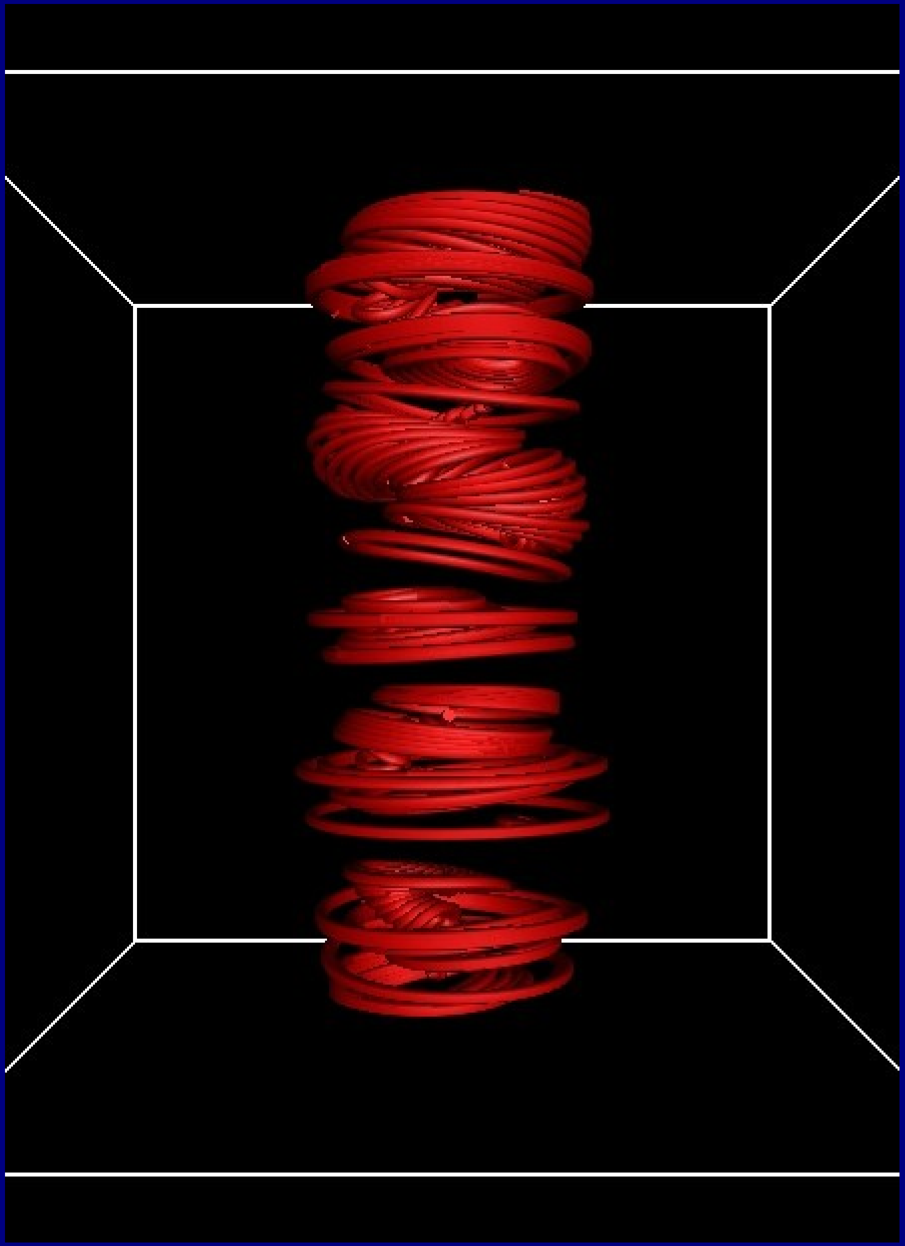

The Tayler instability in stably-stratified stars and a differential-rotation-driven dynamo

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What is the Tayler instability?

