From the complex plane to planet formation

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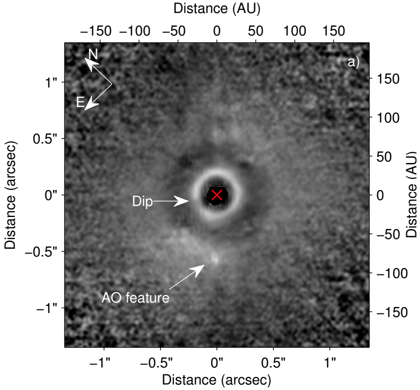
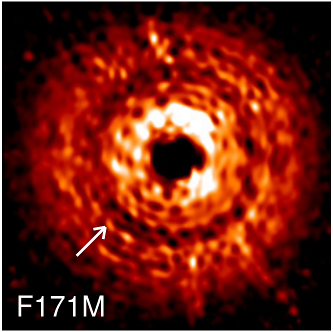
Research interests

- Astrophysical fluid dynamics of accretion/protoplanetary disks
- Disk-planet interactions, orbital migration
- Self-gravitating disks
- Disk instabilities
- Magneto-hydrodynamics (new)
- Non-linear numerical simulations
- Linear hydrodynamics

Today: instabilities and large-scale structures in astrophysical disks

Sub-structures in protoplanetary disks

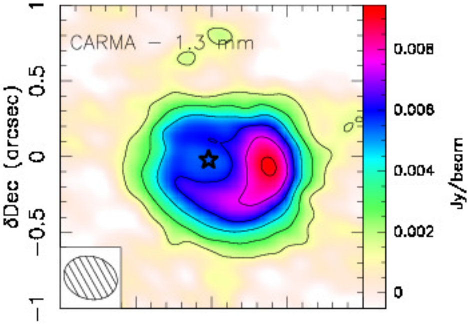




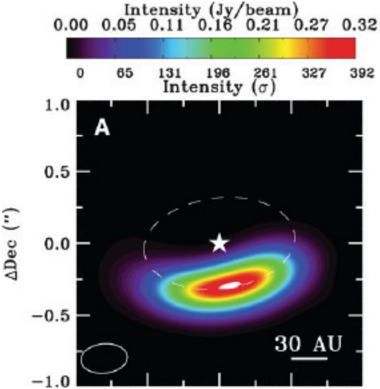
(TW Hya, Debes et al., 2013)

(HD 169142, Quanz et al., 2013)

Non-axisymmetric structures



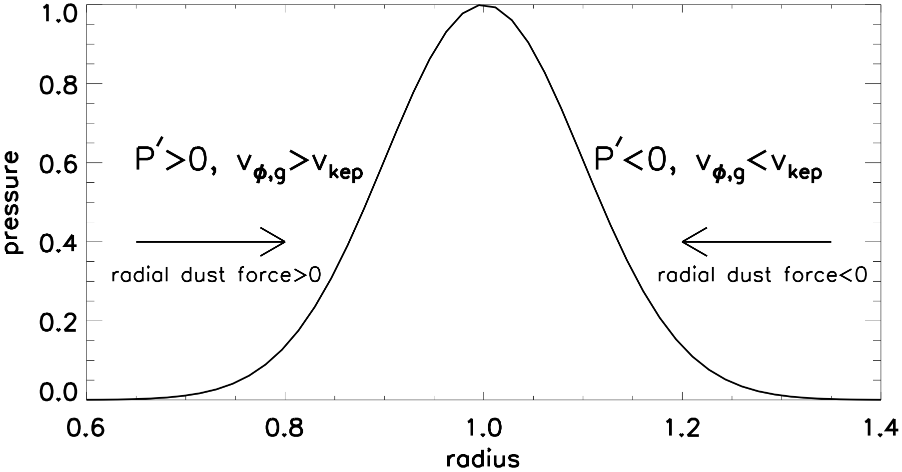
(LkHa 330, Isella et al., 2013)



(Oph IRS 48, van der Marel et al., 2013)

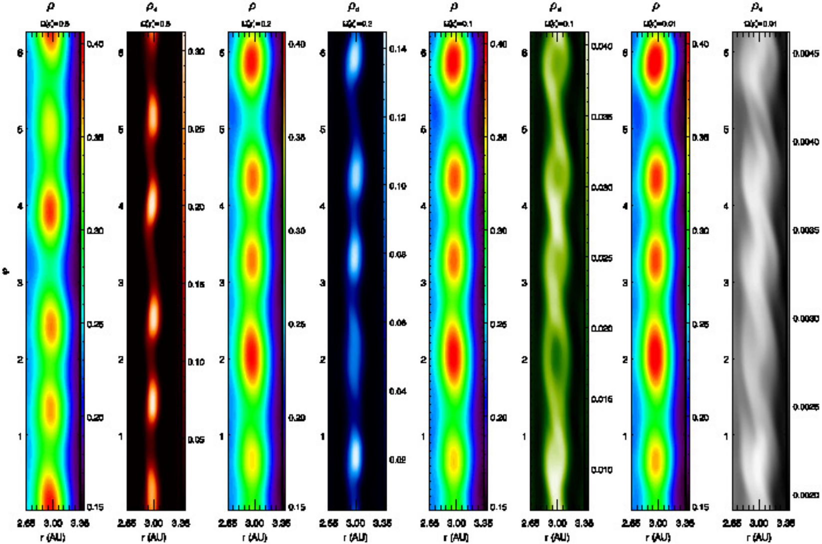
→ Dust-trapping

Dust trapping at pressure maxima

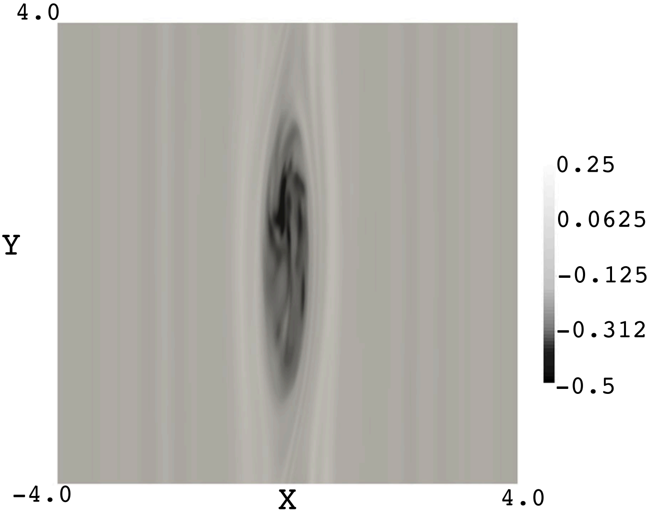


Asymmetric trapping by vortices

Meheut et al. (2012): add dust to disk with vortices



Vortex formation vs. destruction



(Lesur & Papaloizou, 2010)

