

Can inflating braneworlds be stabilized?

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We investigate scalar perturbations from inflation in braneworld cosmologies with extra dimensions. For this we calculate scalar metric fluctuations around five dimensional warped geometry with four dimensional de Sitter slices. The background metric is determined self-consistently by the (arbitrary) bulk scalar field potential, supplemented by the boundary conditions at both orbifold branes. Assuming that the inflating branes are stabilized (by the brane scalar field potentials), we estimate the lowest eigenvalue of the scalar fluctuations—the radion mass. In the limit of flat branes, we reproduce well known estimates of the positive radion mass for stabilized branes. Surprisingly, however, we found that for de Sitter (inflating) branes the square of the radion mass is typically negative, which leads to a strong tachyonic instability. Thus, parameters of stabilized inflating braneworlds must be constrained to avoid this tachyonic instability. Instability of “stabilized” de Sitter branes is confirmed by the BRANECODE numerical calculations of Martin, Felder, Frolov, Peloso, and Kofman. If the model’s parameters are such that the radion mass is smaller than the Hubble parameter, we encounter a new mechanism of generation of primordial scalar fluctuations, which have a scale free spectrum and acceptable amplitude.

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I. INTRODUCTION

One of the most interesting recent developments in high energy physics has been the picture of braneworlds. Higher-dimensional formulations of braneworld models in superstring or M theory, supergravity and phenomenological models of the mass hierarchy have the most obvious relevance to cosmology. In application to the very early Universe this leads to braneworld cosmology, where our (3+1)-dimensional universe is a 3D curved brane embedded in a higher-dimensional bulk [2]. Early Universe inflation in this picture corresponds to 3+1 (quasi) de Sitter brane geometry, so that the background geometry is simply described by the five-dimensional warped metric with four-dimensional de Sitter slices:

$$ds^2 = a^2(w)[dw^2 - dt^2 + e^{2Ht}d\vec{x}^2]. \quad (1)$$

For simplicity we use spatially flat slicing of the de Sitter metric ds_4^2 . The conformal warp factor $a(w)$ is determined self-consistently by the five-dimensional Einstein equations, supplemented by the boundary conditions at two orbifold branes. We assume the presence of a single bulk scalar field φ with the potential $V(\varphi)$ and self-interaction potentials $U_{\pm}(\varphi)$ at the branes. The potentials can be pretty much arbitrary as long as the phenomenology of the braneworld is acceptable. The class of metrics (1) with bulk scalars and two orbifold branes covers many interesting braneworld scenarios including the Hořava-Witten theory [3,4], the Randall-Sundrum model [5,6] with phenomenological stabilization of branes [7,8], supergravity with domain walls, and others [9,10].

We will consider models where by the choice of the bulk and brane potentials the interbrane separation (the so-called radion) can be fixed, i.e. models in which branes could in principle be stabilized. The theory of scalar fluctuations around flat stabilized branes, involving bulk scalar field fluctuations $\delta\varphi$, scalar 5D metric fluctuations and brane displacements, is well understood [11]. Similar to Kaluza-Klein (KK) theories, the extra-dimensional dependence can be separated out, and the problem is reduced to finding the eigenvalues of a second-order differential equation for the extra-dimensional (w -dependent) part of the fluctuation eigenfunctions subject to the boundary conditions at the branes. The lowest eigenvalue corresponds to the radion mass, which is positive $m^2 > 0$ and exceeds the TeV scale or so [12]. Tensor fluctuations around flat stabilized branes are also stable.

Brane inflation, like all inflationary models, generates long wavelength cosmological perturbations from the vacuum fluctuations of all light (i.e. with mass less than the Hubble parameter H) degrees of freedom. The theory of metric fluctuations around the background geometry (1) with inflating (de Sitter) branes is more complicated than that for the flat branes. For tensor fluctuations (gravitational waves), the lowest eigenvalue of the extra-dimensional part of the tensor eigenfunction is zero, $m = 0$, which corresponds to the usual 4D graviton. As it was shown in [21,22], massive KK gravitons have a gap in the spectrum; the universal lower bound on the mass is $m \geq \sqrt{\frac{3}{2}} H$. This means that massive KK tensor modes are not generated from brane inflation. Massless scalar and vector projections of the bulk gravitons are absent, so only the massless 4D tensor mode is generated.

