The stability of self-gravitating gaps

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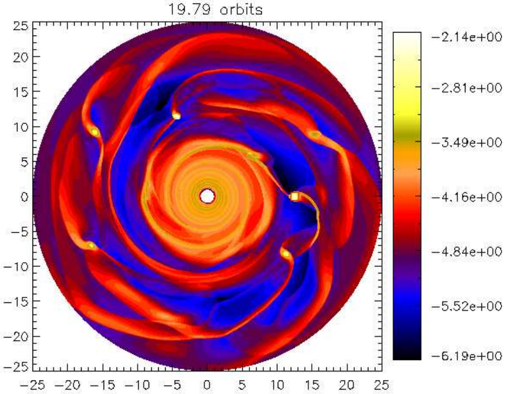
Outline

- Introduction
- Numerical simulations
- Vortices & spirals
- Disc-planet interactions
- Conclusions & future work

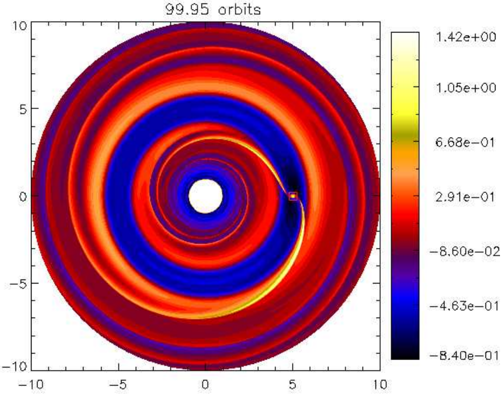
Disc stability

- Discs are ubiquitous in astrophysics.
- Instability is important: accretion, star formation, direct or indirect planet formation.
- Unstable because: magnetic fields, thermodynamics, **disc structure, self-gravity**.



Gaps in protoplanetary discs

- 490+ exo-planets discovered (October 2010).
- Planets form in discs. Sufficiently massive planets opens a gap (Papaloizou & Lin, 1984; Lin & Papaloizou, 1986).
- Dynamical instability at gap edges because of steep gradients.



Self-gravity

- Total gravitational potential: star, planet, disc.
- Usually ignore disc potential because $M_d \sim 0.01 M_*$.
- Massive discs needed for giant planet formation via GI, type III migration.
- How does SG affect gap stability?

Consider a series of disc-planet simulations with increasing M_d:

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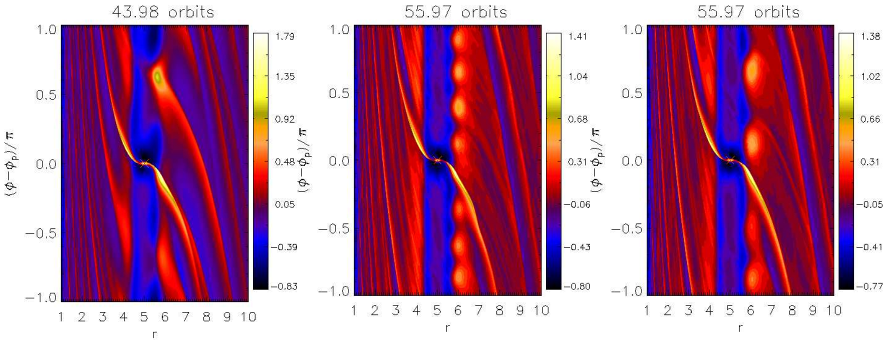
Consider a series of disc-planet simulations with increasing M_d:



$M_d = 0.063 M_*$ $\quad$ $M_d = 0.032 M_*$ $\quad$ $M_d = 0.011 M_*$

Model equations

2D disc in polar co-ordinates centered on primary but non-rotating.

- Hydrodynamic equations with local isothermal equation of state:

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\mathbf{u}\Sigma) = 0,$$
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\Sigma}\nabla P - \nabla \Phi + \frac{\mathbf{f}}{\Sigma}.$$

- Viscous forces $f \propto \nu$, pressure $P = c_s^2 \Sigma$ with $c_s^2 = h^2 GM_*/r$ and potential Φ include star potential, indirect potentials and disc potential Φ_d:

$$\Phi_d = -\int \frac{G\Sigma(r', \phi')}{\sqrt{r^2 + r'^2 - 2rr' \cos(\phi - \phi') + \epsilon_g^2}} r' dr' d\phi'.$$

Linearised equations

- Ignore viscosity, ignore planet & indirect potentials. Set $P = P(\Sigma)$ here.
- Perturb the system, e.g. $\Sigma \rightarrow \Sigma + \delta\Sigma(r)\exp i(\sigma t + m\phi)$.
- Linearise to get ODE for $S = c_s^2\delta\Sigma/\Sigma + \delta\Phi$

$$L(S) = \delta\Sigma$$
$$\delta\Phi = -G\int K_m(r,\xi)\delta\Sigma(\xi)\xi d\xi.$$