Dust trapping vs. diffusion

Particle concentration vs. turbulent diffusion (Lyra & Lin, 2013)

$$\frac{\partial \rho_d}{\partial t} + \nabla \cdot (\rho_d \mathbf{v}_d) = D\nabla^2 \rho_d$$

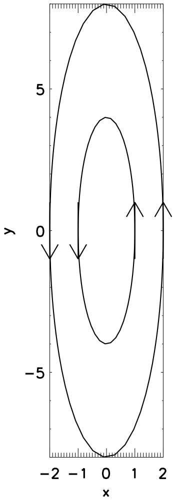
- D: from instability of vortex core
- $\mathbf{v}_d = \mathbf{v}_g + \tau c_s^2 \nabla \ln \rho_g$, isothermal gas
- $\mathbf{v}_g$ from model of an elliptic vortex (e.g. Kida vortex)
- τ friction time

Dust plus fluid equations $\rightarrow$ PDE for $\rho_d(x, y)$

Parameters:

$\delta = D/H^2\Omega$: dimensionless turbulence strength

$\mathrm{St} = \tau\Omega$: dimensionless friction (Stokes number)



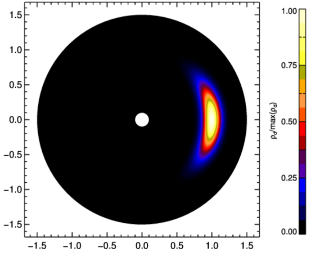
Steady-state dust distribution in elliptic vortices

$$\rho_d(a) \propto \exp\left(-\frac{a^2}{2H_v^2}\right),$$

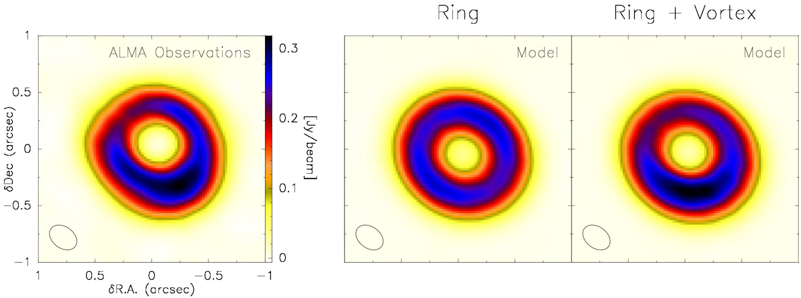
with

$$H_v(\chi, \delta, \mathrm{St}) = \frac{H_g}{f(\chi)}\sqrt{\frac{\delta}{\delta + \mathrm{St}}}.$$

Dust density averaged over an ellipse (semi-minor axis a, aspect-ratio χ)
EXACT solution for certain vortex models
(e.g. Kida vortex with $\chi = 7 \rightarrow$ no pressure gradient along ellipses)



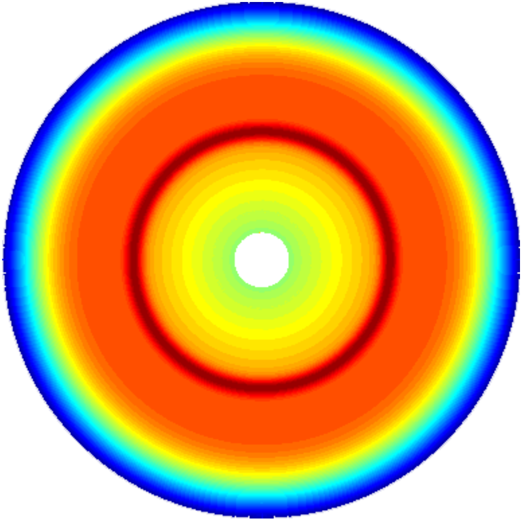
Application to observations



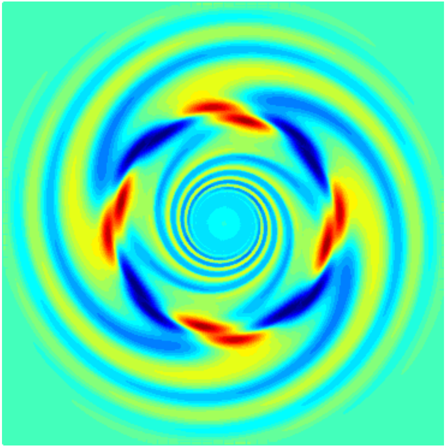
(SAO 206462, Pérez et al., 2014)
$\chi_{\rm obs} \sim 7$, model+ data $\rightarrow v_{\rm turb} \sim 0.22c_s$.

