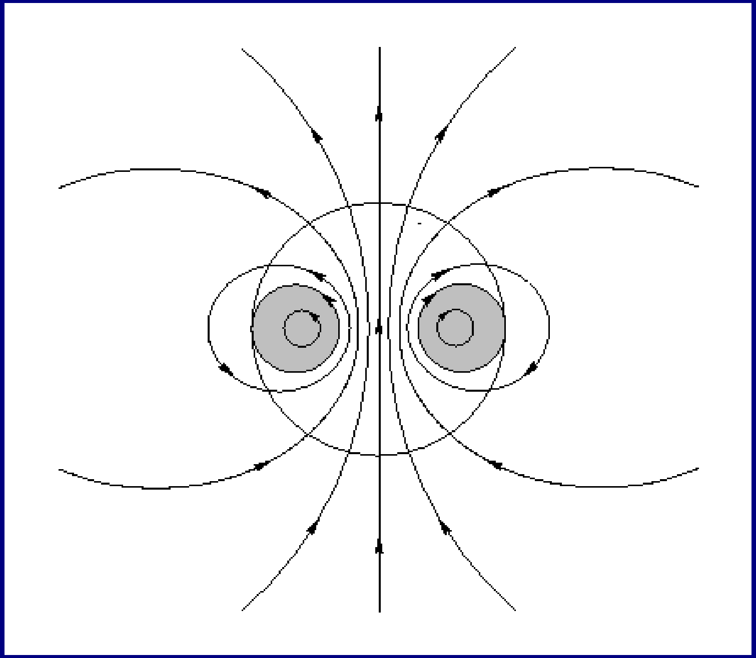
Stable magnetic fields in stars

McGill
16th October 2007



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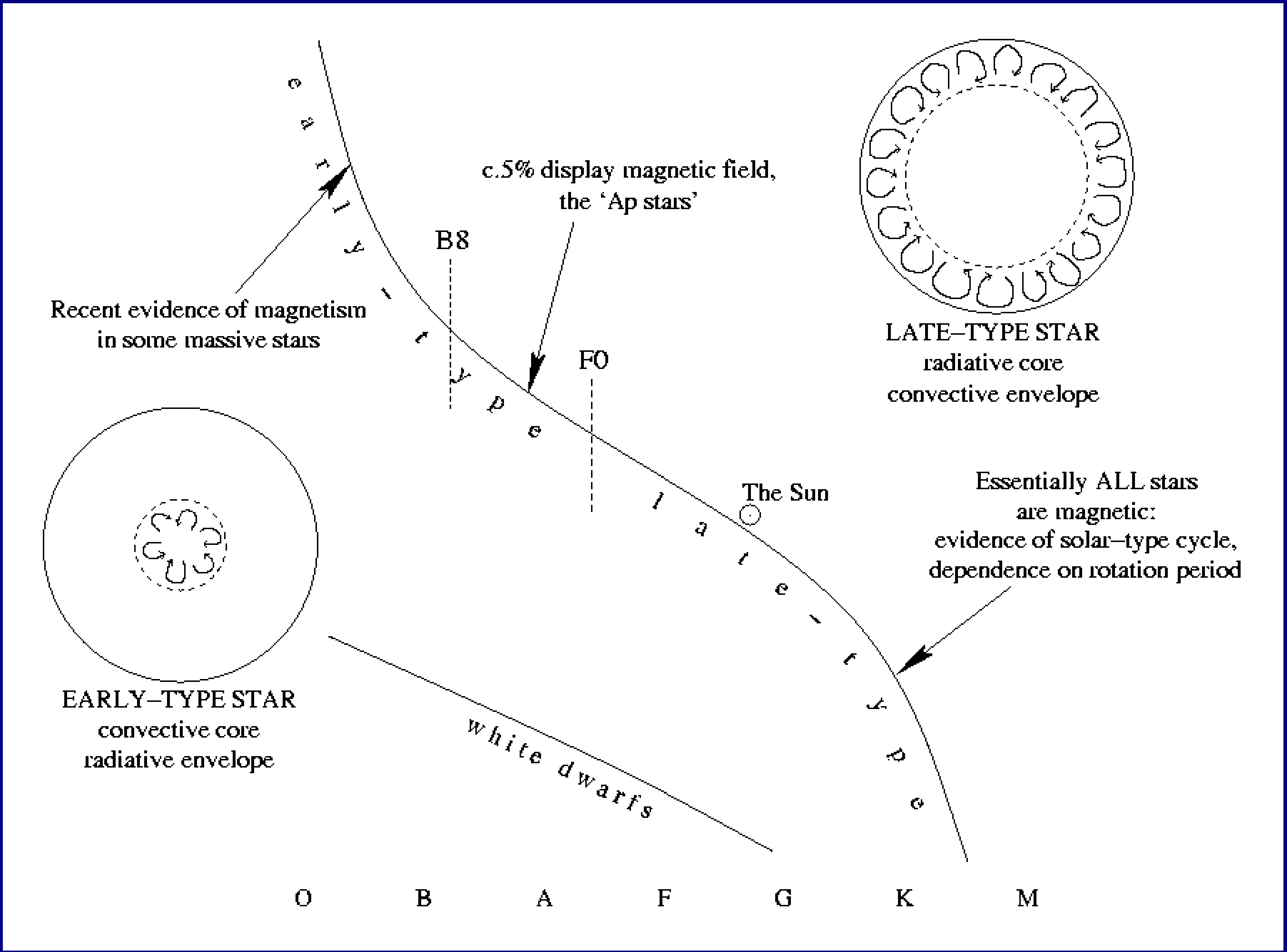
Why study magnetic fields in stars?

- One-dimensional stellar evolution models with nuclear & radiative processes give good results from ZAMS to AGB
- Much of what we don't understand involves magnetic fields and/or rotation
- Both are v. important in processes before and after main-sequence lifetime: star formation, evolved stars, supernovae, planetary nebulae, and compact stellar remnants e.g. neutron stars

