8$



# Linear versus nonlinear

- Strictly isothermal equilibria,  $\gamma = 1.4$



## More details

- *'Vortex and spiral instabilities at gap edges in three-dimensional self-gravitating disc-satellite simulations'*: Lin, M-K., 2012, MNRAS, in press, [astro-ph: 1205.4034](#)
- *'Effects of upper disc boundary conditions on the linear Rossby wave instability'*: Lin, M-K., 2012, MNRAS, accepted (Sept 17), [look out on astro-ph \(PDE solver demo\)](#)
- *'Non-barotropic linear Rossby wave instability in three-dimensional disks'*: Lin, M-K., 2012, ApJ, submitted, [astro-ph: 1209.0470](#)

## References

- Frank J., Robertson J. A., 1988, MNRAS, 232, 1
- Kojima Y., Miyama S. M., Kubotani H., 1989, MNRAS, 238, 753
- Koller J., Li H., Lin D. N. C., 2003, ApJL, 596, L91
- Li H., Li S., Koller J., Wendroff B. B., Liska R., Orban C. M., Liang E. P. T., Lin D. N. C., 2005, ApJ, 624, 1003
- Li H., Lubow S. H., Li S., Lin D. N. C., 2009, ApJL, 690, L52
- Lin M.-K., Papaloizou J. C. B., 2010, MNRAS, 405, 1473
- Lin M.-K., Papaloizou J. C. B., 2011a, MNRAS, 415, 1426
- Lin M.-K., Papaloizou J. C. B., 2011b, MNRAS, 415, 1445
- Lin M.-K., Papaloizou J. C. B., 2012, MNRAS, 421, 780
- Meschiari S., Laughlin G., 2008, ApJL, 679, L135
- Ou S., Ji J., Liu L., Peng X., 2007, ApJ, 667, 1220
- Yu C., Li H., 2009, ApJ, 702, 75