- L is a linear operator

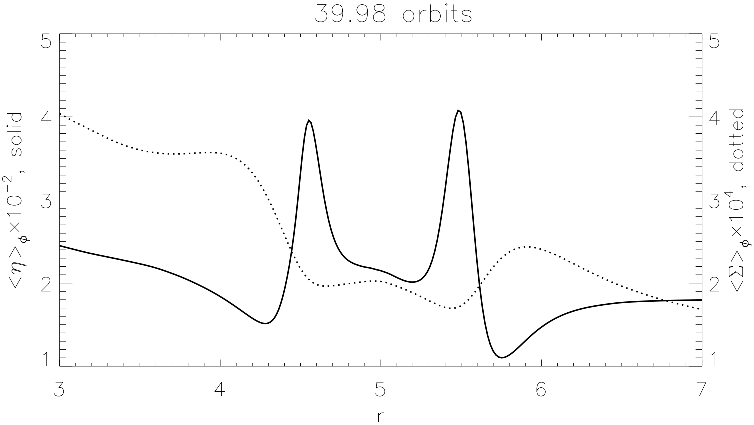
$$L(S) = \frac{mS}{r\bar{\sigma}(1-\bar{\nu}^2)}\frac{d}{dr}\left(\frac{1}{\eta}\right) + \cdots ,$$

$\bar{\sigma} = \sigma + m\Omega$, $\bar{\nu} = \bar{\sigma}/\kappa$ and

$\eta = \kappa^2/2\Omega\Sigma$ is the *vortensity*.

- When $\bar{\sigma}(r_c) = 0$, need $d\eta/dr = 0$ there.

Vortensity profile of gaps



- Disturbances with co-rotation radius near vortensity extrema, $\eta'(r_c) \simeq 0$.
- Consider $\int rS^* L(S)dr = \int rS^* \delta\Sigma dr = \text{energy}.$
- On LHS, only keep vortensity gradient term.

Vortensity profile of gaps

$$\int \frac{m|S|^2}{\bar{\sigma}(1-\bar{\nu}^2)}\frac{d}{dr}\left(\frac{1}{\eta}\right)dr \sim \int c_s^2\frac{|\delta\Sigma|^2}{\Sigma}dr - G\int\int r\xi K_m(r,\xi)\delta\Sigma^*(r)\delta\Sigma(\xi)drd\xi$$

- Assume LHS integral has most contribution near $\bar{\sigma}(r_c) \simeq 0$, then
- Sign of LHS depends on sign of $(1/\eta)''/\Omega'$ at r_c. Note $\Omega' < 0$.
- Insignificant SG: RHS > 0 so r_c is $\min(\eta) \rightarrow$ VORTICES.
- Significant SG: RHS < 0 so r_c is $\max(\eta) \rightarrow$ SPIRALS.

Note: Toomre $Q = \kappa c_s/\pi G\Sigma$. $\max(Q)$ coincides with $\max(\eta)$.

A nice result

Theorem

The perturbative effect of self-gravity through the linear response, $\delta\Phi$, is to stabilise vortex modes and de-stablise spiral modes.

Proof.

- Consider marginally stable mode with $\sigma = -m\Omega(r_c)$. $\eta'(r_c) = 0$.
- Change self-gravity via $G \rightarrow G + \delta G$.
- Perturb eigensolution: $\sigma \rightarrow \sigma + \delta\sigma$ with $\delta\sigma = \delta\sigma_R + i\gamma$; $S \rightarrow S + \delta S$; $\delta\Sigma \rightarrow \delta\Sigma + \delta\Sigma_1$. γ is assumed small negative (unstable).

Can show:

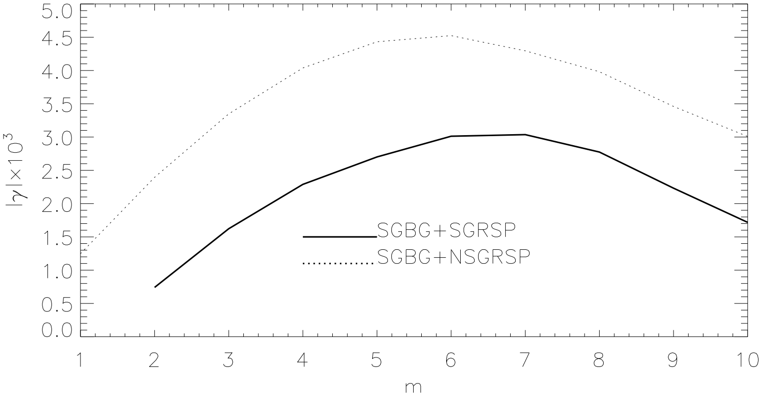
$$\boxed{\gamma = \beta \left.\frac{d^2\eta}{dr^2}\right|_{r_c} \times \delta G,}$$

with $\beta > 0$ for Ω decreasing outwards. Vortex modes have $\eta''(r_c) > 0$. Need $\delta G < 0$ to de-stabilise them, i.e. increasing SG stabilises them. $\square$

Stabilisation of vortex modes

- Solve the linear problem numerically, with local isothermal equation of state.
- Also solved with $\delta\Phi = 0$.