Magnetism on the main sequence



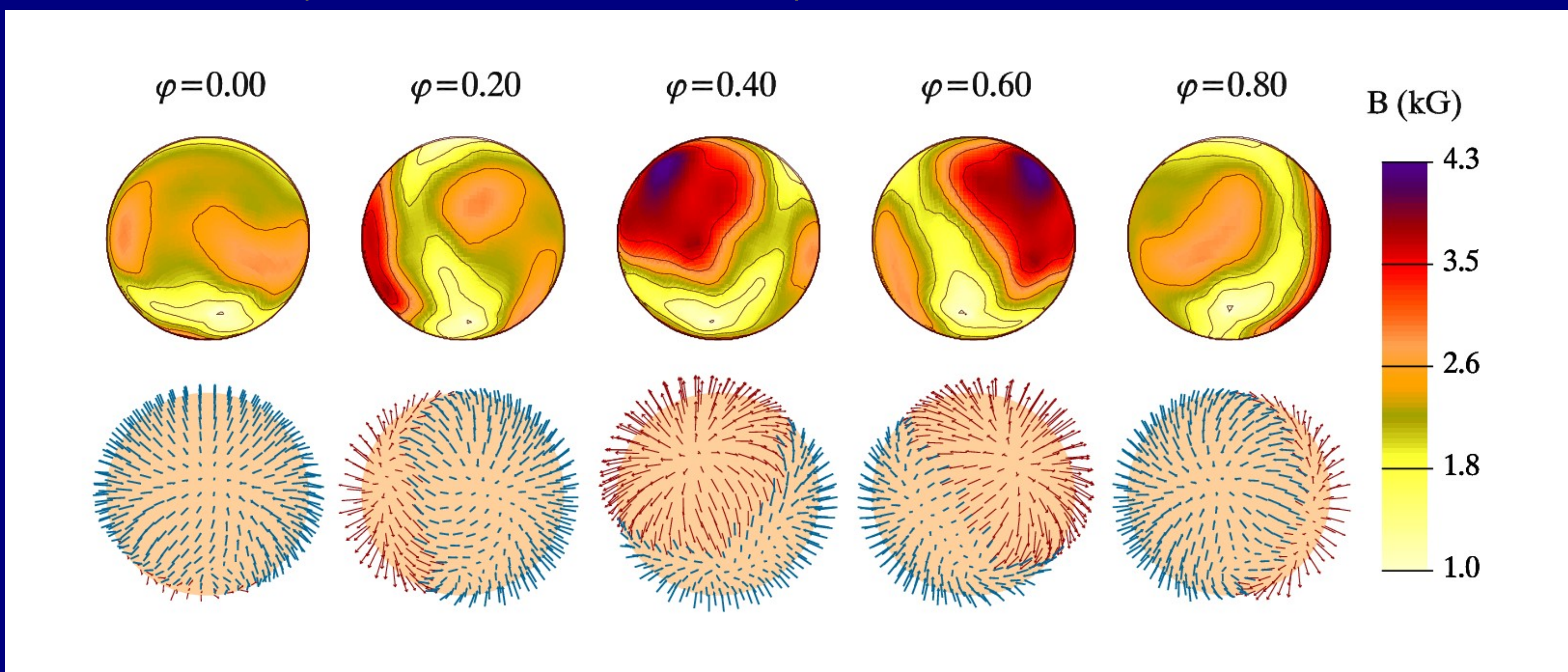
Fields observed on intermediate-mass main-sequence stars (Ap stars)

5% of stars 1.5 – 8 solar masses, the “Ap stars”

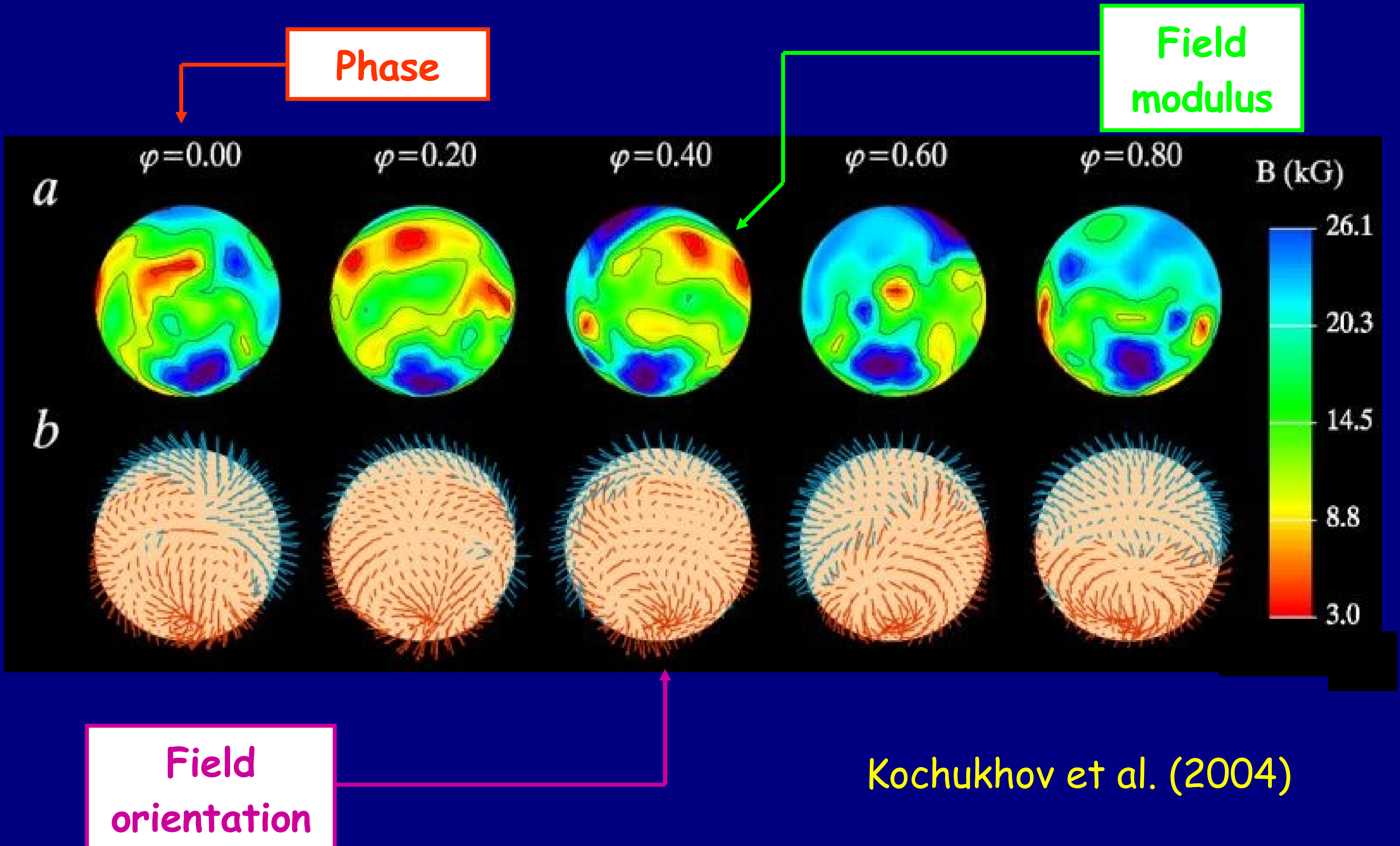
Large-scale fields, stars with 200G – 30kG

No changes seen in field of any Ap star. No differential rotation.

Field on α^2 CVn. (Kochukhov et al., in prep.)



Field on 53 Cam (an Ap star)



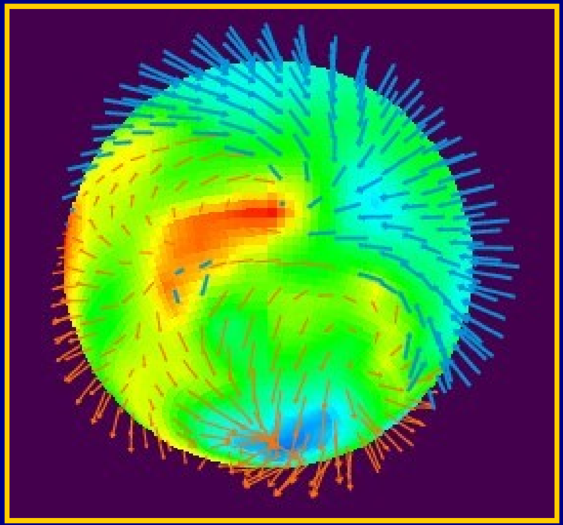
Kochukhov et al. (2004)

Comparison to convective stars

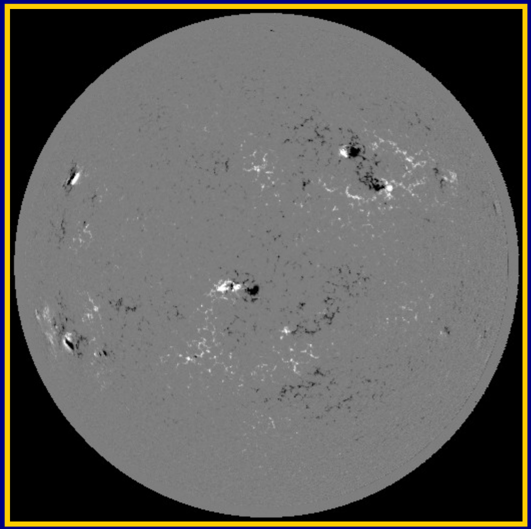
On the Sun we see:
- Small-scale structure (fields up to 3kG)
- Weak large-scale structure (dipole ~ 1G)
- Variation on timescale minutes-weeks-years
- Differential rotation

On other solar-type stars, we see:
- *All* stars have magnetic activity
- Strong rotation-activity correlation

Ap stars have none of these properties!