Scalar cosmological fluctuations from inflation in the braneworld setting (1) have been considered in many important works [13–20]. The theory of scalar perturbations in braneworld inflation with bulk scalars is even more complicated than for tensor perturbations. This is because one has

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to consider 5D scalar metric fluctuations and brane displacements induced not only by the bulk scalar field fluctuations $\delta\varphi$, but also by the fluctuations $\delta\chi$ of the inflaton scalar field χ living at the brane. In fact, most papers on scalar perturbations from brane inflation concentrated mainly on the inflaton fluctuations $\delta\chi$, while the bulk scalar fluctuations were not included. This was partly because in the earlier papers on brane inflation people considered a single brane embedded in an AdS background without a bulk scalar field, and partly because for braneworlds with two stabilized branes there was an expectation that the fluctuations of the bulk scalar would be massive and thus would not be excited during inflation.

In this paper we focus on the bulk scalar field fluctuations, assuming for the sake of simplicity that the inflaton fluctuations $\delta\chi$ are subdominant. We consider a relatively simple problem of scalar fluctuations around curved (de Sitter) branes, involving only bulk scalar field fluctuations $\delta\varphi$. We find the extra-dimensional eigenvalues of the scalar fluctuations subject to boundary conditions at the branes, focusing especially on the radion mass m^2 for the inflating branes. In particular, we investigate the presence or absence of a gap in the KK spectrum of scalar fluctuations in view of the tensor mode result. Our results are a generalization of the known results for flat stabilized branes [11], which we reproduce in the limit where the branes are flattening $H \rightarrow 0$.

II. BULK EQUATIONS

The five-dimensional braneworld models with a scalar field in the bulk are described by the action

$$S = M_5^3 \int \sqrt{-g} d^5x \{ R - (\nabla\varphi)^2 - 2V(\varphi) \} - 2M_5^3 \sum \int \sqrt{-q} d^4x \{ [\mathcal{K}] + U(\varphi) \}, \quad (2)$$

where the first term corresponds to the bulk and the sum contains contributions from each brane. The jump of the extrinsic curvature $[\mathcal{K}]$ provides the junction conditions across the branes [see Eq. (11) below]. Variation of this action gives the bulk Einstein $G_{AB} = T_{AB}(\varphi)$ and scalar field $\square\varphi = V_{,\varphi}$ equations. For the (stationary) warped geometry (1) they are

$$\varphi'' + 3\frac{a'}{a}\varphi' - a^2V' = 0, \quad (3a)$$

$$\frac{a''}{a} = 2\frac{a'^2}{a^2} - H^2 - \frac{\varphi'^2}{3}, \quad (3b)$$

$$6\left(\frac{a'^2}{a^2} - H^2\right) = \frac{\varphi'^2}{2} - a^2V, \quad (3c)$$

where the prime denotes the derivative with respect to the extra dimension coordinate w . The first two equations are dynamical, and the last is a constraint. The solutions of Eqs. (3) were investigated in detail in [10].

Now we consider scalar fluctuations around the background (1). The perturbed metric can be written in the longitudinal gauge as

$$ds^2 = a(w)^2[(1 + 2\Phi)dw^2 + (1 + 2\Psi)ds_4^2]. \quad (4)$$

The linearized bulk Einstein equations and scalar field equation relate two gravitational potentials $\Phi(x^A)$, $\Psi(x^A)$ and bulk scalar field fluctuations $\delta\varphi(x^A)$. The off-diagonal Einstein equations require that

$$\Psi = -\frac{\Phi}{2}, \quad (5)$$

similar to four-dimensional cosmology, although the coefficient is different.

The symmetry of the background guarantees separation of variables, so that perturbations can be decomposed with respect to four-dimensional scalar harmonics, e.g.

$$\Phi(x^A) = \sum_m \Phi_m(w) Q_m(t, \vec{x}), \quad (6)$$

where the eigenvalues m (constant of separation) appear as the four-dimensional masses ${}^4\square Q_m = m^2 Q_m$, where ${}^4\square$ is the D'Alembert operator on the 4D de Sitter slice. The four-dimensional massive scalar harmonics Q_m can be further decomposed as $Q_m(t, \vec{x}) = \int f_k^{(m)}(t) e^{i\vec{k}\vec{x}} d^3k$. The temporal mode functions $f_k^{(m)}(t)$ obey the equation

$$\ddot{f} + 3H\dot{f} + (e^{-2Ht}k^2 + m^2)f = 0, \quad (7)$$

where dot denotes time derivative, and we dropped the labels k and m for brevity.