Vortex formation from the Rossby wave instability

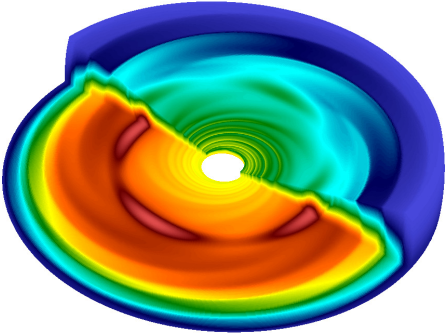
- Kelvin-Helmholtz instability in a rotating disk (Lovelace et al., 1999)
- Requires a radially-structured disk (e.g. planet gaps)



Numerical examples of the RWI



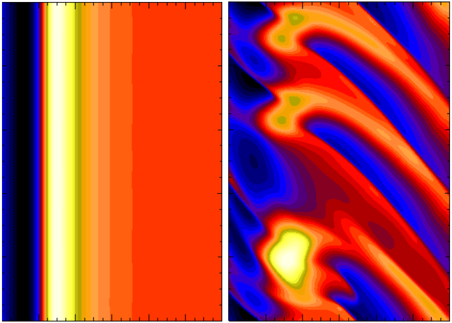
ATHENA code: 3D disk in a box



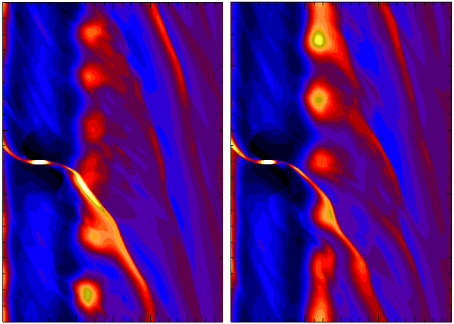
ZEUS code: 3D self-gravitating adiabatic disk, spherical grid

Numerical examples of the RWI

PLUTO code



3D disk with viscosity jump in radius



3D self-gravitating disk-planet simulation

[Note: global simulations plotted in a box ($r \to x$, $\phi \to y$)]

Step 1: linear problem

1. Steady, axisymmetric, vertically hydrostatic density bump at $r = r_0$
2. Perturb fluid equations, e.g. $\rho \to \rho + \delta\rho(r, z) \exp i(m\phi + \sigma t)$
3. Combine linear equations to get equation for $W \equiv \delta p/\rho$:

$$\boxed{L(r, z; \sigma) W = 0.}$$

- $W \to$ eigenfunction ; $\sigma \to$ eigenvalue

Co-rotation singularity where $\bar{\sigma} \equiv \sigma + m\Omega = 0$,

$$L = \cdots + \frac{1}{\bar{\sigma}} \frac{d}{dr} \left(\frac{\Sigma\Omega}{\kappa^2} \right).$$

RWI: $\boxed{\mathrm{Re}[\bar{\sigma}(r_0)] \simeq 0 \textit{ and } \left. \frac{d}{dr} \left(\frac{\Sigma\Omega}{\kappa^2} \right) \right|_{r_0} \simeq 0}$

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Pole in complex plane $\to$ linear instability $\to$ vortices $\to$ dust-trapping $\to$ planetesimal formation $\to$ planets

Example: RWI in 3D polytropic disk ($p \propto \rho^{1+1/n}$)

Convert to 1D problem,

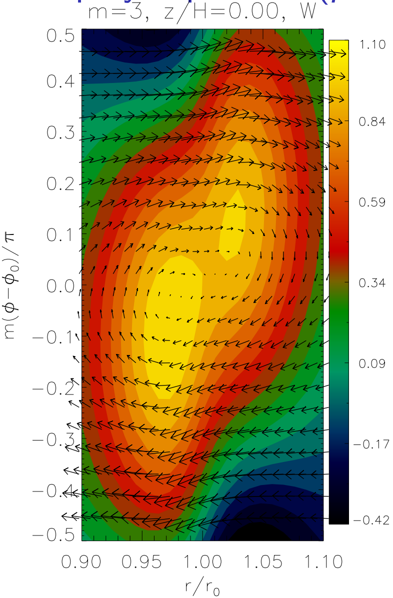
$$W(r,z) = \sum_{l=0}^{\infty} W_l(r)\mathcal{C}_l^{\lambda}(z/H),$$

where $\mathcal{C}_l^{\lambda}(x)$ are Gegenbauer polynomials. (Generalized Legendre/Chebyshev.)

$$L(r,z;\sigma)W = 0 \to A_l(W_l) + B_l(W_{l-2}) + C_l(W_{l+2}) = 0.$$

- Technical but neat
- No freedom in upper disk boundary conditions

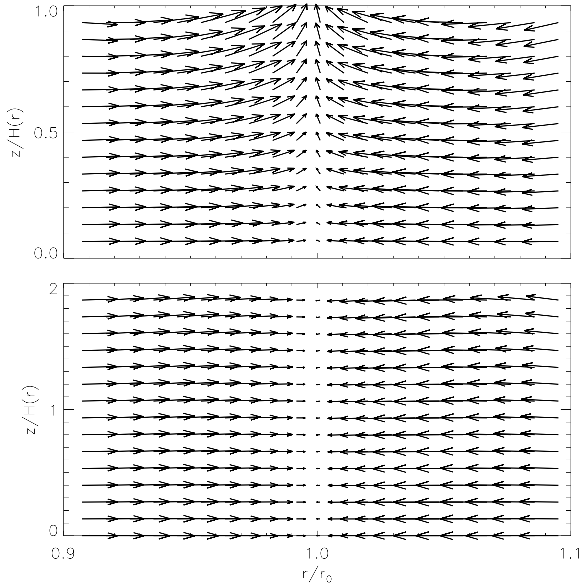
Example: RWI in 3D polytropic disk ($p \propto \rho^{1+1/n}$)



Anti-cyclonic motion associated with over-density.