Kochukhov et al. (2004)



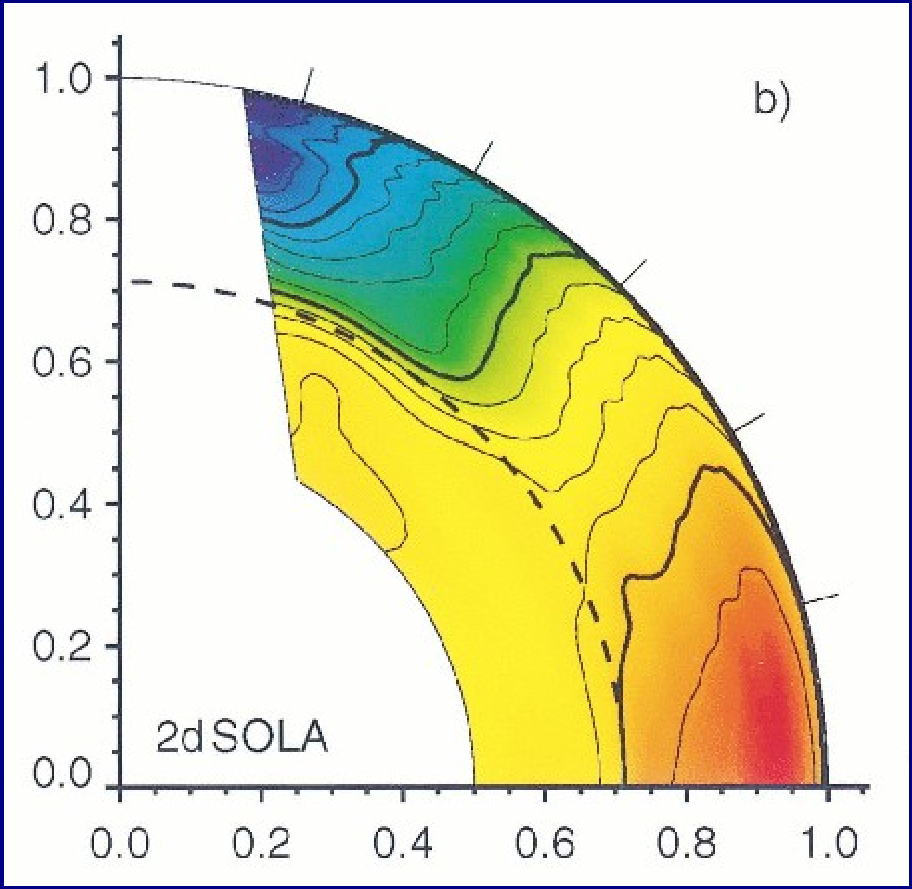
Which stars are non-convective?

- Main-sequence stars above 1.5 solar masses
- White dwarfs
- Neutron stars
- Solar core, etc.
- Same principles apply to all, but magnetic field observation of intermediate-mass main-sequence stars (by the Zeeman effect) is easiest.

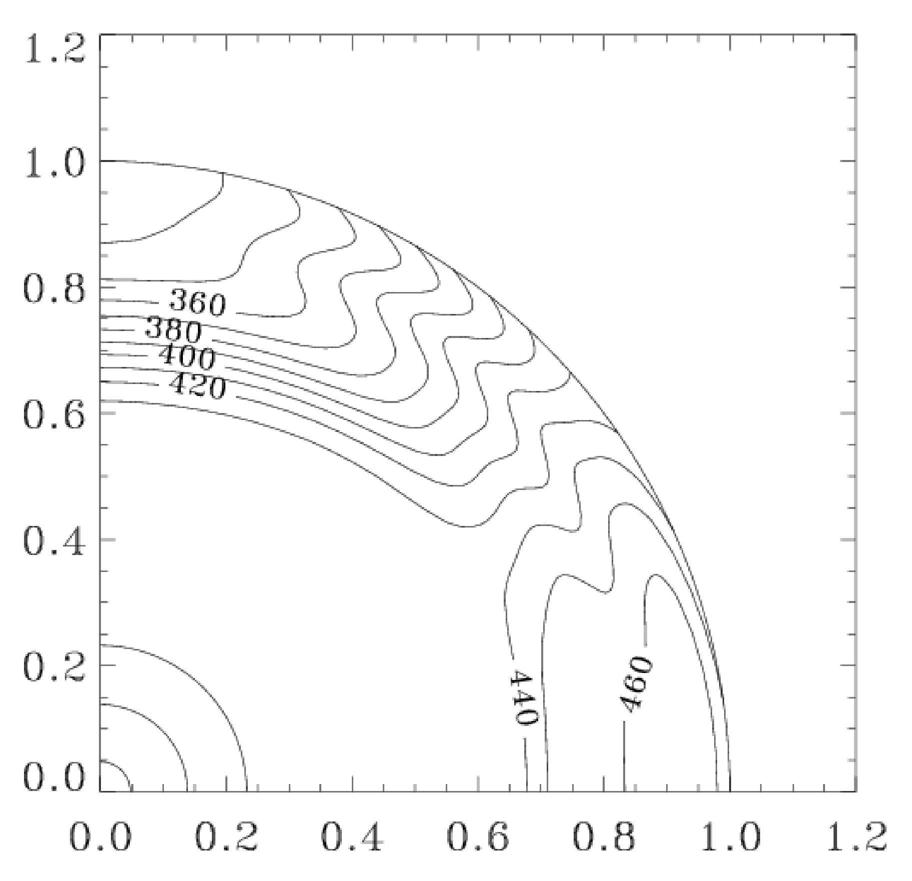
Compact stars

- White dwarfs:
  - similar shapes & fluxes to Ap stars. 10^4 – 10^9G (e.g. Wickramasinghe & Ferrario 2000).
  - no changes seen since observations began.
- Neutron stars:
  - Dipole components in range 10^8 – 10^{15} G inferred from spin-down rate.
  - Little observational constraint on shape and higher multipoles.
  - Some changes seen, but generally on timescales much longer than the Alfvén timescale

Differential Rotation in the Sun: Observations



Schou et al. 1998



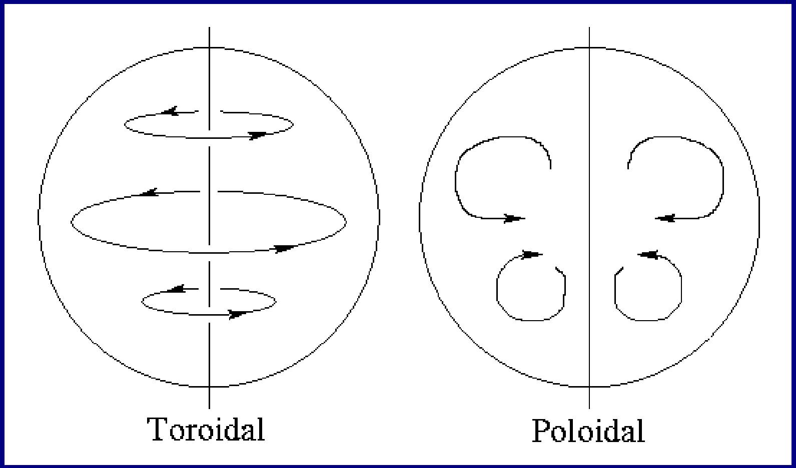
Charbonneau et al. 1999

Summary

- Non-convective stars show a tendency for:
  - steady, large-scale magnetic fields
  - large range of field strengths between different stars
  - lack of differential rotation
- From this, we infer:
  - Fields are evolving on a diffusive timescale, not on the Alfvén timescale
  - Lack of suitable self-regenerative (dynamo) processes
- We need a field in stable equilibrium

Equilibrium and stability

- A non-equilibrium field will evolve on the Alfvén timescale
- Various equilibrium fields can be constructed
- Certain equilibrium field configurations have been shown to be unstable, including:
  - All purely toroidal fields[1],
  - All purely poloidal fields[2].