Out of the remaining linearized Einstein equation we get the following equations for the extra-dimensional mode functions $\Phi_m(w)$ and $\delta\varphi_m(w)$:

$$(a^2\Phi)' = \frac{2}{3}a^2\varphi'\delta\varphi, \quad (8a)$$

$$\left(\frac{a}{\varphi'}\delta\varphi\right)' = \left(1 - \frac{3}{2}\frac{m^2 + 4H^2}{\varphi'^2}\right)a\Phi, \quad (8b)$$

where we again omitted the label m for transparency.

These are very similar to the scalar perturbation equations in four-dimensional cosmology with a scalar field [23], except for some numerical coefficients and powers of $a(w)$ (because the spacetime dimensionality is higher), and up to time to extra spatial dimension exchange. Indeed, we can introduce the higher-dimensional analog of the Mukhanov's variable. However, in the presence of the curvature term H^2 , the eigenvalue m^2 enters the second order equation for it in a complicated way, similar to that in the 4D problem with nonzero spatial curvature, see e.g. [24]. We can introduce another convenient variable $u_m = \sqrt{3/2}(a^{3/2}/\varphi)\Phi_m$. Then the two first order differential equations (8) can be combined into a single Schrödinger-type equation

$$u_m'' + [m^2 + 4H^2 - V_{\text{eff}}(w)]u_m = 0 \quad (9)$$

with the effective potential $V_{\text{eff}} = (z''/z) + \frac{2}{3}\varphi'^2$, where we defined $z = (\frac{2}{3}a\varphi'^2)^{-1/2}$.

There are two main differences relative to the four-dimensional cosmology. First, in the latter case, FRW geometry with *flat* 3D spatial slices is usually considered, while the five-dimensional brane inflation metric has *curved* 4D slices, which results in extra terms like $4H^2$ in Eq. (9). Second, here we are dealing not with an *initial* but a *boundary* value problem, with associated boundary conditions for perturbations at the branes on the edges. After we derive the boundary conditions, we will calculate the KK spectrum of the eigenvalues m .

III. BRANE EMBEDDING AND BOUNDARY CONDITIONS

The embedding of each brane is described by $w = w_{\pm} + \xi_{\pm}(x^a)$, where ξ_{\pm} is the transverse displacement of the perturbed brane and w_{\pm} is the position of the unperturbed brane. Holonomic basis vectors along the brane surface are $e_{(a)}^A \equiv \partial x^A / \partial x^a = (\xi_{,a}, \delta_a^A)$, while the unit normal to the brane is $n_A = a(1 + \Phi, -\xi_{,a} \delta_A^a)$. The induced four-metric on the brane $d\sigma^2 = q_{ab} dx^a dx^b$ does not feel the brane displacement (to linear order) and is conformally flat

$$d\sigma^2 = a^2(1 - \Phi)[-dt^2 + e^{2Ht} d\vec{x}^2]. \quad (10)$$

The junction conditions for the metric and the scalar field at the brane are

$$[\mathcal{K}_{ab} - \mathcal{K}q_{ab}] = U(\varphi)q_{ab}, \quad [n \cdot \nabla \varphi] = \frac{\partial U}{\partial \varphi}, \quad (11)$$

where the extrinsic curvature is defined by $\mathcal{K}_{ab} = e_{(a)}^A e_{(b)}^B n_{A;B}$. We will only need its trace, which up to linear order in perturbations is

$$\mathcal{K} = 4\frac{a'}{a^2} - 2\frac{(a^2\Phi)'}{a^3} - \frac{4\Box\xi}{a}. \quad (12)$$

For the background geometry (under the assumption of reflection symmetry across the branes), Eqs. (11) reduce to

$$\frac{a'}{a^2} = \mp \frac{U}{6}, \quad \frac{\varphi'}{a} = \pm \frac{U'}{2}. \quad (13)$$

For the perturbed geometry, the traceless part of the extrinsic curvature must vanish in the absence of matter perturbations on the brane. Since it contains second cross-derivatives of ξ , the brane displacement ξ is severely restricted. Basically, this means that the oscillatory modes of brane displacement are not excited without matter support at the brane. While there could possibly be global displacements of the brane, they do not interest us, so in the following we set $\xi = 0$. Of course, for the more complete problem which includes fluctuations $\delta\chi$ of the ‘‘inflaton’’ field on the brane, the displacement ξ does not vanish.