Example: RWI in 3D polytropic disk ($p \propto \rho^{1+1/n}$)

Magnitude of vertical motion decreases with increasing n (more compressible)

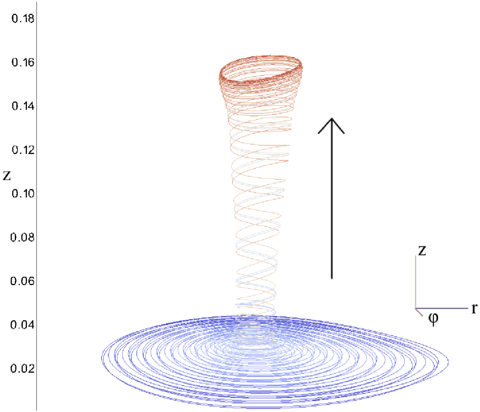


← $n = 1.0$ polytrope

← vertically isothermal disk ($n = \infty$, special treatment with Hermite polynomials)

Comparison to non-linear simulations

Upward motion seen in non-linear hydrodynamic simulations of Meheut et al. (2012):



Meheut et al. (2012) → mm dust lifted to disk surface

Extension to adiabatic 3D disks

- $p \propto \rho^{\Gamma}$ in basic state only
- Energy equation $Ds/Dt = 0$, $s \equiv p/\rho^{\gamma} \propto \rho^{\Gamma-\gamma}$
- $\gamma \geq \Gamma \geq 1$, density bump $\rightarrow$ entropy dip

$$V_1 W + \overline{V}_1 Q = 0$$
$$V_2 W + \overline{V}_2 Q = 0$$

- $W = \delta p/\rho \rightarrow$ pressure perturbation
- $Q = c_s^2 \delta\rho/\rho \rightarrow$ density perturbation

Finite-difference/pseudo-spectral method:

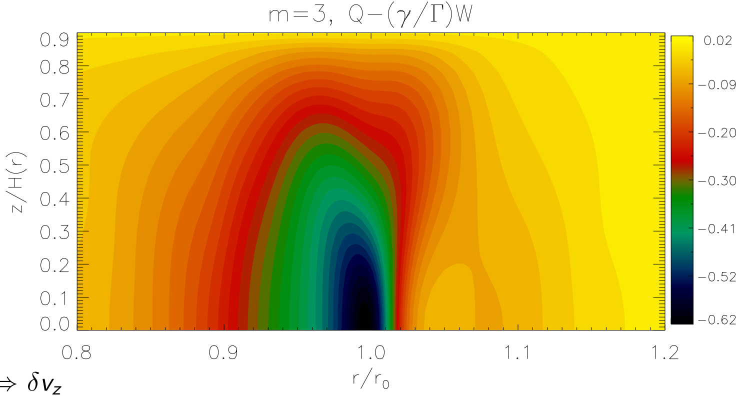
$$W(r_i, z) \equiv W_i(Z) = \sum_{k=1}^{N_z} w_{ki} \psi_k (Z/Z_{\max})$$

[$\psi_k = T_{2(k-1)}$ are Chebyshev polynomials]

Baroclinity/buoyancy, $\nabla p \times \nabla \rho \neq 0$

$$\overline{S} \equiv Q - \frac{\gamma}{\Gamma} W$$

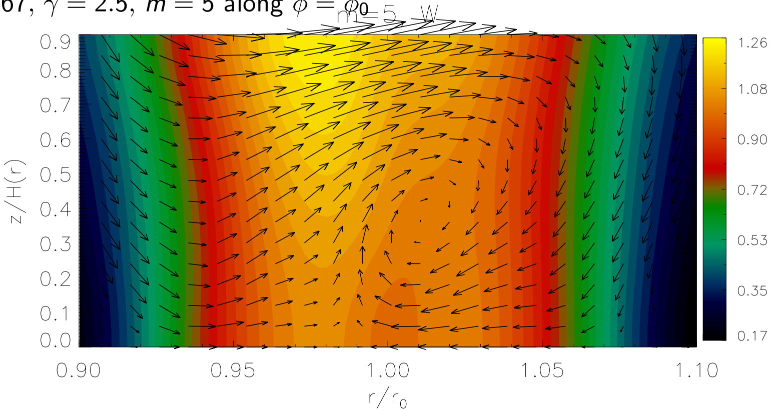
$\rightarrow$ a measure of baroclinity ($= 0$ if $\Gamma = \gamma$)



- $\bar{S} \Rightarrow \delta v_z$
- $\nabla \bar{S} \Rightarrow (\nabla \times \boldsymbol{\delta v})_\phi$

Example: meridional vertical motion in adiabatic disks

$\Gamma = 1.67$, $\gamma = 2.5$, $m = 5$ along $\phi = \phi_0$



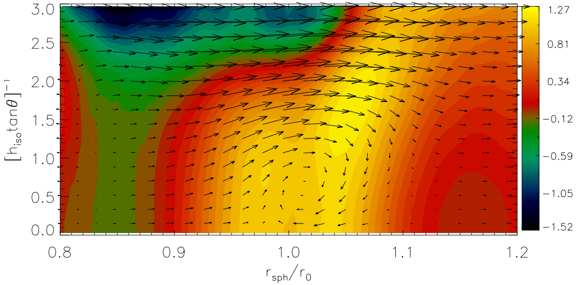
Buoyancy > pressure at large heights, since

$$\frac{\text{pressure}}{\text{buoyancy}} \sim \frac{\Omega^2}{N_z^2}\frac{\partial \ln W}{\partial \ln z},$$

and vertical frequency $N_z \to \infty$ as $z \to H$. 'Swirling' because of entropy pert.

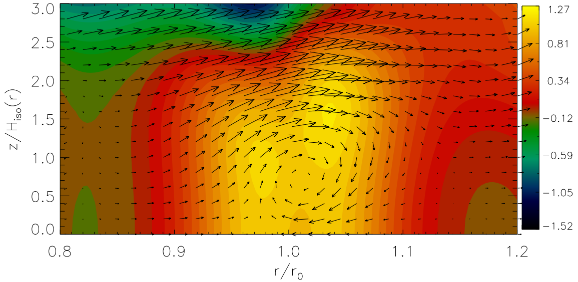
Comparison with hydrodynamic simulations

- Isothermal disk, adiabatic evolution ($\Gamma \equiv 1$, $\gamma = 1.4$)



← ZEUS simulation

$$\mathrm{Re}(\sigma) = -0.99 m\Omega_0$$
$$\mathrm{Im}(\sigma) = -0.194\Omega_0$$