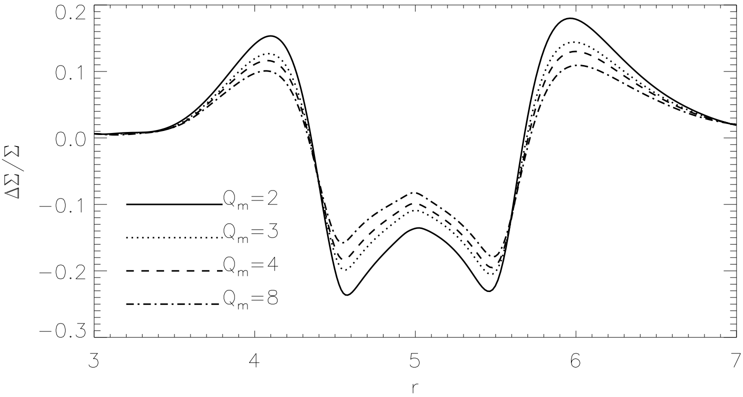
Growth rate $|\gamma|$ as a function of azimuthal wave-number m:



Solid: with SG in response. Dotted: no SG in response.

Effect of SG via the background

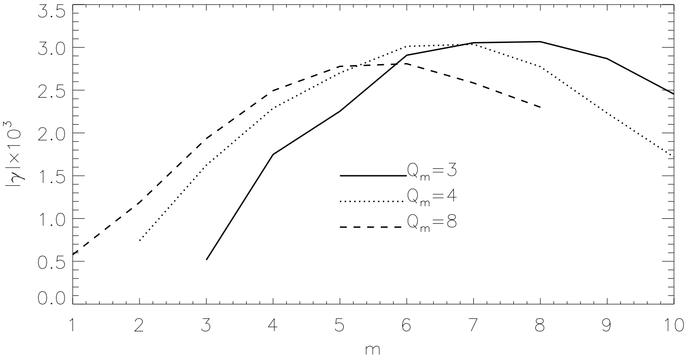
Altering SG affect the gap profile set up by simulation:



$Q_m \propto 1/M_d$. Deeper gaps with increasing SG → more unstable. Effect *diminishes* with lowering m.

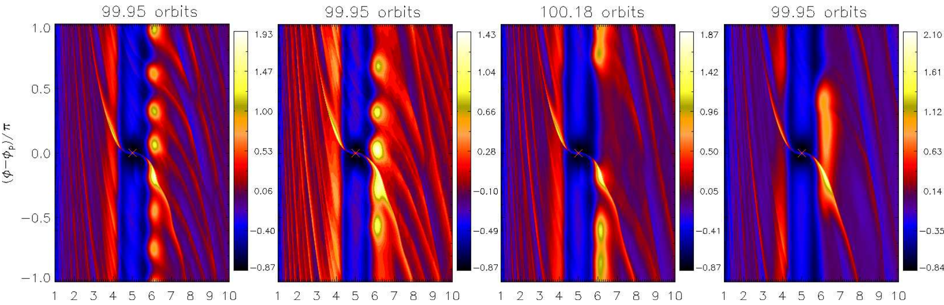
Getting more vortices

Now include SG all the way in linear problem. Recall $Q_m \propto 1/M_d$.



- Higher m preferred with increasing SG. Get more vortices.
- Loss of low m (stabilisation by SG in response).
- Enable higher m (effect of SG via basic state).

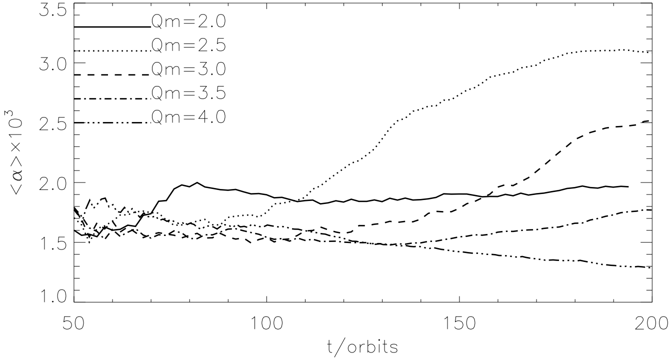
Effect of SG on vortex evolution



$M_d = 0.032M_*$ $\quad$ $M_d = 0.027M_*$ $\quad$ $M_d = 0.024M_*$ $\quad$ $M_d = 0.012M_*$

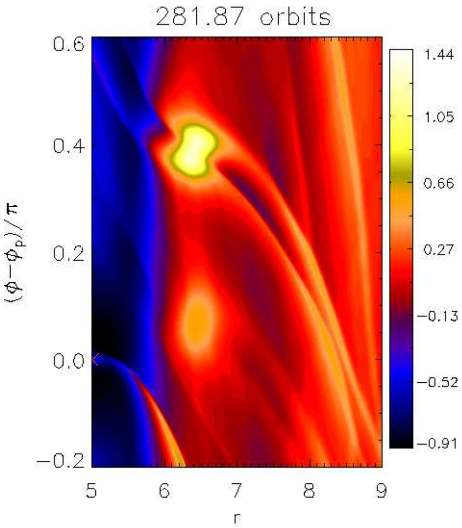
- Multiple-vortices configuration is sustained longer with increasing SG.