Using expression (12) for the trace of the extrinsic curvature, the first of Eqs. (11) gives us the junction condition for linearized perturbations at the two branes $(a^2\Phi)'|_{w_{\pm}} = \pm \frac{1}{3}U'a^3\delta\varphi|_{w_{\pm}}$. However, this junction condition does not really place any further restrictions on the bulk field perturbations, as it identically follows from the bulk perturbation equations (8) and the background junction condition (13). Rather, this junction condition would relate the brane displacement ξ to the matter perturbations on the brane if they were not absent.

The second of Eqs. (11) gives us a physically relevant boundary condition for the bulk field perturbations

$$(\delta\varphi' - \varphi'\Phi)|_{w_{\pm}} = \pm \frac{1}{2}U''a\delta\varphi|_{w_{\pm}}. \quad (14)$$

Using the bulk equations (8), this can be rewritten in a more suggestive form

$$\left(\frac{a}{\varphi'}\delta\varphi\right)|_{w_{\pm}} = \frac{3}{2}\frac{m^2 + 4H^2}{a\varphi'^2} \frac{a^2\Phi}{\frac{a^2V'}{\varphi'} - 4\frac{a'}{a} \mp aU''}|_{w_{\pm}}. \quad (15)$$

The eigenvalues m^2 of bulk perturbation equations subject to the boundary condition (15) form a KK spectrum, which we find numerically. We considered several examples of the potentials V and U_{\pm} , and found no universal positive mass gap. Moreover, for the most interesting models we found negative m^2 .

To understand the KK spectrum of m^2 , we make a simplification of the boundary condition (15) which will allow us to treat the eigenvalue problem analytically, and which well corresponds to a spirit of brane stabilization [7]. Indeed, *rigid stabilization* of branes is thought to be achieved by taking U'' (i.e. the brane mass of the field) very large, so that the scalar field gets pinned down at the positions of the branes. In this case, the right hand side of Eq. (15) becomes very small, which leads to the boundary condition

$$\delta\varphi|_{w_{\pm}} = 0. \quad (16)$$

This by itself *does not guarantee stability*, or vanishing of the metric perturbations on the brane for that matter, as perturbations live in the bulk and only need to satisfy Eq. (16) on the branes. This poses an eigenvalue problem for the mass spectrum of the perturbation modes, which we study next.

IV. KK MASS SPECTRUM

Unlike the situation with gravitational waves [22], for the scalar perturbations there is no zero mode with $m=0$, nor is there a ‘‘supersymmetric’’ factorized form of the ‘‘Schrödinger’’-like Eq. (9). To find the lowest mass eigenvalue, we have to use other ideas. Powerful methods for analyzing eigenvalue problems exist for normal self-adjoint systems [25]. To use them, we transform our eigenvalue

problem (8) and (16) into the self-adjoint form. While the second order differential equation (9) is self-adjoint, the boundary conditions for u are not. Therefore, we introduce a new variable $Y = u/z = a^2\Phi$ and impose the boundary conditions (16) to obtain the boundary value problem

$$DY \equiv -(gY')' + fY = \lambda gY, \quad (17a)$$

$$Y'(w_{\pm}) = 0, \quad (17b)$$

where we have introduced the short-hand notation $f = 1/a$, $g = z^2 = (\frac{2}{3}a\varphi'^2)^{-1}$, and $\lambda = m^2 + 4H^2$. Since the boundary value problem (17) is self-adjoint, it is guaranteed that the eigenvalues λ are real and non-negative, $\lambda \geq 0$. To estimate the lowest eigenvalue λ_1 of the eigenvalue problem (17), we apply the Rayleigh's formula [25], which places a rigorous upper bound on λ_1

$$\lambda_1 \leq \frac{\int FDFdw}{\int gF^2dw}, \quad (18)$$

where F can be *any* function satisfying the boundary conditions (17b), and does not have to be a solution of Eq. (17a). Taking a trial function $F = 1$, we have

$$\lambda_1 \leq \frac{\int fdw}{\int gdw}. \quad (19)$$

This bound on the lowest mass eigenvalue is our main result:

$$m^2 \leq -4H^2 + \frac{2}{3} \frac{\int \frac{dw}{a}}{\int \frac{dw}{a\varphi'^2}}. \quad (20)$$

In practice, $F = 1$ is a pretty good guess for the lowest eigenfunction, so the bound (20) is usually close to saturation (up to a few percent accuracy in some cases), as we have observed in direct computations using a numerical eigenvalue finder.