- Instability in toroidal fields in stably-stratified stars
- Of interest in differentially-rotating stars where any weak field can be wound up into a predominantly toroidal field

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- Of interest in differentially-rotating stars where any weak field can be wound up into a predominantly toroidal field
- Tayler (1973) used the energy method of Bernstein et al. (1958) to obtain necessary and sufficient stability conditions, showing that any purely toroidal magnetic field is unstable at least somewhere in the star
- Stability conditions local in meridional plane, global in azimuthal direction
- Instability should always be present near the axis of symmetry

Form of instability near axis

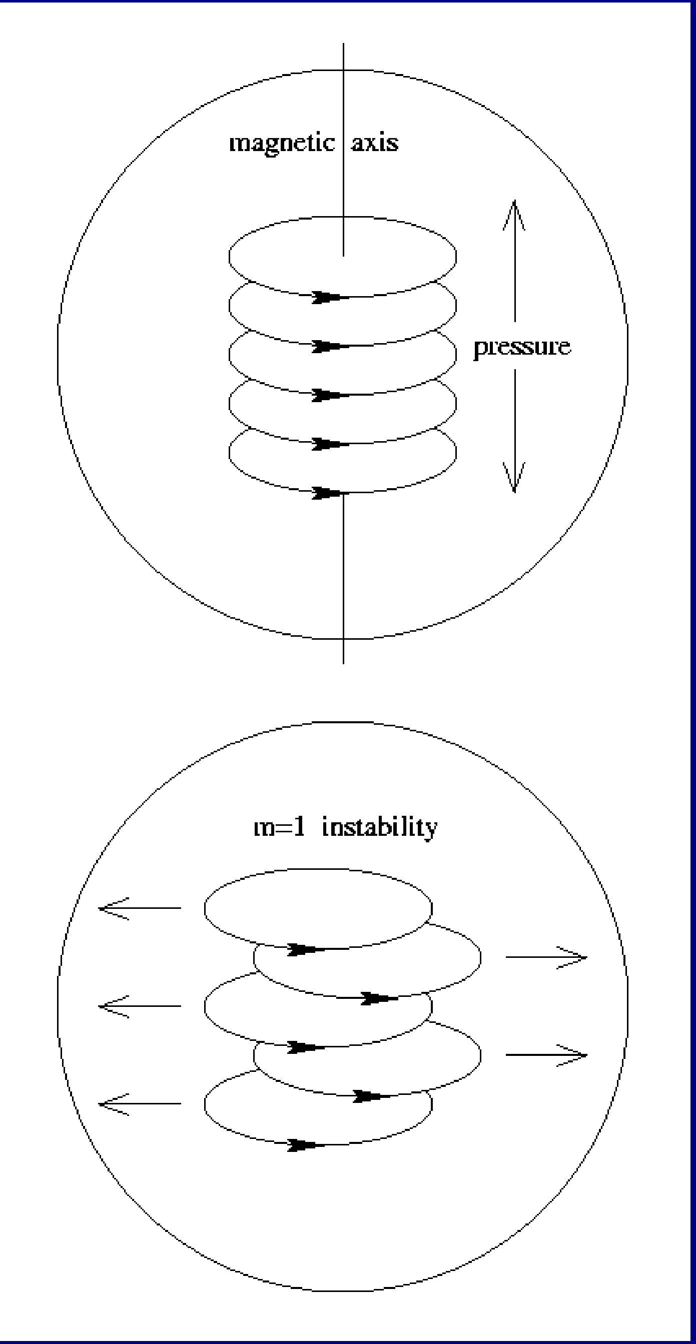
A magnetic field exerts a positive pressure perpendicular to its direction.

In a toroidal field, this creates a situation like that in a compressed spinal column.

Consequently: any purely toroidal field is subject to instability on or near the magnetic axis.