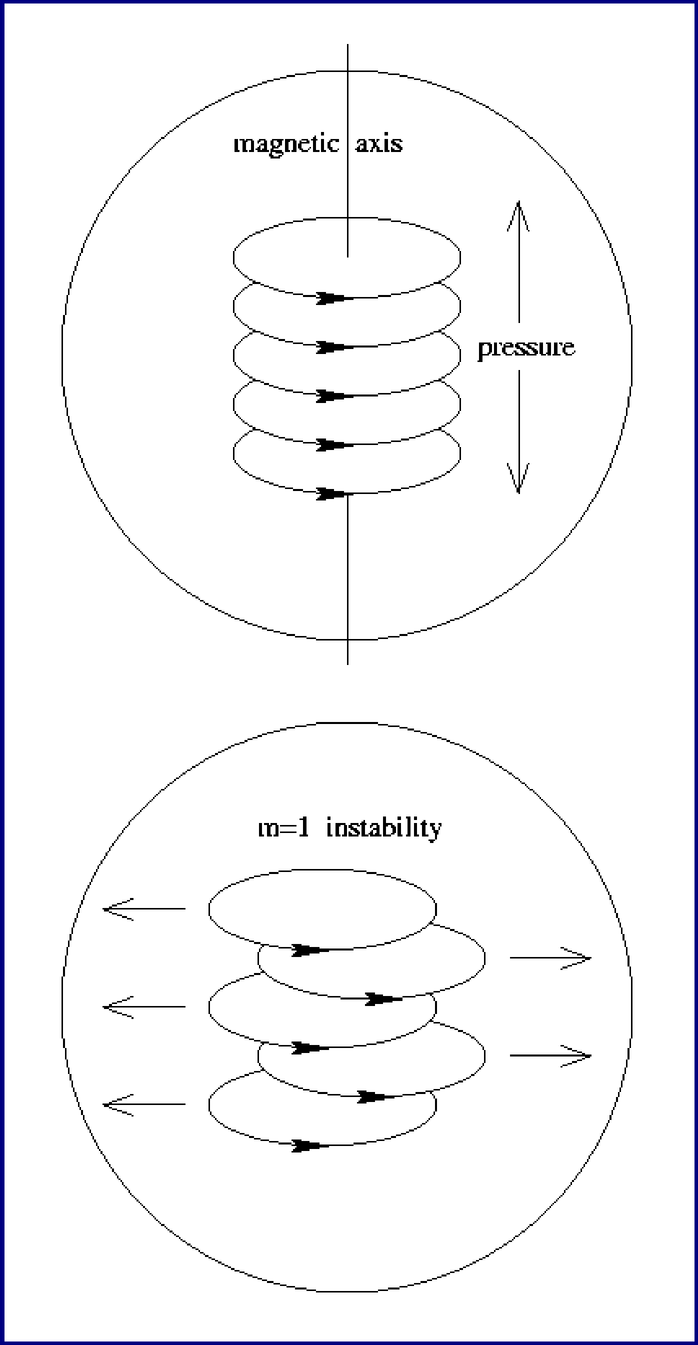
1. Wright 1973, Markey & Tayler 1973.
2. Tayler 1973.

Instability of toroidal field

A magnetic field exerts a positive pressure perpendicular to its direction.

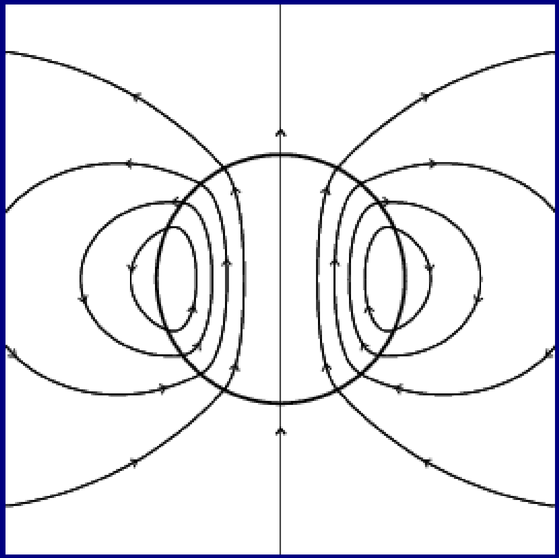
In a toroidal field, this creates a situation like that in a compressed spinal column.

Consequently: any purely toroidal field is subject to instability on or near the magnetic axis.

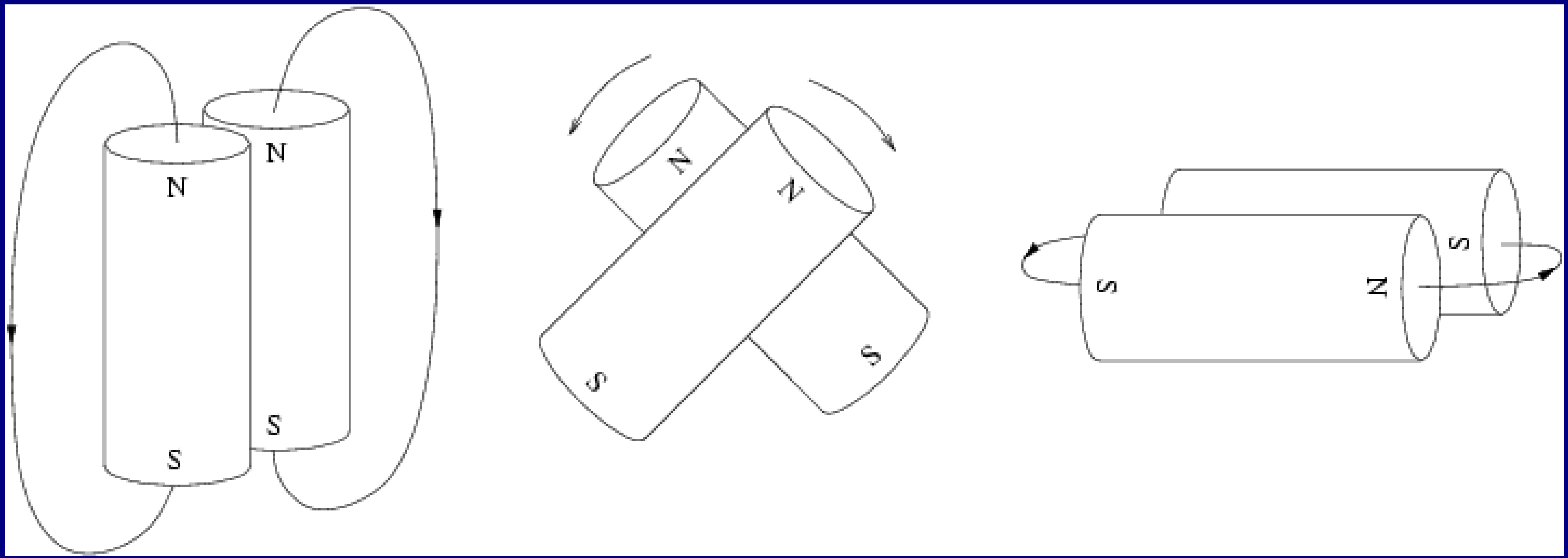


Instability of a poloidal magnetic field

A purely poloidal field is unstable, as one half of the star can rotate with respect to the other, and the magnetic energy outside the star goes down.