Effect of SG on vortex evolution

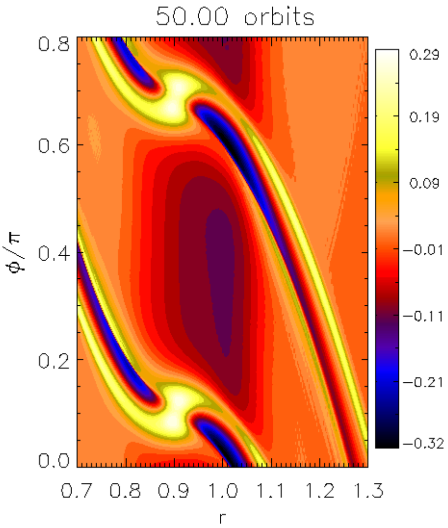


- Multiple-vortices configuration is sustained longer with increasing SG.
- Get α growth for intermediate range of disc mass.

Vortices as co-orbital planets

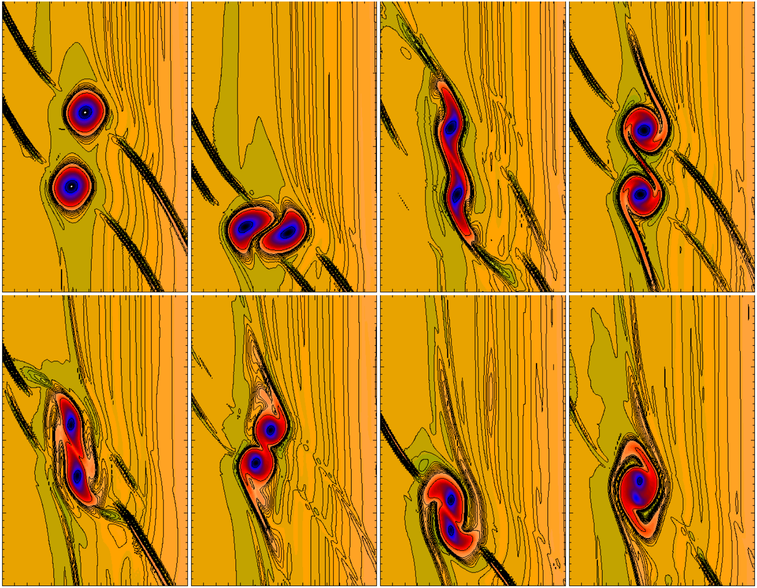


Gap edge vortex-pair

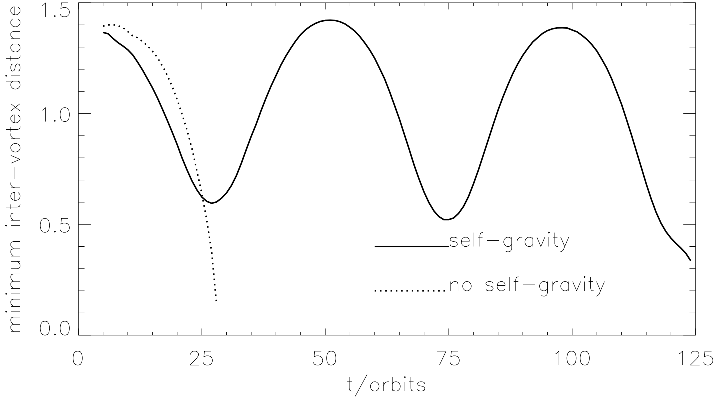


Kida vortex pair

Vortices as co-orbital planets



Vortices as co-orbital planets

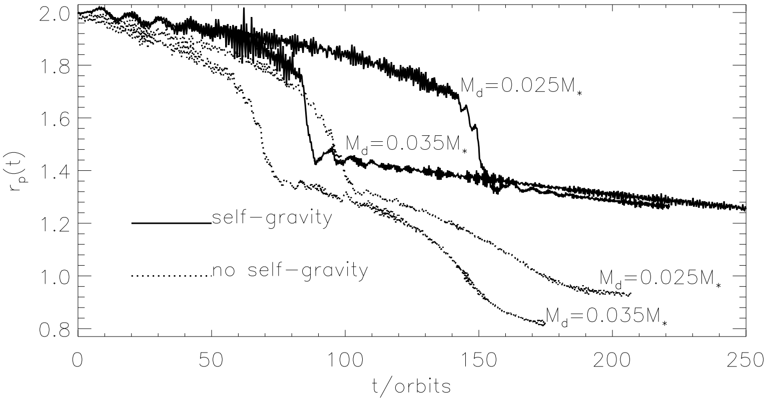


SG on/off

- SG imply minimum inter-vortex distance. If still larger than critical → no merging.