The dominant mode is m=1 ('kink' mode)

Growth rate $\sigma \sim \omega_A$



Stability conditions near the axis

- If B $\propto \varpi^p$, i.e. $p \equiv \partial \ln B / \partial \ln \varpi$, instability conditions are (Tayler 1957)

$$p > \frac{m^2}{2} - 1 \quad (m \neq 0) \qquad\qquad p > 1 \quad (m = 0)$$

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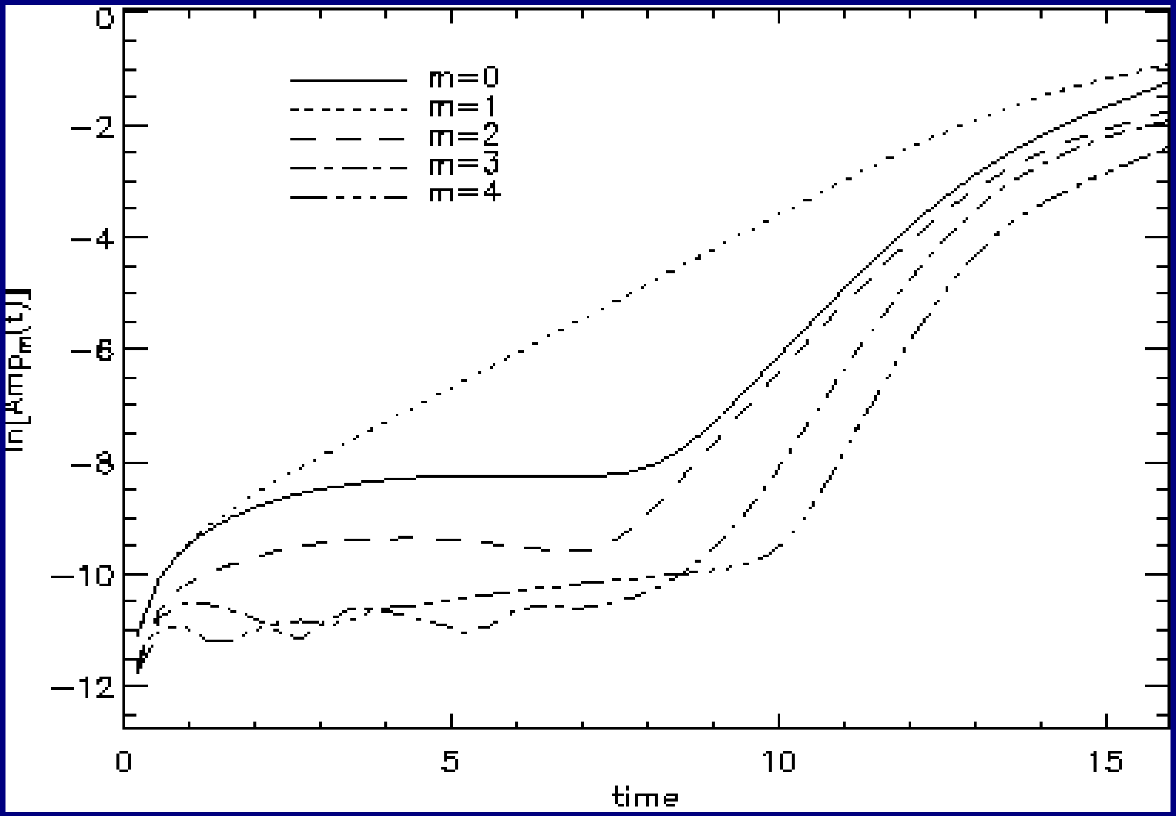
  where m is the azimuthal wavenumber
- A field wound up by differential rotation has p=1 near the axis of rotation; only m=1 mode unstable
- Along axis (i.e. parallel to gravity), wavenumber n restricted by gas pressure/stratification and diffusion to

$$\frac{\sigma}{\eta} > n^2 > \frac{N^2}{\omega_A^2 r^2}$$

Simulations

- Instead of modelling the whole star, model part of the star around the magnetic axis
- $\mathbf{B} = B(\varpi, z)\,\mathbf{e}_\varpi$ and $B \propto \varpi$ at small ϖ
- Field is initially in equilibrium; small perturbation is added at t=0
- Use Åke Nordlund's “stagger code”

Unstable modes



Amplitude (in displacement field) against time
Only the m=1 ('kink') mode is unstable

Growth rate ~ 0.9 ω_A where $\omega_A \equiv v_A / \varpi$

Now investigate the effects of diffusion, gravity and rotation...