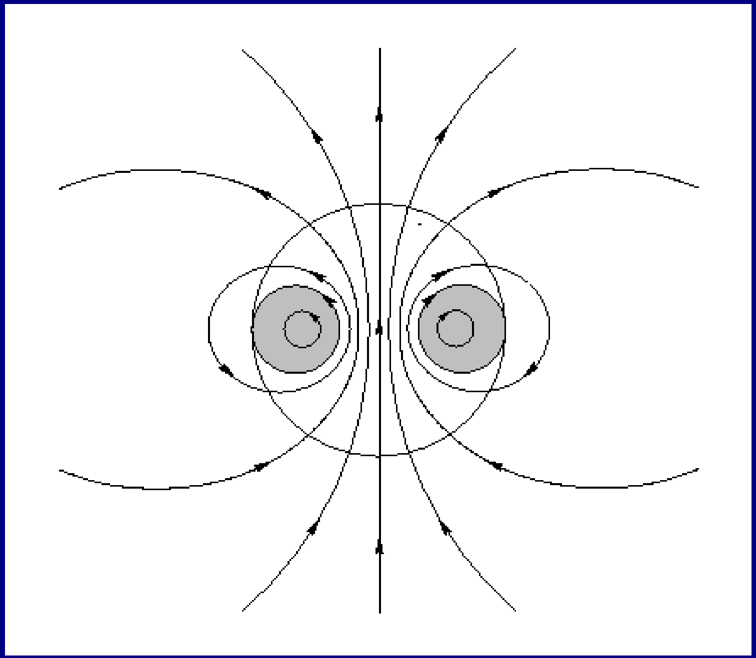


A poloidal field

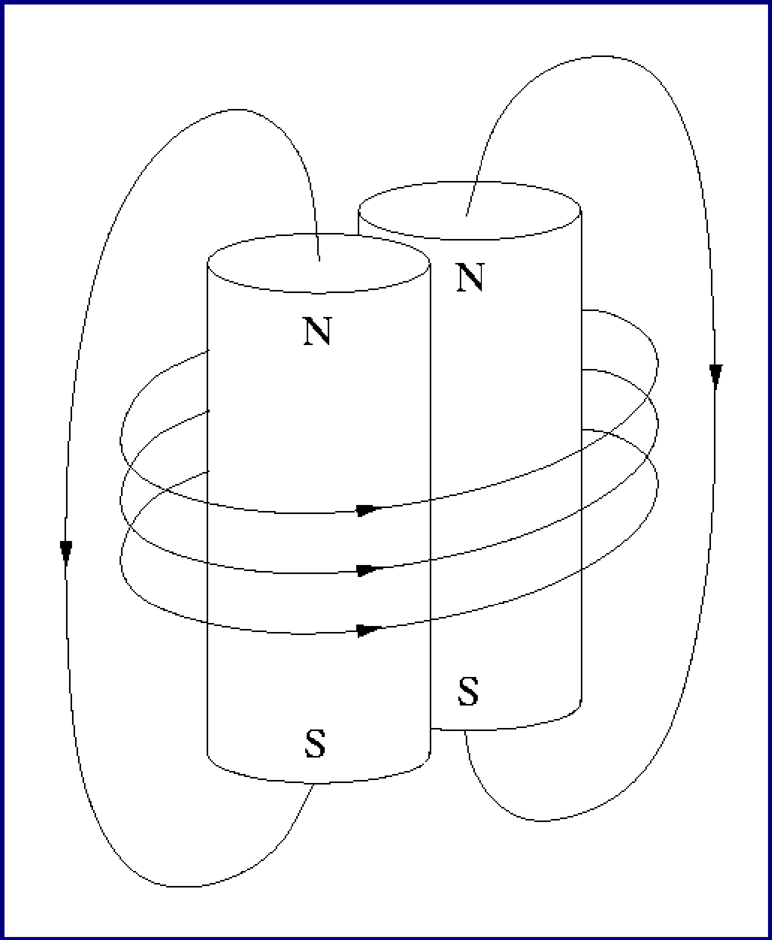


Finding a stable field configuration

A stable equilibrium needs both poloidal and toroidal components



A mixed poloidal-toroidal field. Shading represents toroidal component



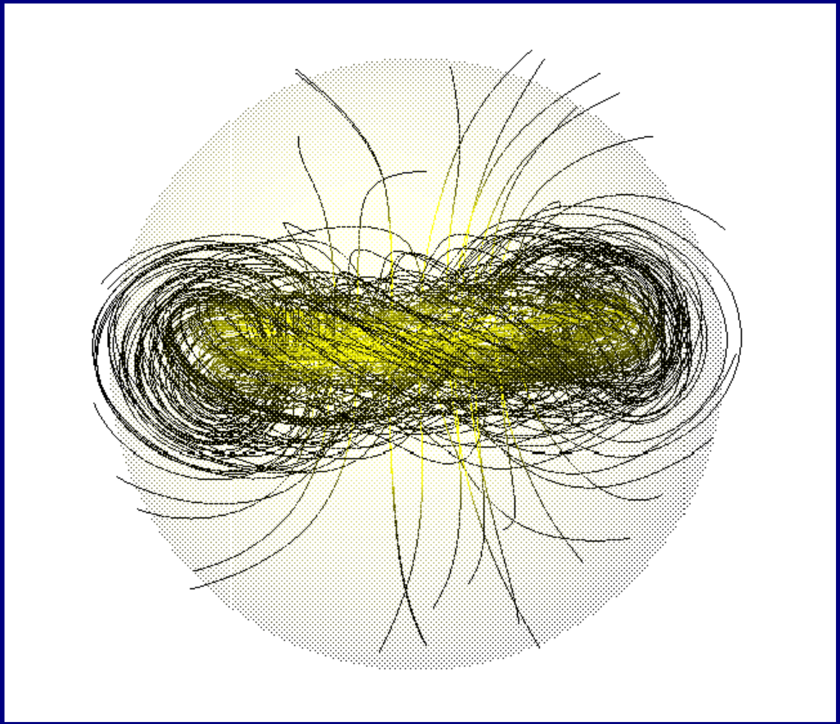
By adding a toroidal component, rotation of magnets is prevented.