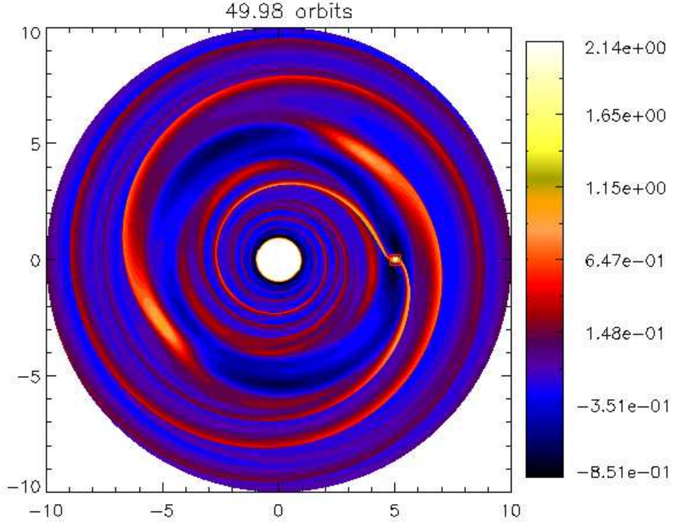
Application to vortex-induced migration

Repeat Lin & Papaloizou (2010)'s simulations with SG.



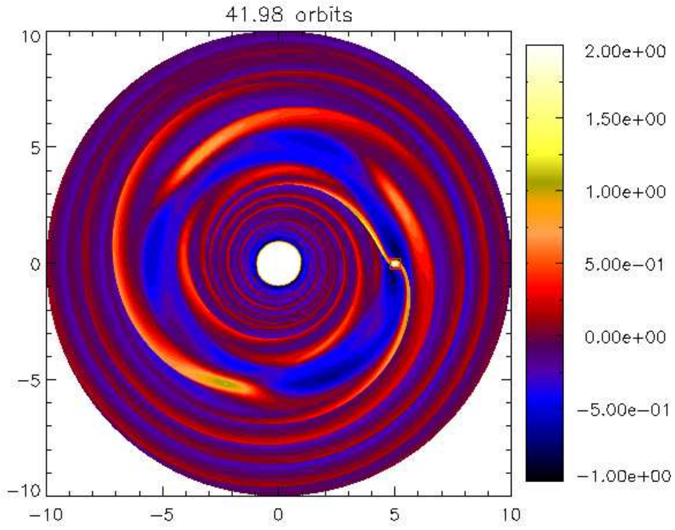
Strong self-gravity

A case with $M_d = 0.06M_*$. Co-rotation radius at $r \simeq 5.5$, local vortensity *maximum*. Confirms Meschiari & Laughlin (2008).



Strong self-gravity

An example with $M_d = 0.08M_*$.



Requirement for edge modes

- Recall ODE:
  $L(S) = \delta\Sigma$, $S = c_s^2\delta\Sigma/\Sigma + \delta\Phi$. Only keep vortensity gradient term in L. Get...

  $$\lambda\mathcal{H}(r) = \int \mathcal{R}_m(r,\xi)\mathcal{H}(\xi)d\xi.$$

  with $\lambda = 1$. $S \rightarrow \mathcal{H}$ (new eigenfunction); $K_m \rightarrow \mathcal{R}_m$ (new kernel).

- But there is a $\max(\lambda)$. So if $\max(\lambda) < 1$ then no mode.
- Can show $\max(\lambda) < \Lambda$ and estimate Λ:

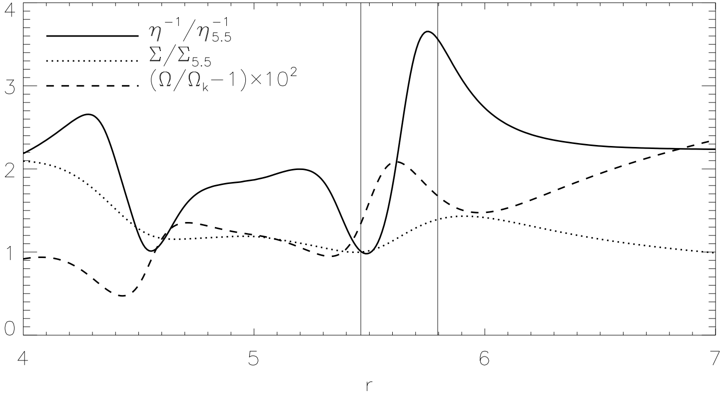
  $$\Lambda \sim \frac{2GK_0(m\epsilon_g/r_c)}{r_c}\left|\frac{1}{\Omega'}\left(\frac{2\Omega\Sigma}{\kappa^2}\right)''\right| L_c.$$

  K_0: Bessel function; L_c length-scale of edge.

- Translate to Toomre Q, need $Q < Q_*$ but $Q_* \sim 4.5$ for fiducial parameter values. Classic GI $Q_* \sim 1$.