Unstable vertical wavelengths

- We expect instability at the following (vertical) wavenumbers:
- $\omega_A / \eta > n^2 > N^2 / \omega_A^2 r^2$
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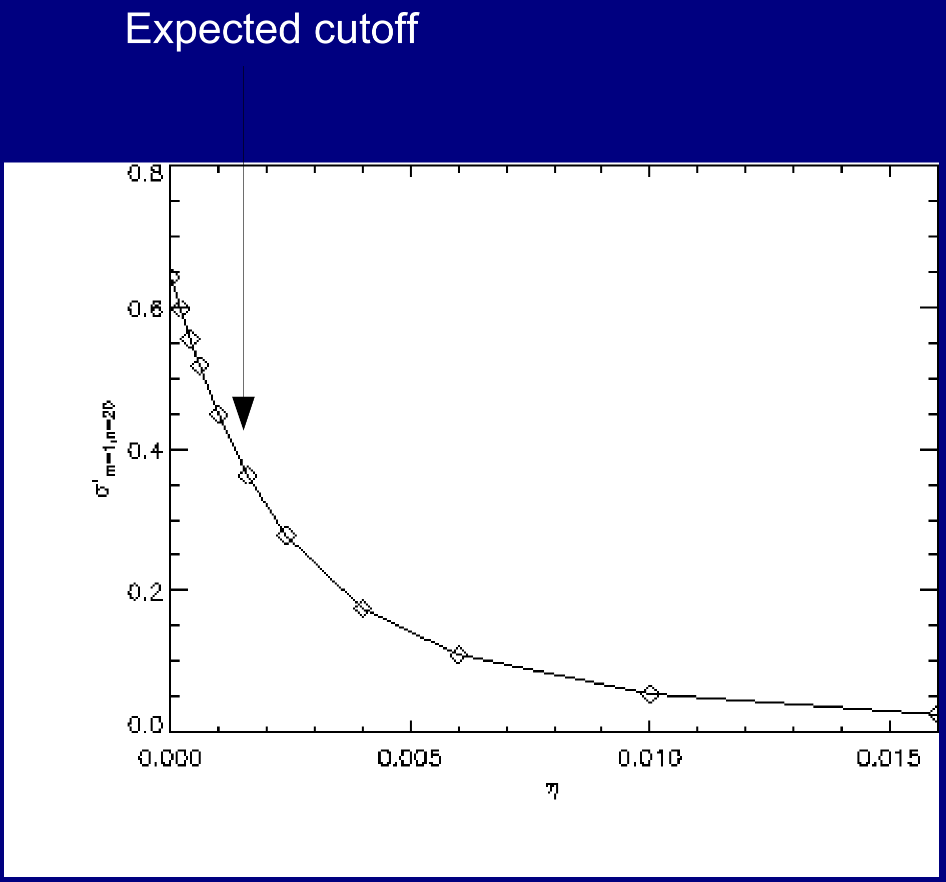
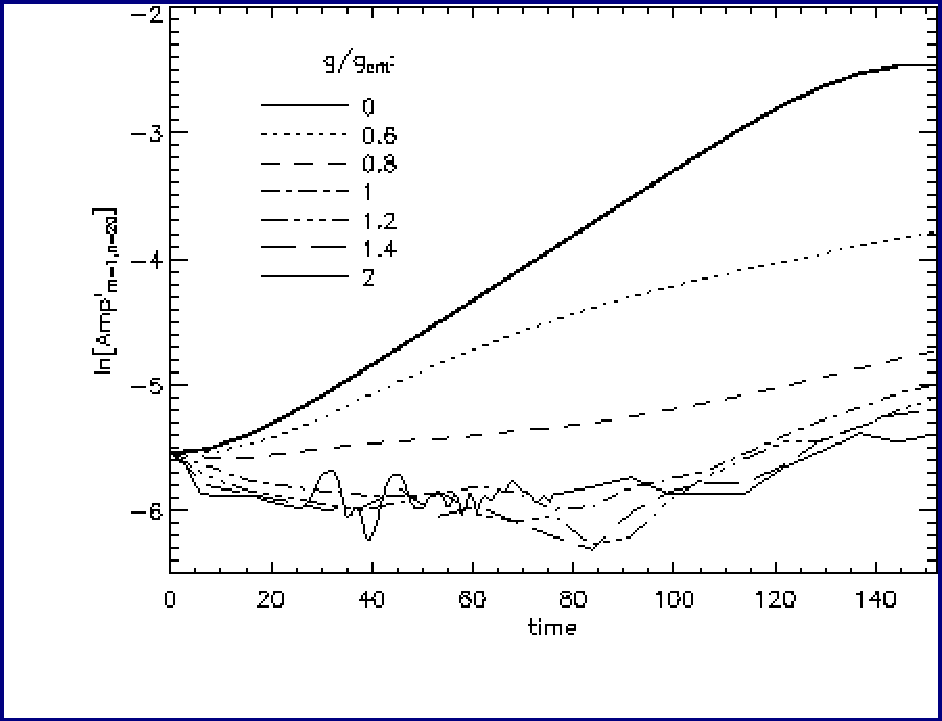
- We expect instability at the following (vertical) wavenumbers:
- $\omega_A / \eta > n^2 > N^2 / \omega_A^2 r^2$
- Magnetic diffusion provides a maximum unstable wavenumber
- Run code with various values of magnetic diffusivity η and measure growth rate at a particular wavenumber
- $\omega_A/\eta = n^2$ if $\eta = 1.6 \times 10^{-3}$
- Result not quite as expected, the instability is not entirely suppressed, perhaps to do with zero thermal conductivity