The right hand side of Eq. (20) has the structure $-4H^2 + m_0^2(H)$, where the second term is a functional of H (including the implicit H -dependence of the warp factor a). In the limit of flat branes $H \rightarrow 0$ we have only the second, positive term. In this limit our expression agrees with the estimation of the radion mass m_0^2 for flat branes, obtained in various approximations [11,12,20]. A nonvanishing H alters m^2 through both terms. The most drastic alteration of m^2 due to H comes from the big negative term $-4H^2$. For the particular case of two de Sitter branes embedded in 5D AdS without a bulk scalar this negative term was noticed in [19].

V. TACHYONIC INSTABILITY OF THE RADION FOR INFLATING BRANES

The most striking feature of the mass bound (20) is that m^2 for de Sitter branes is typically negative. Trying, for instance, to do Goldberger-Wise stabilization of braneworlds with inflating branes while taking bulk gradients φ'^2 small enough to ignore their backreaction (as it is commonly done for flat branes) is a sure way to get a tachyonic radion mass: an estimate of the integrals gives $m^2 \leq -4H^2 + O(\varphi'^2)$, which will go negative if the bulk scalar field is negligible $\varphi'^2 \ll H^2$.

In what follows we consider two situations. In this section, we consider braneworld models where m^2 is negative and mostly defined by the first term $-4H^2$ in Eq. (20). In the next section, we consider the case where both terms in Eq. (20) are tuned to be comparable and the net radion mass is smaller than the Hubble parameter $|m^2| \leq H^2$. In the last section we will discuss how these two cases may be dynamically connected.

Suppose we start with a braneworld with curved de Sitter branes, and we find the mass squared of the radion to be negative. The extra-dimensional eigenfunction $\Phi_m(w)$ is regular in the interval $w_- \leq w \leq w_+$. Let us turn, however, to the four-dimensional eigenfunction $Q_m(t, \vec{x})$. Bearing in mind the evolution of the quantum fluctuations of the bulk field, we choose the positive frequency vacuum-like initial conditions in the far past $t \rightarrow -\infty$, $f_k(t) \simeq (1/\sqrt{2k})e^{ik\eta}$, $\eta = \int dt e^{-Ht}$. For the tachyonic mode $m^2 < 0$ the solution to Eq. (7) with this initial condition is given in terms of Hankel functions $f_k^{(m)}(\eta) = (\sqrt{\pi}/2)H|\eta|^{3/2}\mathcal{H}_\mu^{(1)}(k\eta)$, with the index $\mu = \sqrt{\frac{9}{4} + |m^2|/H^2}$. The late-time asymptotic of this solution diverges exponentially as $t \rightarrow \infty$ ($\eta \rightarrow 0$)

$$f_k^{(m)}(t) \propto \exp\left[\left(\sqrt{\frac{9}{4} + \frac{|m^2|}{H^2}} - \frac{3}{2}\right)Ht\right]. \quad (21)$$

Thus the negative tachyon mass of the radion $|m^2| \sim 4H^2$ leads to a strong exponential instability of scalar fluctuations $\Phi \propto e^{Ht}$. This instability is observed using a completely different method in the accompanying BRANECODE paper [1], where we give a fully nonlinear numerical treatment of inflating branes which were initially set to be stationary by the potentials $U_{\pm}(\varphi)$, and without any simplifications like approximating boundary condition (15) with (16).

Tachyonic instability of the radion for inflating branes means that, in general, *braneworlds with inflation are hard to stabilize*. From the point of view of 4D effective theory one would expect brane stabilization at energies lower than the mass of the flat brane radion m_0^2 , which is roughly equal to the second term in Eq. (20). If the energy scale of inflation H is larger than m_0 , $H^2 \gg m_0^2$, this expectation is incorrect.