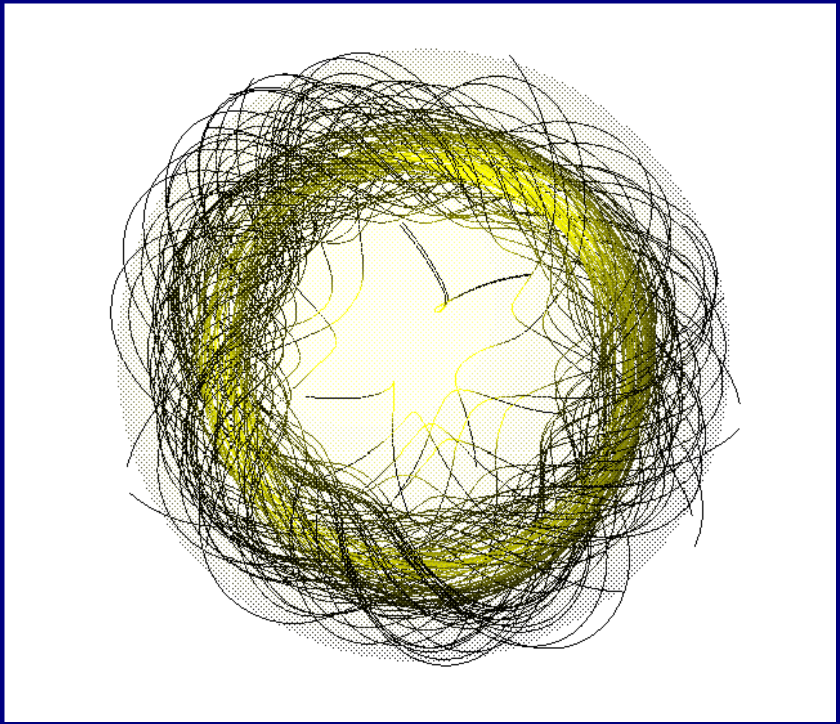
Simulations to investigate magnetic field equilibria & stability

- Use numerical magnetohydrodynamics (MHD) to model star as ball of self-gravitating ideal gas (n=3 polytrope) in a box
- Follow evolution of an initially random “turbulent” field: it will either decay to nothing or find a stable equilibrium
- Use Åke Nordlund's “stagger-code”, a high-order finite-difference Cartesian MHD code.

Shape of stable torus field

Forms in a star, out of an arbitrary initial magnetic field





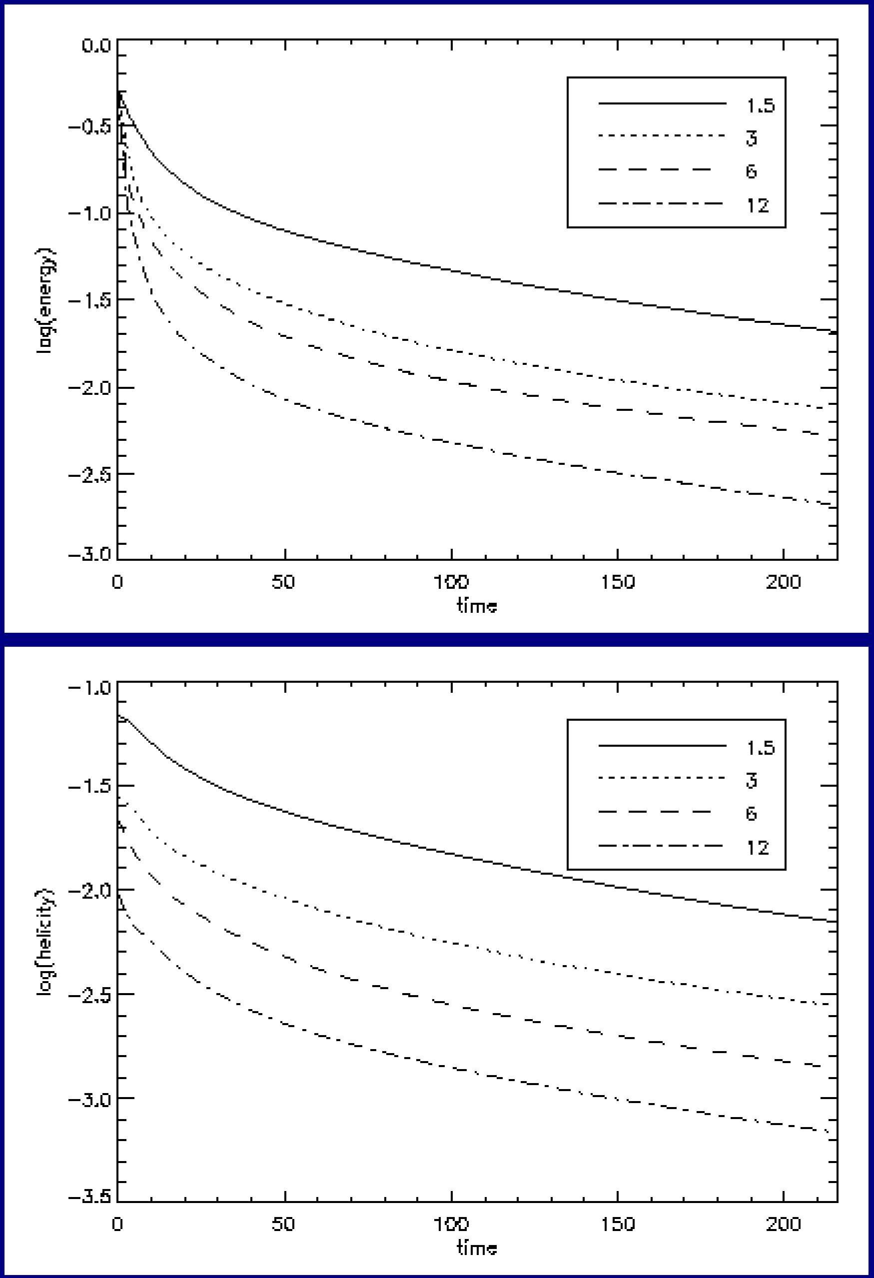
Braithwaite & Nordlund 2006

Conclusions & questions

- Axisymmetric twisted-torus is simplest, most fundamental stable equilibrium
- It can form from an initially chaotic field
- Are there other, more complex stable equilibria? (cf. observations of non-dipolar Ap stars)
- If so, which initial turbulent fields will evolve into which equilibria? And what strength field is produced? Possible factors include:
  - central concentration, i.e. radial profile of field strength
  - coherence length of turbulent field

Magnetic helicity of initial field

- Initial random field contains wavenumbers up to k_{max}
- Simulations run with $R_* k_{max}/2\pi$ = 1.5, 3, 6 and 12.
- Helicity defined as $H \equiv \int A.B\, dV$, where B = curl A. It is conserved in the limit of infinite conductivity
- Higher k_{max} means lower helicity, because different regions cancel each other out
- Lower initial helicity results in a lower-energy field, since the equilibrium is the lowest energy state at that value of helicity



Above right: magnetic energy against time
Below right: magnetic helicity against time

Figures by Brandon Helfield

Radial profile of initial field

- Run simulations where initial field is tapered as B ~ ρ^p
  (simulations run by B. Helfield)
- If star forms from a uniform magnetised cloud and flux loss fraction is independent of radius, we expect p=2/3

Simulations with different values of p

- $B \sim \rho^p$
- If $p > 0.5$:
  - dipolar torus field forms on an Alfvén timescale
  - field then diffuses outwards on an Ohmic timescale
  - at some point, it goes into a non-axisymmetric equilibrium, and continues to evolve on a diffusive timescale
- If $p < 0.5$:
  - a more complicated (generally non-axisymmetric) equilibrium is reached on an Alfvén timescale
  - This also evolves further on the diffusive timescale
- Non-axisymmetric equilibria consist of a twisted flux tube(s) close to the surface of the star