Growth rate of the m=1, n=20 mode against magnetic diffusivity

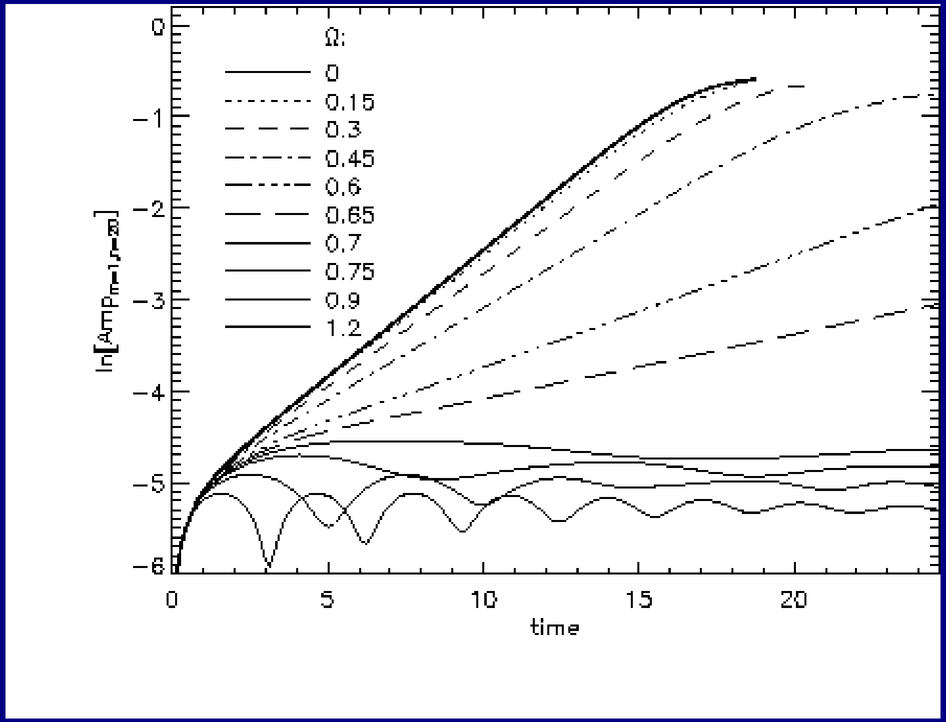
Gravity

- Instability does work against gravity, which is highest at long vertical wavelengths. Stratification stabilises longer wavelengths
- Run code with various values of buoyancy frequency N
- Expected cutoff at N = n ω_A r
- Result as expected



Effect of rotation on instability (adiabatic case)

- Rotation parallel to magnetic axis: instability found to be suppressed if $\Omega > \omega_A$
- Rotation perpendicular to magnetic axis has no effect on stability
- But: when diffusion is present, rotation no longer provides stabilisation



Amplitude of unstable m=1 mode against time, with different rotation speeds