Successful inflation (lasting more than $65H^{-1}$) requires the radion mass m^2 to be not too negative

$$m^2 \gtrsim -\frac{H^2}{20}. \quad (22)$$

This is possible if both terms in Eq. (20) are of the same order. In the popular braneworld models the radion mass in the low energy limit, m_0 , is of order of a TeV. For these models the scale of “stable” inflation would be the same order of magnitude, $H \sim \text{TeV}$. Although there is no evidence that this scale of inflation is too low, it is not a comfortable scale from the point of view of the theory of primordial perturbations from inflation.

It is interesting to note that the system of curved branes may dynamically reconfigure itself to reach a state where the condition (22) is satisfied. In the case of the bulk scalar field φ acting alone, for quadratic potentials U_{\pm} suitable for brane stabilization, there may be two stationary warped geometry solutions (1) with two different values of H . The solution with the larger Hubble parameter H might be dynamically unstable due to the tachyonic instability of the radion, which we described above. The second solution with the lower H which satisfies Eq. (22) might be stable. A fully nonlinear study of this model was performed numerically with the BRANECODE and is reported in the accompanying paper [1]. It shows that, indeed, the tachyonic instability violently reconfigures the starting brane state with the larger H into the stable brane state with the lower H . This reconfiguration of the brane system has a spirit of the Higgs mechanism.

If we add an “inflaton” scalar field χ located at the brane, its slow roll contributes to the decrease of H .

Thus, for the “stable” brane we have a radion mass (22). This condition includes the case when the radion is lighter than H , $|m^2| < H^2$. Even if the radion tachyonic instability is avoided, the light radion leads us to the other side of the story, a new mechanism of generation of scalar fluctuations from inflation associated with the radion.

VI. INDUCED SCALAR METRIC PERTURBATIONS AT THE OBSERVABLE BRANE

Suppose that the radion mass is smaller than H , $|m^2| \ll H^2$, so that from Eq. (21) we get the amplitude of the temporal mode function $f_k^{(m)}(t)$ in the late time asymptotic frozen at the level $f_k^{(m)}(t) \simeq H/(\sqrt{2}k^{3/2})$. This is nothing but the familiar generation of inhomogeneities of a light scalar field from its quantum fluctuations during inflation. Therefore an observer at the observable brane will encounter long wavelength scalar metric fluctuations generated from braneworld inflation.

The four-dimensional metric describing scalar fluctuations around an inflating background is usually written as

$$d\sigma^2 = -(1 + 2\tilde{\Phi})d\tilde{t}^2 + (1 - 2\tilde{\Psi})e^{2\tilde{H}\tilde{t}}d\tilde{x}^2, \quad (23)$$

where $\tilde{\Phi}$ and $\tilde{\Psi}$ are scalar metric fluctuations. The induced four-metric on the brane (10) in our problem can be rewritten in this standard form (23) if we absorb the (constant) warp factor $a(w_+)$ in the redefined time $\tilde{t} = at$ and spatial coordinates $\tilde{x} = a\vec{x}$ and rescale the Hubble parameter $\tilde{H} = H/a$. Then we see that the induced scalar perturbations on the brane are

$$\tilde{\Psi} = -\tilde{\Phi} = \frac{1}{2}\Phi. \quad (24)$$

The sign of the first equality here is opposite to what we usually have for $(3+1)$ -dimensional inflation with a scalar field. It implies that the 4D Weyl tensor of the induced metric vanishes, as the induced fluctuations are conformally flat. The conformal structure of fluctuations (24) is typical [28] for a R^2 inflation in the Starobinsky model [26]. It is not a surprise, because for the scale of inflation comparable to the mass m_0 of the flat brane radion we expect higher derivative corrections to the 4D effective gravity on the brane. Indeed, the massive radion corresponds to a higher derivative 4D gravity [20].

The amplitude and spectrum of induced fluctuations are defined by $\tilde{\Phi}$. From the mode decomposition (6) we get

$$k^{3/2}\tilde{\Phi}_k \simeq \Phi_m(w_+) \frac{H}{M_4}, \quad (25)$$

where $\Phi_m(w_+)$ is the amplitude of the extra-dimensional eigenmode at the observable brane, normalized in such a way that the fluctuations $\Phi(w, t, \vec{x})$ are canonically quantized on the 4D slice, namely $M_4^3 \int_{\frac{3}{2}}^{\frac{3}{2}} (a^3/\varphi'^2) |\Phi_m(w)|^2 dw = 1$. The normalization M_4 of the 4D mode functions follows from canonical quantization of the perturbed action (2); the usual 4D Planck mass M_p is expected to be recovered in the effective field theory on the observable brane [11].