Linear calculations of edge modes

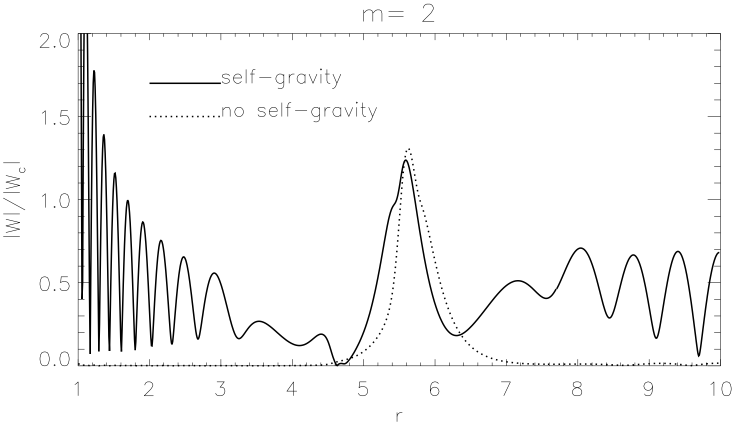
Solve the ODE with and without SG in response. Background has SG.



Solid line is $1/\eta$! NSG $r_c = 5.80$ and SG $r_c = 5.46$.

Linear calculations of edge modes

Here are the eigenfunctions:



Analogy with disc-planet system

- Edge disturbance acts as external forcing on smooth part of the disc.
- Edge disturbance is like a planet. Does its potential vary on global scale?

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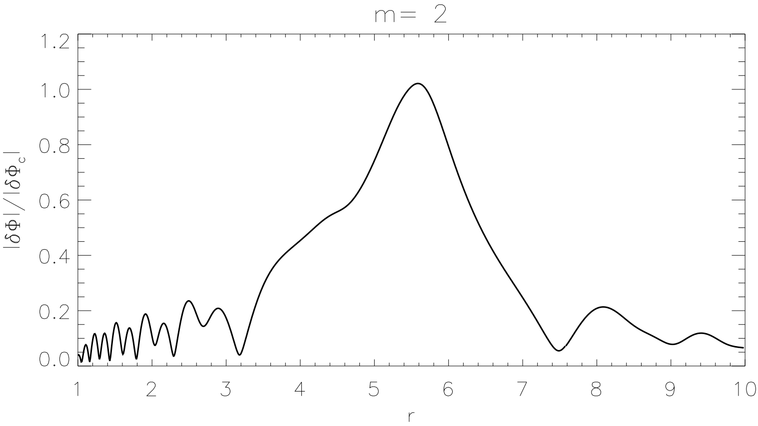
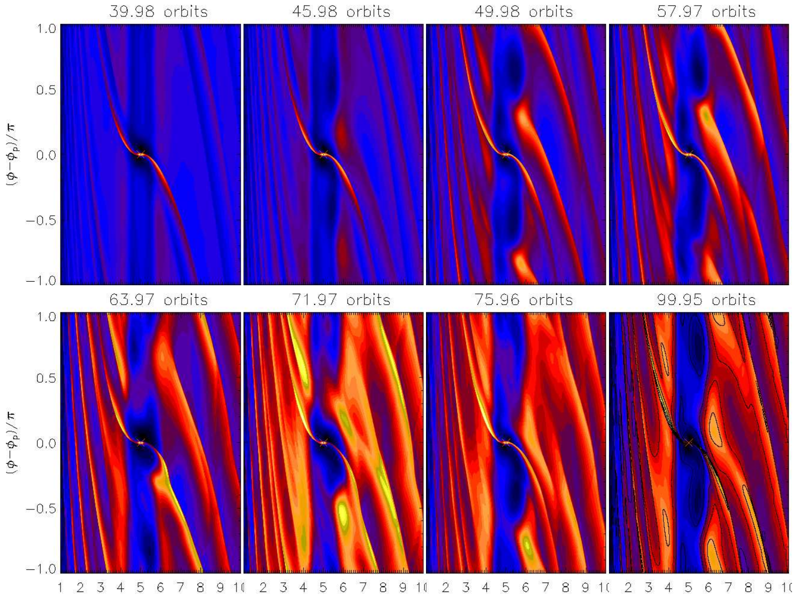
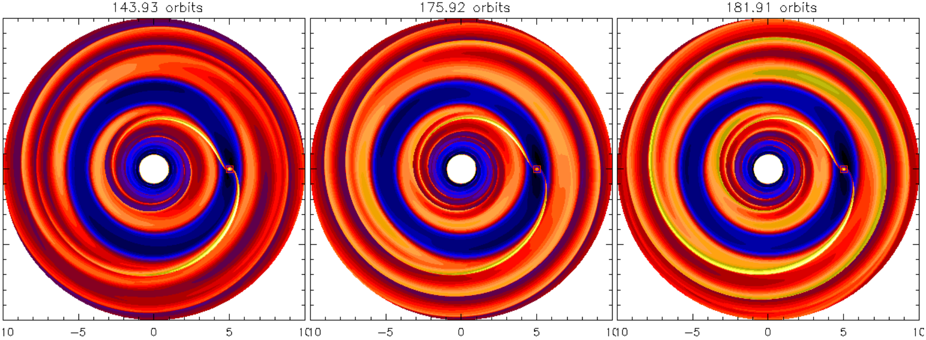


Back to hydrodynamics



Eccentric gap



Viscosity

- Need low viscosity to get vortices ($\nu < 10^{-6}$ or $\alpha < 10^{-4}$).
- Standard viscosity $\alpha = 10^{-3}$ can't kill edge modes.