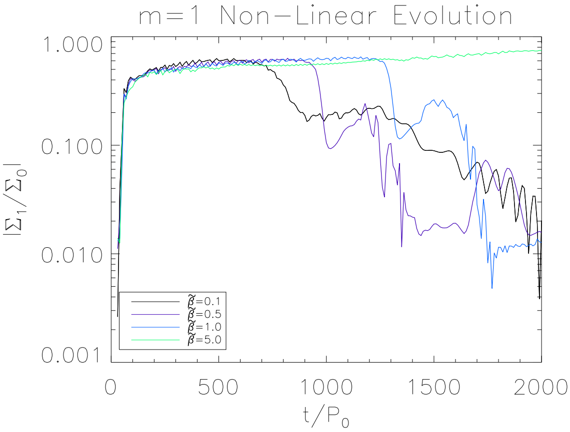
← linear code

$$\mathrm{Re}(\sigma) = -0.9896 m\Omega_0$$
$$\mathrm{Im}(\sigma) = -0.1937\Omega_0$$

Gap formation and stability in non-isothermal disks

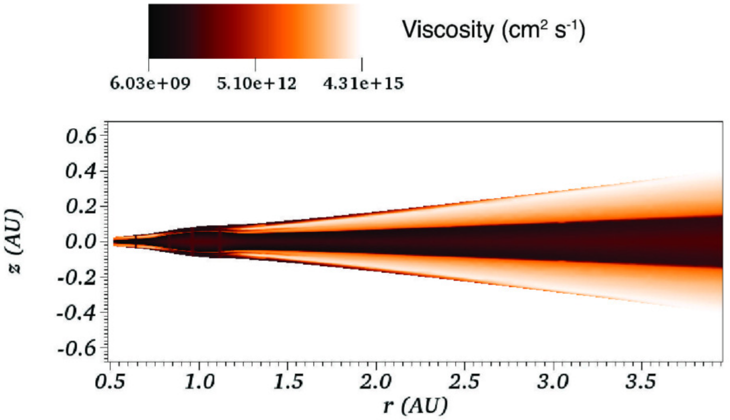
[2014 CITA Summer student program]

- FARGO 2D, disk-planet
- $t_{\mathrm{cool}} = \tilde{\beta} \times t_{\mathrm{libration}}$



Very long cooling actually gives *shorter* vortex lifetime (R. Les & Lin, in prep.)

Vortex-formation in layered-accretion disks?



(Axisymmetric model from Landry et al., 2013)

- RWI requires low viscosity, but only have dead zone near midplane
- Rossby vortices have weak vertical structure (vorticity columns)

Linear RWI in layered disks

First task for any linear problem: equilibrium state. But:

- Need localized radial gradient for RWI
- Want a viscous atmosphere

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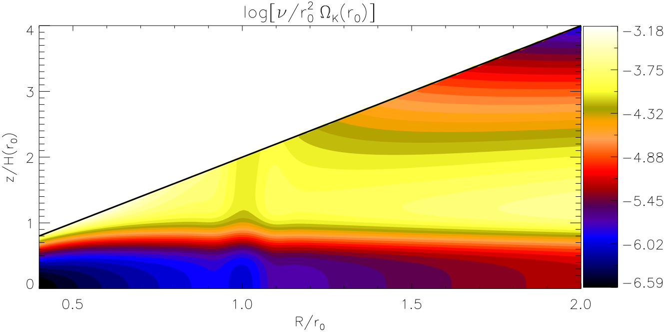


(Lin, 2014b)

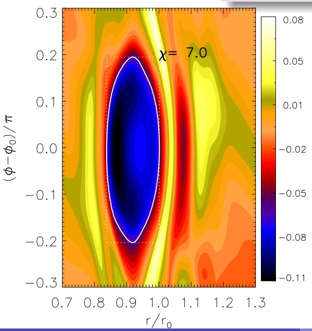
- Choose viscosity and v_R s.t. $R\rho v_R = \dot{M}(z)$
- Strictly isothermal gas

PLUTO simulations of layered disks

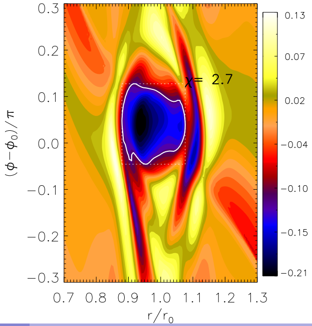
Spherical grid, $z \in [0, 2H]$ at $R = r_0$.
ν decreases by 10^2 from active (upper) to dead (lower) layer.

- Case 1: all dead, linear growth rate $= 0.199\Omega$
  ($\alpha \sim 10^{-4}$)
- Case 2: half dead, linear growth rate $= 0.182\Omega$
  ($\alpha \sim 10^{-4}$ for $z \in [0, H]$; $\alpha \sim 10^{-2}$ for $z \in [H, 2H]$)