Outwards diffusion of axisymmetric torus field

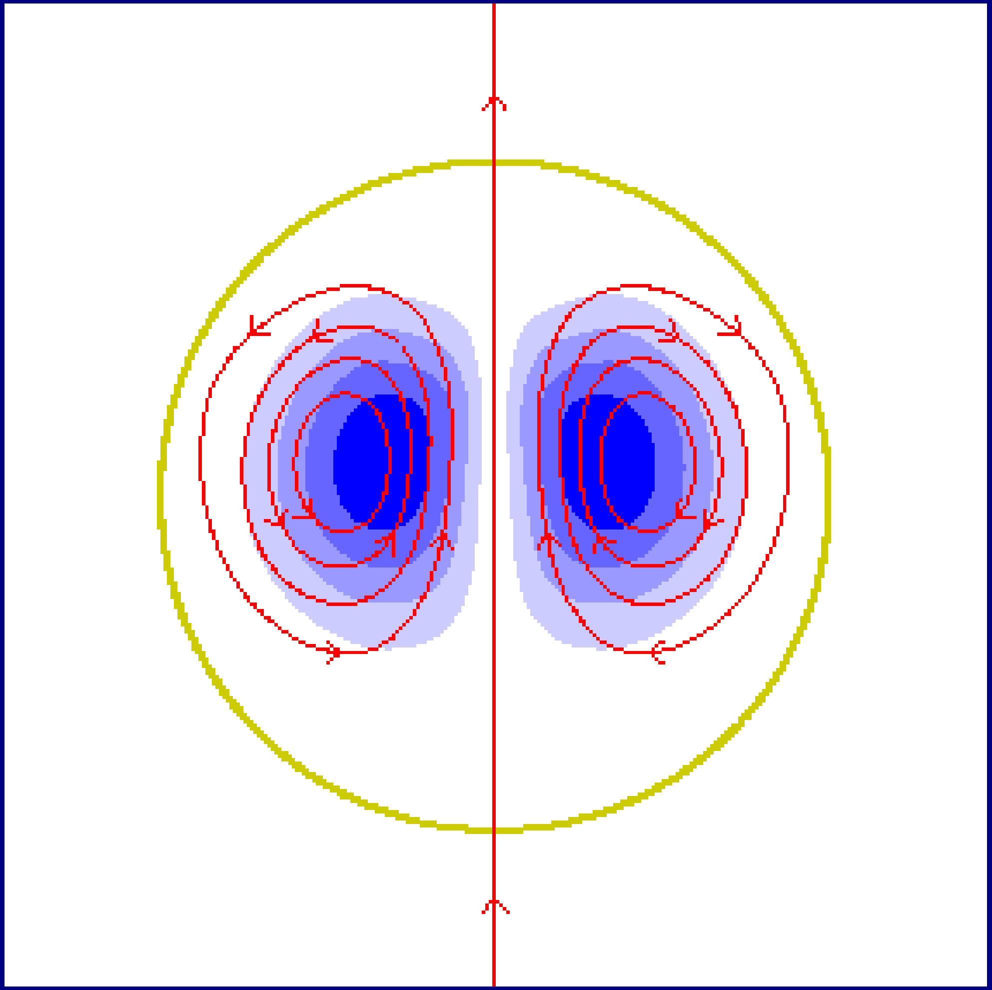
- Owing to finite conductivity, field decays on an Ohmic diffusion time-scale (~10^{10} years in a main-sequence star)
- As it decays, it moves outwards
- Field strength on surface may increase even though total magnetic energy is falling

Field diffusing outwards

Azimuthal average of poloidal (in red) and toroidal (in blue) components.

Toroidal field threads through loops of poloidal field.

As field diffuses outwards, poloidal component extends into atmosphere.



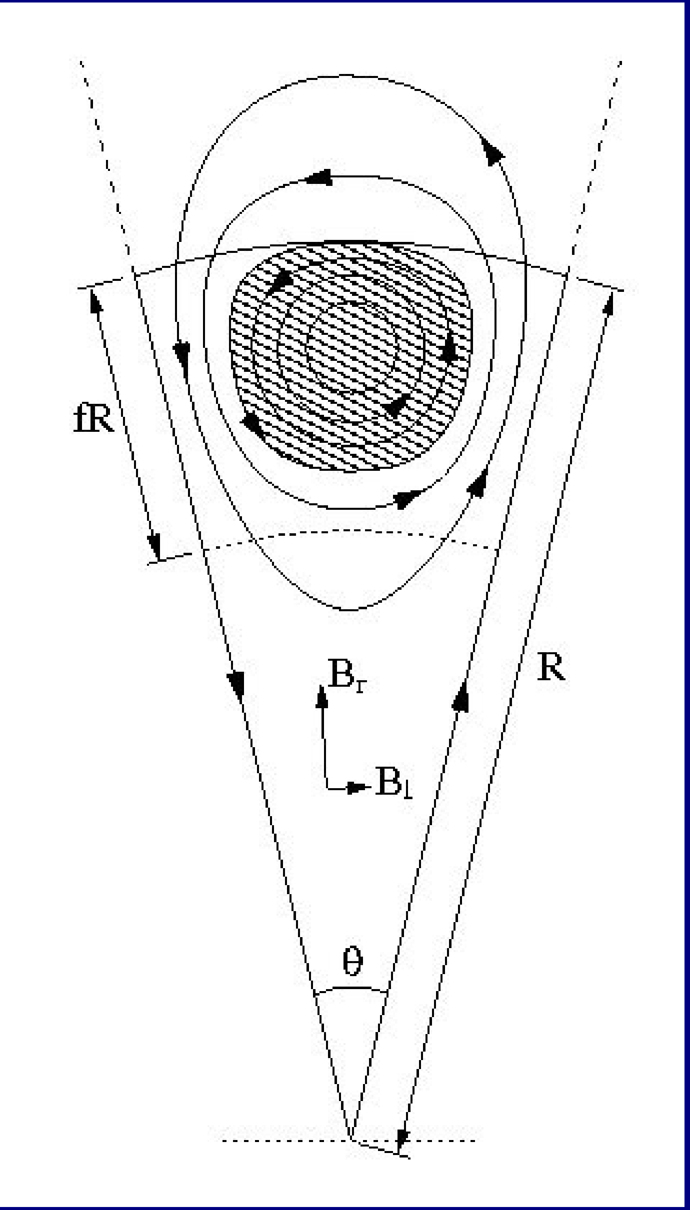
Properties of non-axisymmetric equilibria

- Consist of twisted flux tube(s) below the stellar surface
- Toroidal flux confined to largest closed poloidal loop
- Energy of tube, $E \approx \frac{ARf}{8\pi}(B_l^2 + B_r^2 + B_t^2)$
- If we allow length & width of tube to change adiabatically (with fixed length x width), we find that since

$$\frac{\partial \ln B_l}{\partial \ln \theta} = 1 \qquad \frac{\partial \ln B_r}{\partial \ln \theta} = 0 \qquad \frac{\partial \ln B_t}{\partial \ln \theta} = -1$$