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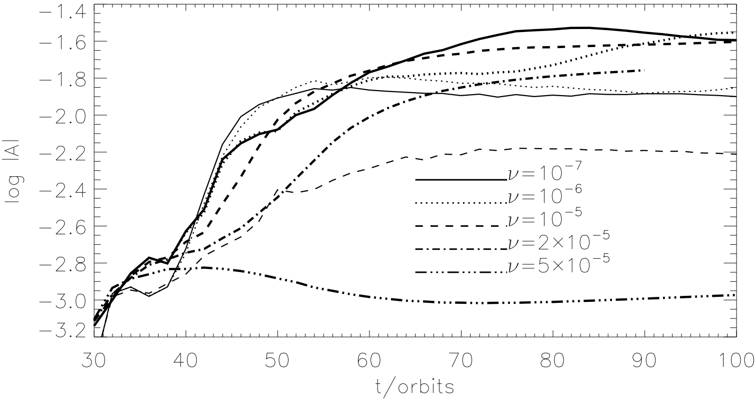
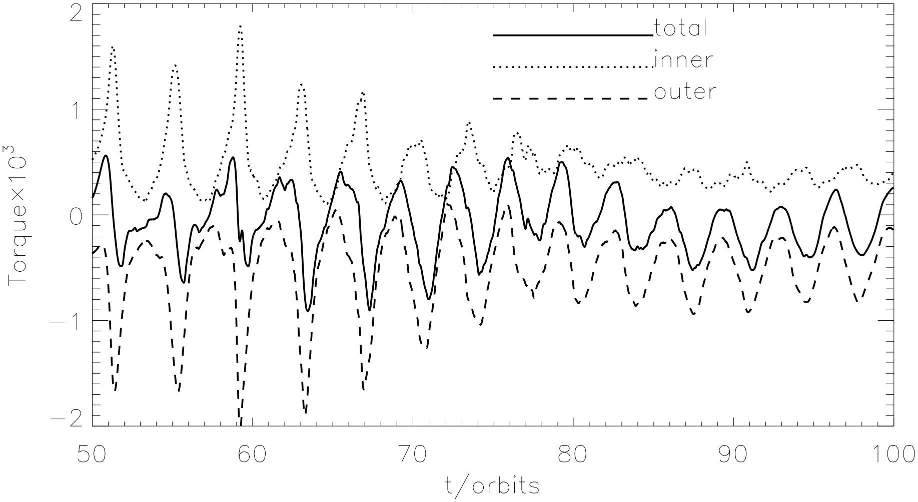


- Lower viscosity → higher *m* (sharper profiles).
- Eventually surpressed but because no local vortensity maximum.

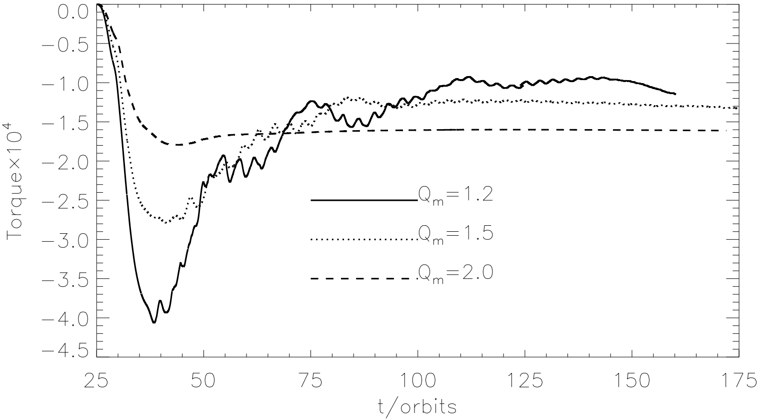
Application to disc-planet interaction

Disc-planet torque for fiducial case:



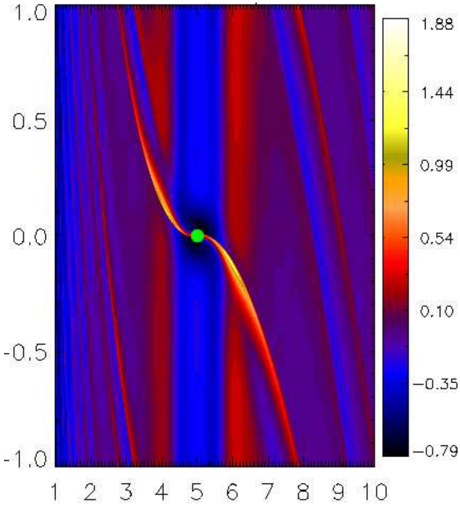
Application to disc-planet interaction

Time-averaged disc-on-planet torques:



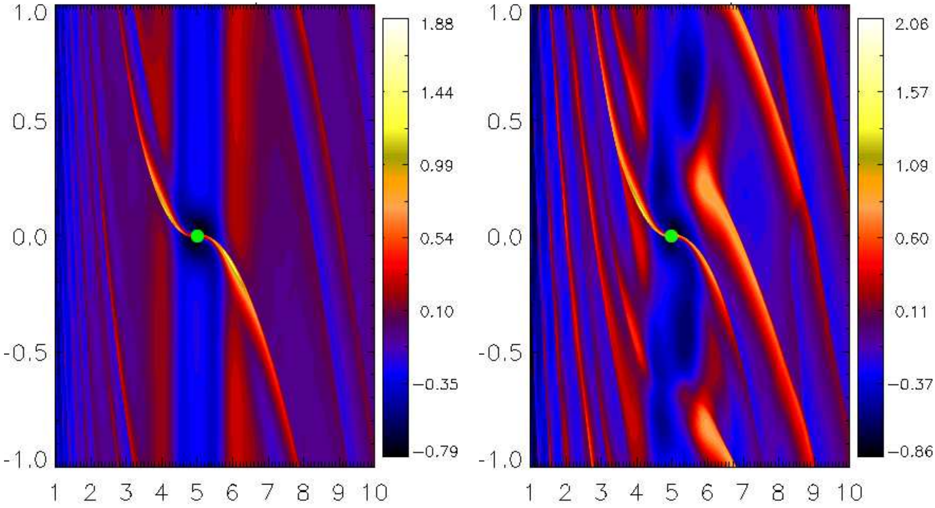
If unstable → average torque more positive with increasing disc mass.

Diluting the outer torque



Stable disc.

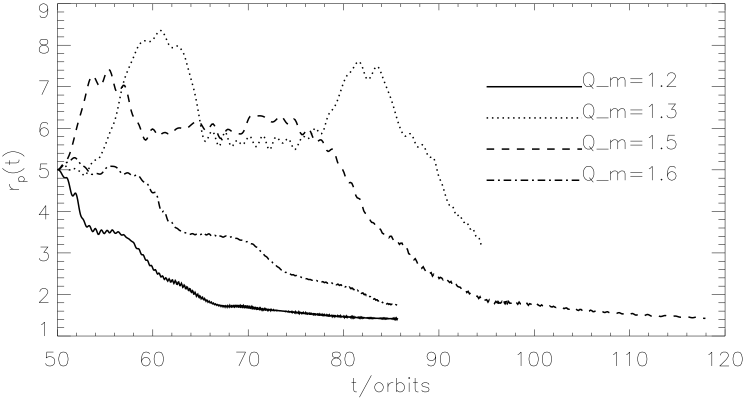
Diluting the outer torque



Stable disc. Unstable disc.

Migration in massive disks

- Numerically difficult: lots of parameters.
- Need better set up.



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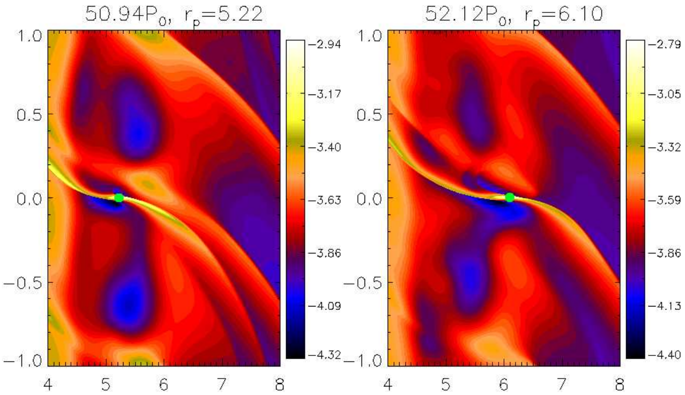

Spiral-induced type III migration.

Summary & future work

- Planetary gaps support two types of instability: vortices & spirals.
- The level of self-gravity determines which type is preferred.
- Increasing SG produce more vortices and merging is resisted.
- Global spiral modes with enough SG. They occur *during* gap formation.

Summary & future work

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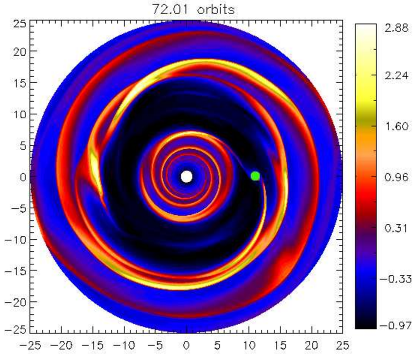
Future topics:

- *'Planetary migration in the presence of large-scale spiral arms'*



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