Local viscous time $H^2/\nu \gg t_{\mathrm{RWI}}$

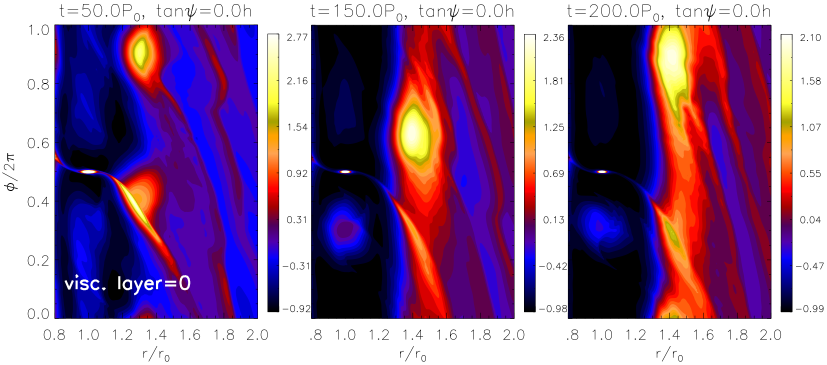


←Rossby numbers→
← Case 1
Case 2 →



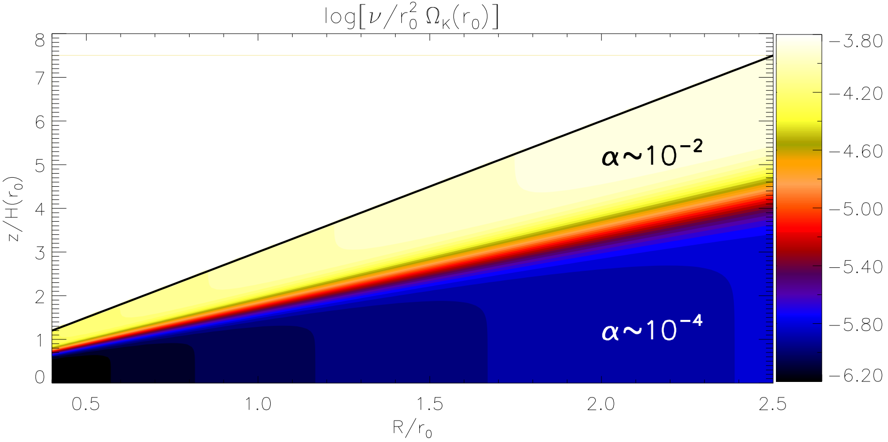
Disk-planet interaction in layered disks

Standard result for Jupiter-mass planet in a low viscosity unlayered disk ($\alpha \sim 10^{-4}$)



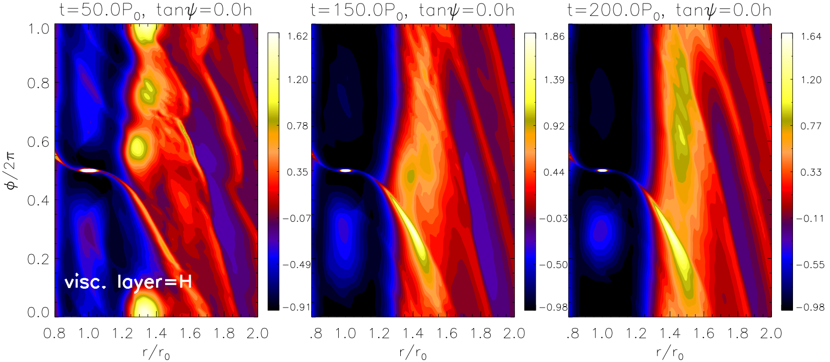
Disk-planet interaction in layered disks

Repeat simulation with layered viscosity



Disk-planet interaction in layered disks

Rossby vortex does not survive against viscous layer

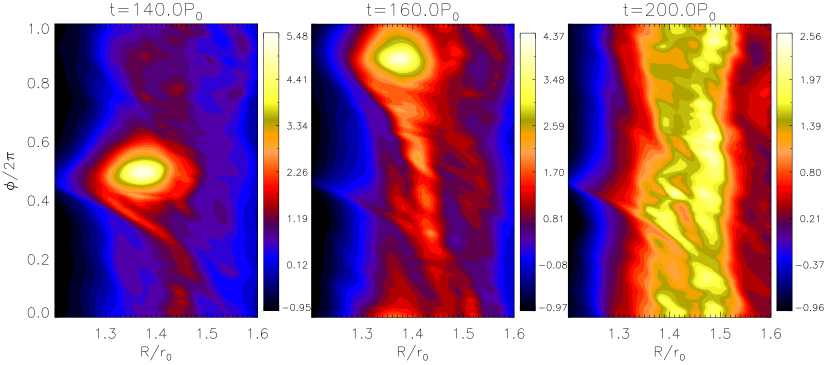


Vertical domain size: $z \in [0, 3H]$, viscous layer $z \in [2, 3H]$, $\Sigma_{\rm visc}/\Sigma \sim 0.04$

- Lesson: long term vortex formation sensitive to disk vertical structure
- Next step: back-reaction on α

Disk-planet interaction in layered disks

Restart a low-viscosity simulation with a viscous atmosphere



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- Lesson: long term vortex formation sensitive to disk vertical structure
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Magnetized massive astrophysical disks

Principle routes to turbulent angular momentum transport in rotating disks:

Magneto-rotational instability

and

Gravitational instability

May have MRI and GI in:

- Early evolution of protoplanetary disks
- Layered accretion in PPDs
- Black hole accretion disks
- Galactic disks

Latest work in PPDs: Fromang et al. (2004) using full simulations

Step 1: linear stability (Lin, 2014a)

Shearing box resistive MHD plus Poisson, linearize $\rightarrow$

$$\frac{\mathrm{i}\sigma}{c_s^2}W + \mathrm{i}k_x\delta v_x + (\ln\rho)'\,\delta v_z + \delta v_z' = 0,$$

$$\mathrm{i}\sigma\delta v_x - 2\Omega\delta v_y = -\mathrm{i}k_x\tilde{W} + \frac{B_z}{\mu_0\rho}\left[\delta B_x' - \mathrm{i}k_x\left(\delta B_z + \epsilon\delta B_y\right)\right],$$

$$\mathrm{i}\sigma\delta v_y + \frac{\kappa^2}{2\Omega}\delta v_x = \frac{B_z}{\mu_0\rho}\delta B_y',$$

$$\mathrm{i}\sigma\delta v_z = -\tilde{W}' - \frac{B_y}{\mu_0\rho}\delta B_y',$$

$$\mathrm{i}\bar{\sigma}\delta B_x = B_z\delta v_x' + \eta\delta B_x'' + \eta'\delta B_x' - \mathrm{i}k_x\eta'\delta B_z,$$

$$\mathrm{i}\bar{\sigma}\delta B_y = B_z\delta v_y' - B_y\Delta - S\delta B_x + \eta\delta B_y'' + \eta'\delta B_y',$$

$$\mathrm{i}\bar{\sigma}\delta B_z = -\mathrm{i}k_xB_z\delta v_x + \eta\delta B_z'',$$

$$\delta\Phi'' - k_x^2\delta\Phi = \frac{\rho}{c_s^2 Q}W.$$

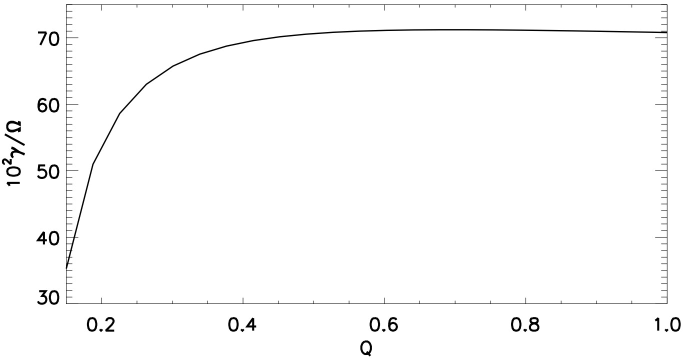
Pseudo-spectral method reduces numerical complexity