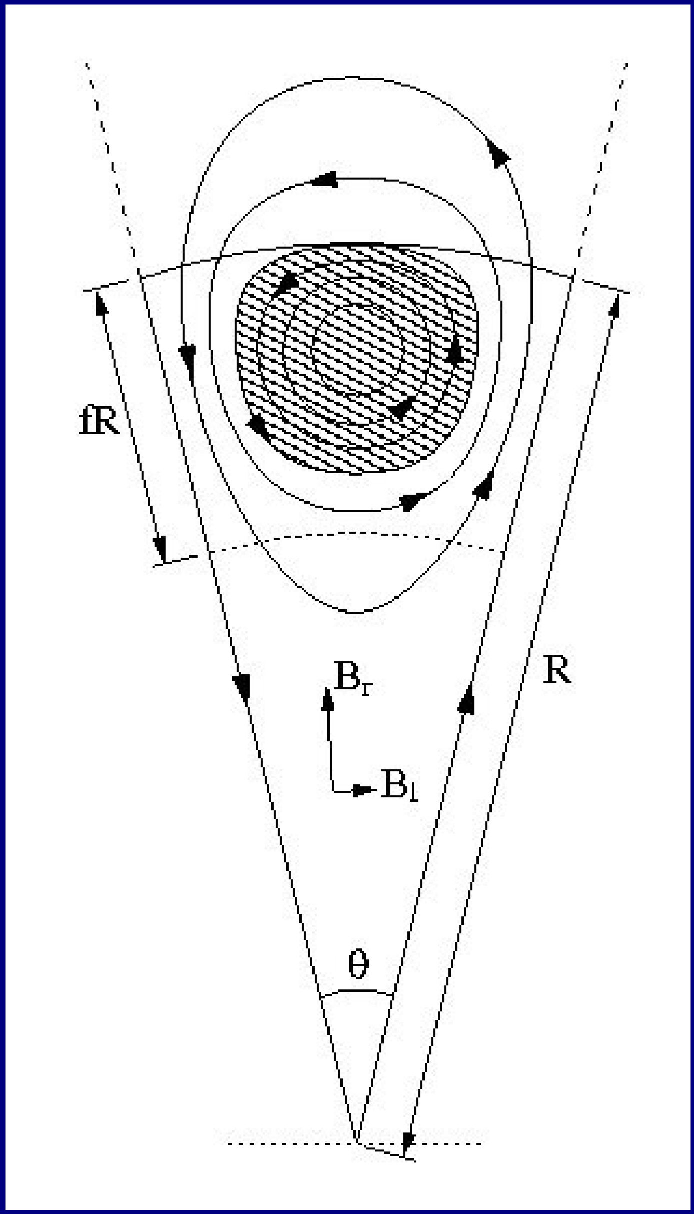
  we have $\frac{\partial E}{\partial \theta} \approx \frac{ARf}{4\pi}\left(\frac{B_l^2}{\theta} - \frac{B_t^2}{\theta}\right)$

- Therefore: $B_l \approx B_t$
- Energy lowest for circular tube, i.e. f=θ, so $B_r \approx B_l \approx B_t$



- Roughly equal toroidal & poloidal fluxes:
  - Too strong toroidal field ⇒ flux tube contracts and widens ⇒ toroidal field weakens
  - Too strong poloidal field ⇒ flux tube lengthens & becomes narrower ⇒ poloidal field becomes weaker
- At equilibrium:

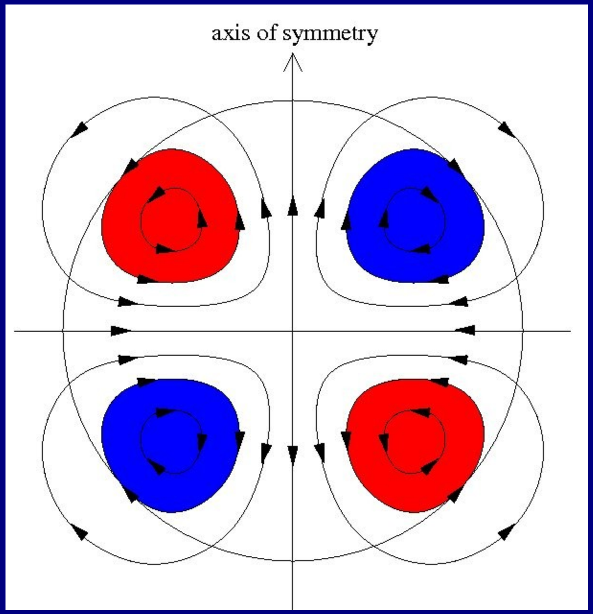
$$\theta^2 \approx \frac{A}{2R^2}\frac{\phi_t}{\phi_p}$$



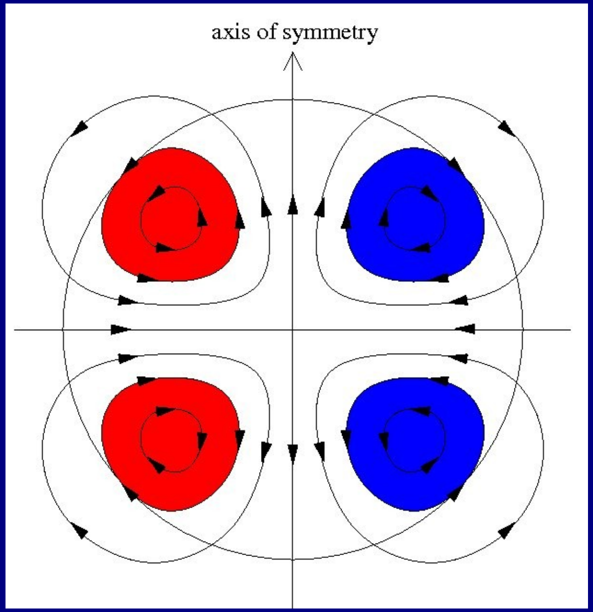
Evolution of non-axisymmetric equilibria

- Equilibria are stable on Alfven timescale but evolve over diffusive timescale
- Flux tubes rise to surface
- Toroidal flux destroyed faster than poloidal
- Flux tubes become longer and thinner
- As this happens, the pattern of tubes below the surface becomes more complicated. Sudden (Alfvén timescale) readjustments conceivable

A zero-helicity equilibrium?



- Helicity is conserved, so stable equilibria should be at an energy minimum for that value of helicity
- In simple terms, helicity is product of toroidal and poloidal fluxes
- Lowest zero-helicity energy state is zero field!
- Magnetic field consisting of two flux tubes of equal and opposite helicities is nonetheless stable



Cross-sections of simple equlilbria consisting of two flux tubes
Top: finite helicity
Bottom: zero helicity

Conclusions: part 1

- Random initial field will evolve into a stable equilbrium
- Equilibrium is either axisymmetric or non-axisymmetric, depending on central concentration of field
- Equilibria consist of twisted flux tube(s) covering surface of star
- With thermal and magnetic diffusion, flux tubes (slowly) become narrower and longer
- If more than one tube present, sign of their helicities not relevant for stability
- Axisymmetric fields: poloidal component cannot be much stronger than toroidal, but opposite not ruled out.

Side effects of magnetic field

- Magnetic field contributes pressure but not mass:
  - changes mass distrubution, generally to non-spherical
- Stellar magnetic field also important for, e.g.
  - Torque on surface from magnetic wind
  - Accretion
  - Energy source for magnetars
-

Moment of inertia

- Magnetic field changes star's moment of inertia (e.g. Wentzel 1961) $$\frac{I_3 - I_1}{I_1} \sim \frac{E}{T}$$
- If damping timescale is long, star should undergo free precession (e.g. Melatos 1999)
- Energy falls to minimum while angular momentum is conserved: star rotates about largest moment of inertia
  - dominant toroidal field, star prolate $\Rightarrow$ perpendicular rotator
  - dominant poloidal field, star oblate $\Rightarrow$ aligned rotator
- Fast-spinning neutron star with toroidal field should emit gravitational waves! (e.g. Cutler 2002)

Ongoing and future projects, open questions

- Stability and longevity of non-axisymmetric equilibria
- Axisymmetric equilibria: stable toroidal/poloidal ratios, and likely occurrence in nature
- Gravitational radiation from magnetically-deformed neutron stars
- Why are only ~5% of intermediate-mass stars magnetic?
- Why such a large range in field strengths in white dwarfs and neutron stars?
- Helicity generation by dynamo action in protostars & new-born neutron stars & its effect on likely equilibria after convective phase ends