(Braithwaite 2006, A&A 453, 687)

Tayler instability and dynamo effect

- Dynamo loop:
  - Weak seed field is wound up by differential rotation, creating a predominantly toroidal field
  - Toroidal field decays via Tayler instability, creating new poloidal field
  - New poloidal field is wound up by differential rotation
- Differential rotation will eventually disappear unless it is continuously replenished
- This can be investigated by means of numerical MHD

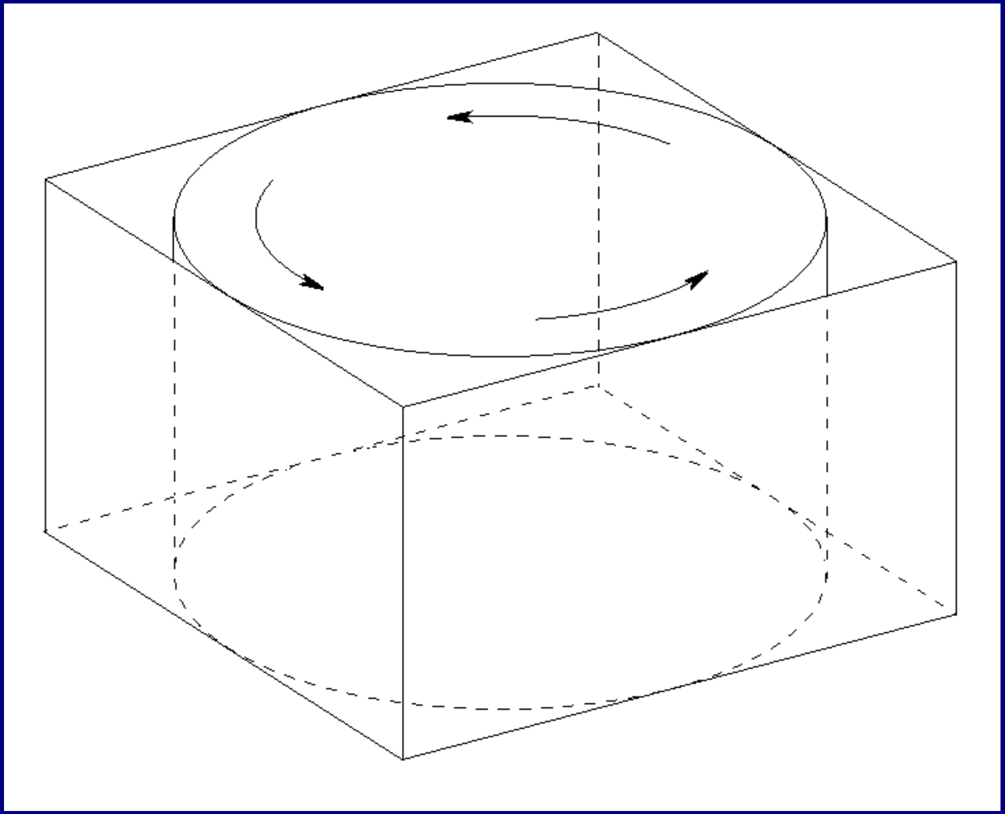
The model

Gas in box made to rotate by applying a force

$$\boldsymbol{F}(\varpi, z) = (\boldsymbol{v_0} - \boldsymbol{v}) / \tau_f$$

$$\boldsymbol{v_0} = (\Omega_0 + \frac{d\,\Omega}{dz} z)\,\varpi\,\boldsymbol{e_\phi}$$