Effect of SG on the MRI through the background

Rule of thumb for MRI:

$$\boxed{\lambda_{\mathrm{MRI}} \lesssim 2H}.$$

$\lambda_{\mathrm{MRI}} \sim \frac{v_A}{\Omega} = \frac{B_z}{\sqrt{\mu_0 \rho}}\frac{1}{\Omega}$, $H = H(Q)$ decreases with increasing SG
So, for fixed $\beta \equiv c_s^2/v_A^2$,



Note: $Q = 0.5 \longleftrightarrow Q_{\mathrm{2D}} = 1.5$.

Limiting field strength

Example: polytropic disk with $p \propto \rho^2$, then $\lambda_{\mathrm{MRI}} \lesssim 2H \rightarrow$

$$\beta^{-1/2} \lesssim \frac{\sqrt{15}}{4\pi}\sqrt{Q}\,\mathrm{arccos}\left(\frac{Q}{1+Q}\right).$$

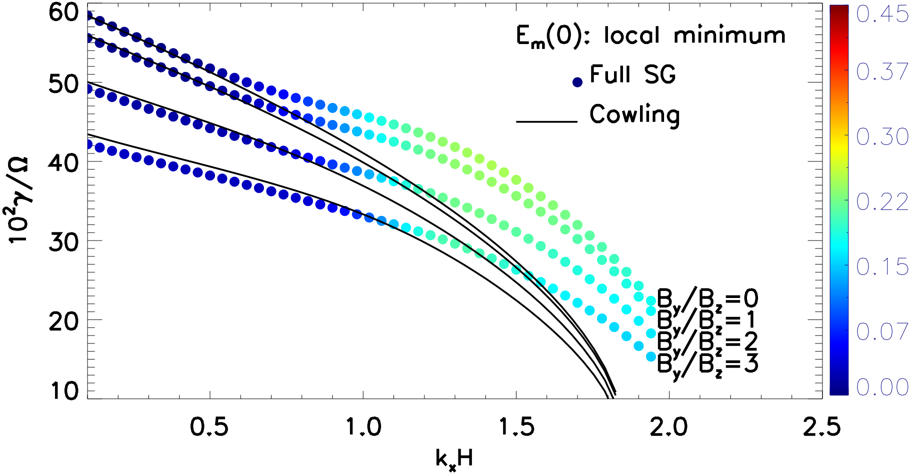
For $Q \ll 1$,

$$\frac{B_z}{c_{s0}\Omega}\sqrt{\frac{\pi G}{\mu_0}} \lesssim \frac{\sqrt{15}}{16}.$$

Both $v_A, \lambda_{\mathrm{MRI}} \rightarrow 0$ and $H \rightarrow 0$ as $\rho(0) \rightarrow \infty$.

Effect of SG on the MRI through the perturbation

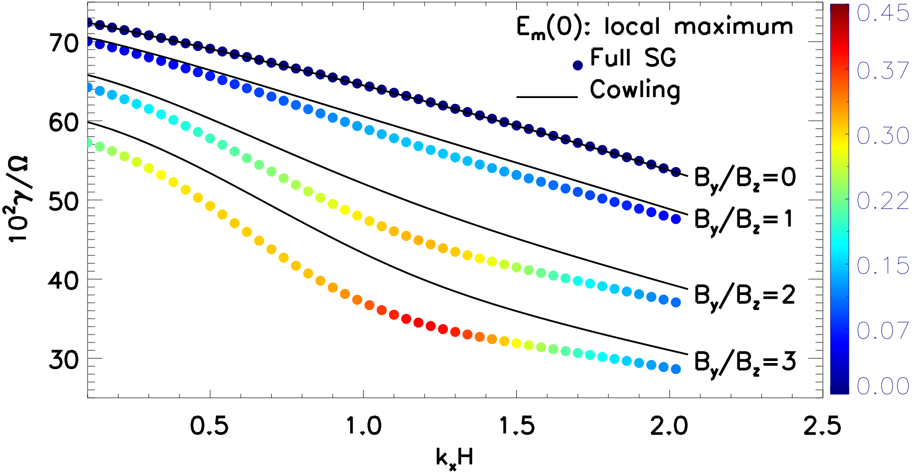
Modes with perturbed magentic energy *minimized* at $z = 0$



$\delta\rho'(0) = 0$ for $B_y = 0$
Colourbar: $E_{\mathrm{grav}}/\left(E_{\mathrm{grav}} + E_{\mathrm{mag}}\right)$