The scalar metric fluctuations induced by the bulk scalar field fluctuations are scale-free and have the amplitude $k^{3/2}\tilde{\Phi}_k \propto H/M_p$, with the numerical coefficient depending on the details of the warped geometry. The nature of these fluctuations is very different from those in $(3+1)$ -dimensional inflation, where the inflaton scalar field is time dependent. Induced scalar fluctuations do not require “slow-roll” properties of the potentials V and U_{\pm} . The underlying background bulk scalar field has no time-dependence, but only y dependence. Thus, generation of induced scalar metric fluctuations from braneworld inflation is a new mechanism for producing cosmological inhomogeneities.

If we add another, inflaton field χ localized at the brane, we should expect that fluctuations of both fields, the bulk scalar $\delta\varphi$ and the inflaton $\delta\chi$, contribute to the metric perturbations. We can conjecture that the net fluctuations will be similar to those derived in the combined model with R^2 gravity and a scalar field [27].

VII. DISCUSSION

Let us discuss the physical interpretation and the meaning of our result. Stabilization of flat branes is based on the balance between the gradient ϕ' of the bulk scalar field and the brane potentials $U(\phi)$ which keeps ϕ pinned down to its values ϕ_i at the branes. The interplay between different forces becomes more delicate if the branes are curved. The warped configuration of curved branes has the lowest eigenvalue for scalar fluctuations around it,

$$m^2 = -4H^2 + m_0^2(H). \quad (26)$$

The term $m_0^2(H)$ is a functional of H , and depends on the parameters of the model. If parameters are such that m^2 becomes negative due to excessive curvature $\sim H^2$, the brane configuration becomes unstable. This is analogous to an instability of a simple elastic mechanical system supported by the balance of opposite forces, which arises for a certain range of the underlying parameters.

Tachyonic instability of curved branes has serious implications for the theory of inflation in braneworlds. It may not be so easy to have a realization of inflation in the braneworld picture without taking care of parameters of the model. Inflation where m^2 in Eq. (26) is negative and $|m^2|$ is larger than H^2 is a short-lived stage because of this instability. After inflation, the late time evolution should bring the brane configuration to (almost) flat stabilized branes in the low energy limit. This by itself requires fine tuning of the potentials V and U_{\pm} to provide stabilization. Stabilization at the inflation energy scale requires extra fine tuning to get rid of the tachyonic effect.

Working with a single bulk scalar field, it is probably not easy to simultaneously achieve stabilization not only at low energy, but also at the high energy scale of inflation, to insure that $|m^2| \ll H^2$, and to provide a graceful exit from inflation. One may expect that introduction of another scalar field χ on the brane can help to have stabilization both at the scale of inflation and in the low energy limit. If we can achieve brane stabilization during inflation by suppression of the tachyonic instability, we encounter a byproduct effect. Light modes of radion fluctuations inevitably contribute to the induced scalar metric perturbations. Therefore the theory of braneworld inflation has an additional mechanism of generation of primordial cosmological perturbations. This new mechanism is different from that of the usual 4D slow roll inflation.

It appears that one of the most interesting potential applications of our effect is a mechanism for reducing the 4D

effective cosmological constant at the brane. Indeed, in terms of brane geometry, the 4D cosmological constant is related to the 4D curvature of the brane. Suppose we have two solutions of the background equations (3) with higher and lower values of the curvature of de Sitter brane, which is proportional to H^2 . (The existence of two solutions for certain choices of parameters of the Goldberger-Wise type potentials used for brane stabilization can be demonstrated, see [1,10].) Suppose that the solution with the larger value of brane curvature is unstable. Then the brane configuration will violently restructure into the other static configuration, which is characterized by the lower value of brane curvature where the tachyonic instability is absent. The branes are flattening, which for a 4D observer means the lowering of the cosmological constant. It will be interesting to investigate how this mechanism works for brane configurations with several scalar fields or potentials which can admit more than two static solutions.

The problem of the cosmological constant from a braneworld perspective (as a flat brane) was discussed in the literature. There was a suggestion that the flat brane is a special solution of the bulk gravity/dilaton system with a single brane [29,30], a claim which was later dismissed [31]. In our setup, we consider two branes in order to screen the naked bulk singularity, which was one of the factors spoiling the models [29,30]. The new element which emerges from our study is the instability of the curved branes.

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