Begin with a weak seed field

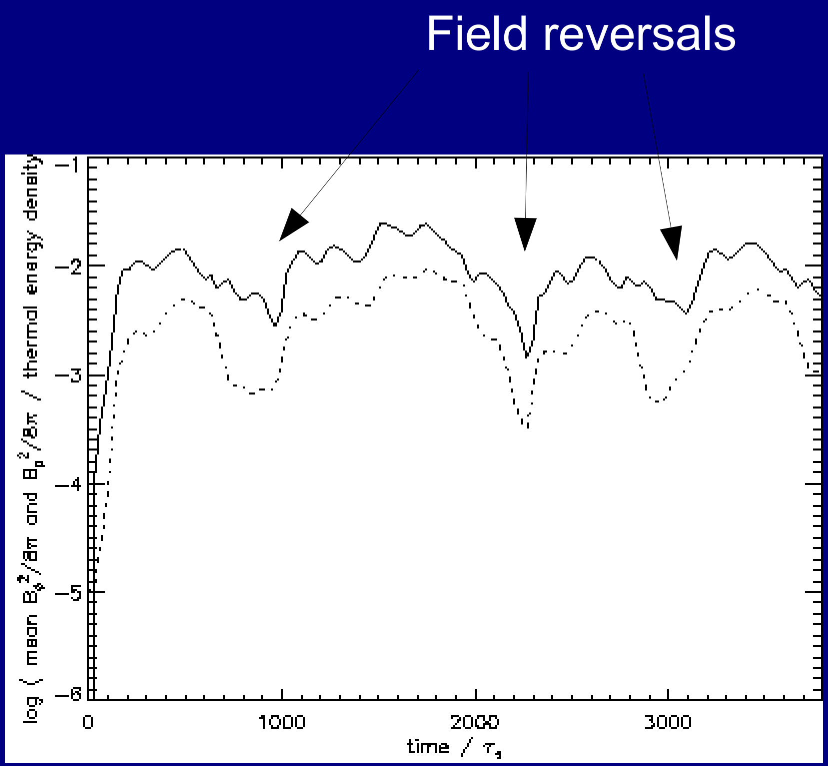


Results

(with Ω_0 = 0 and g = 0)

At first, the toroidal component grows from its zero initial value until it is stronger than the poloidal field.

Then the poloidal field strength starts to increase. Both increase until saturation is reached.
A self-sustaining field has been created.



Log energy density in toroidal (solid line) and poloidal (dashed line) components, against time

Some applications of dynamo process

- Origin of solid-body rotation in solar core and other non-convective stars (e.g. Charbonneau et al. 1999)
- Core-envelope coupling in AGB stars and supernova progenitors: rotation rates of white dwarfs and neutron stars (e.g. Heger et al. 2005, Yoon & Langer 2005)
- Gamma-Ray Bursts/core-collapse supernovae: are hypermassive neutron stars formed? If so, by what process is the differential rotation damped, and how long does that take?
- Differential rotation during x-ray bursts on accreting neutron stars (e.g. Payne & Melatos 2007)

Conclusions

- Simulations of Tayler instability confirm some analytic predictions:
  - long-wavelength limit from stratification
  - effect on short-wavelengths from magnetic diffusivity
  - effect of rotation
- Differential-rotation-driven dynamo (aka Tayler-Spruit dynamo) seen in simulations, but details uncertain.. more work required!