Effect of SG on the MRI through the perturbation

Modes with perturbed magentic energy *maximized* at $z = 0$



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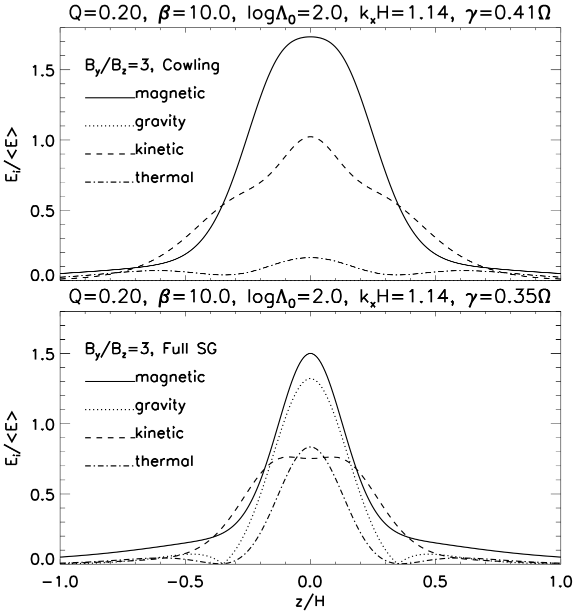
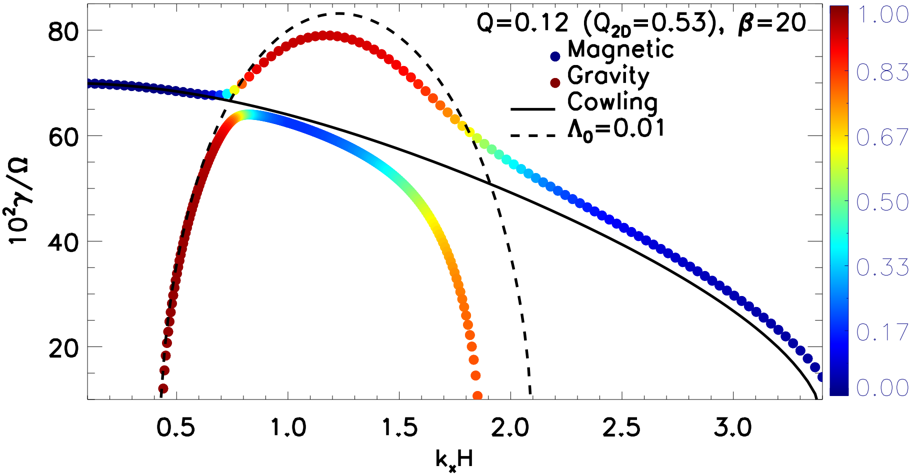


Co-existence of MRI and GI



- ‘Avoided crossing’ of modes
- MRI-GI interaction only possible with same parity

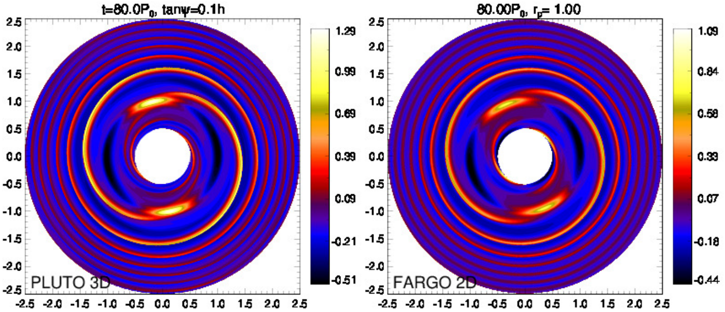
Future

Linear

Non-axisymmetric modes in magnetized self-gravitating disks

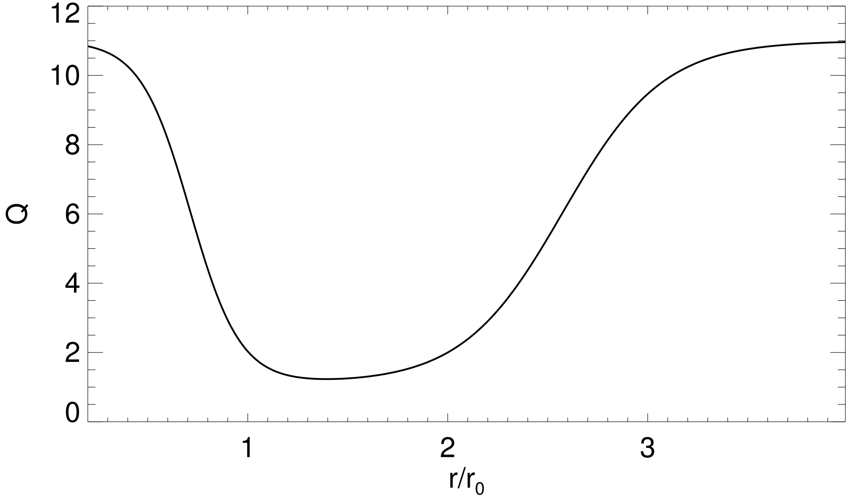
- 2D global, 3D local
- Numerically technical (integro-differential equation eigenvalue problem)
- Effect of magnetic field/MRI on angular momentum transport by gravity, global structures

3D edge instabilities with self-gravity



Future

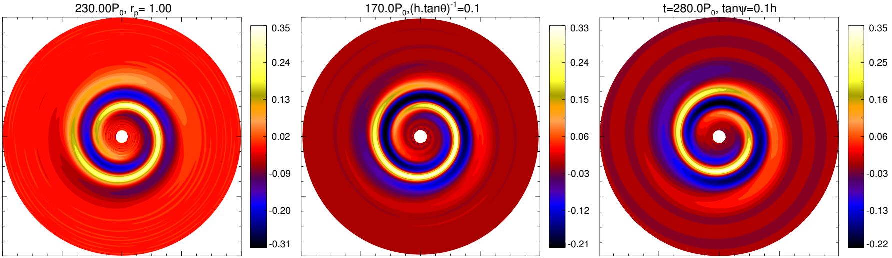
Non-linear



Future

Non-linear

Eccentric modes in radially-structured disks (?)



FARGO 2D ZEUS 3D PLUTO 3D

- Finite difference
- Poisson: 2D FFT

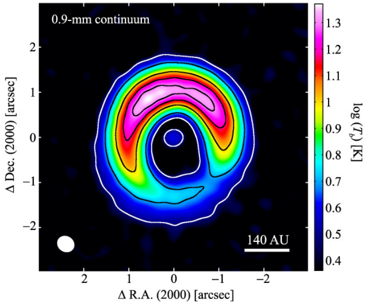
- Finite difference
- Poisson: linear solver

- Godunov
- Poisson: spherical harmonic expansion

Future

Non-linear

- Observational relevance: HD 142527



Estimated $Q \sim 1$—2 (Fukagawa et al., 2013; Christiaens et al., 2014)

- Theoretical relevance: episodic accretion
  GI in dead zone $\rightarrow$ MRI, but what kind of GI? Turbulence or large-scale waves?

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