Shining Through the Darkness: detecting dark matter with stellar astrometry

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Overview

- I. Our clumpy dark matter halo What do numerical simulations tell us about substructure?
- II. Astrometric microlensing by subhalos What happens when a subhalo passes between us and a star?
- III. High-precision astrometry Can we measure stellar separations in microarcseconds?
- IV. Cross sections, event rates, and detection prospects How close does the star need to be to the subhalo center? What hope do we have of observing these events?

Simulated Dark Matter Halos

Via Lactea II





Diemand et al. 2008

One halo with I.I billion particles
 4,100 Msun per particle



- One halo with 4.2 billion particles
- I,712 Msun per particle
- Five additional halos with lower res.



Springel et al. 2008

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Subhalos are Gravitational Lenses

When galaxies produce multiple images of a quasar; subhalos can modify the properties of these images.

- subhalos magnify one image, leading to flux-ratio anomalies. Mao & Schneider 1998; Metcalf & Madau 2001; Chiba 2002; Dalal & Kochanek 2002
- subhalos alter the time delays between images

Keeton& Moustakas 2009; Congdon et al. 2010

subhalos deflect one image

Koopmans et al. 2002; Chen et al. 2007; Williams et al. 2008; More et al. 2009

subhalos can split one image into two

Yonehara et al. 2003; Inoue & Chiba 2005; Zackrisson et al. 2008; Riehm et al. 2009

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Only the last possibility can detect individual subhalos, and it's unlikely. Astrometric lensing is more promising:

- split images are hard to resolve; changes in image position are much easier
- Iarger impact parameters can give detectable image deflections
- we're looking for a dynamical signature from a local subhalo

Astrometric Microlensing



• thin lens approximation: lens thickness is small compared to $d_{\rm L}, d_{\rm S}$ • weak lensing: one image with $\alpha \ll \beta$ so that $\xi \simeq d_{\rm L}\beta$

• thin lens equation: $\vec{\alpha} = \frac{d_{\rm LS}}{d_{\rm S}} \left[\frac{4GM_{\rm 2D}(\xi)}{\xi} \right] \hat{\xi}$

Deflection Angle $\propto \frac{\text{Projected mass enclosed}}{\text{Distance from lens center}}$

Lens: Singular Isothermal Sphere

Density profile: $\rho(r) \propto r^{-2}$ Constant Projected mass enclosed: $M_{2D}(r) \propto r$ Deflection Angle!

Lens: Singular Isothermal Sphere



Lens: Singular Isothermal Sphere



If the SIS is infinite, the star traces a semi-circle on the sky.

Constant

The radius of the semi-circle is the Einstein angle:

$$\theta_{\rm E}^{\rm SIS} = 10\,\mu{\rm as}\,\left(\frac{\sigma_v}{0.6\,{\rm km/s}}\right)^2 \left(1-\frac{d_{\rm L}}{d_{\rm S}}\right)$$

- Truncating the SIS completes the image path.
- Larger impact parameter: more circular image path, slower image movement.



Singular Isothermal Sphere



Singular Isothermal Sphere

Star field over 4 years



We need a (projected) close encounter between the star and the subhalo center.

Singular Isothermal Sphere

Star field over 4 years



Lens mass: 5 Msun Lens radius: 0.02 pc (85'') Lens distance: 50 pc

We need a (projected) close encounter between the star and the subhalo center. • the subhalo center must pass

- within 0.03 pc of the star
- at these small impact parameters, only the innermost region of the subhalo affects the lensing
- we only need to know the density profile within 0.1 pc of the subhalo center
- the truncation of the subhalo is not important; only the mass enclosed in the inner 0.1 pc matters

Unfortunately, even the best simulations can only probe the density profiles of the largest subhalos ($M_{
m sub} \gtrsim 10^8 M_{\odot}$), and the inner 350 pc are unresolved.

• Via Lactea II: $\rho(r) \propto r^{-(\gamma \simeq 1.24)}$ for large subhalos. Diemand et al. 2008 • Aquarius: $\rho(r) \propto r^{-(\gamma < 1.7)}$ for large subhalos. Springel et al. 2008

• Simulations of first halos: Earth-mass halos at a redshift of 26 have $\rho(r) \propto r^{-(1.5 < \gamma < 2.0)}$ extending to within 20 AU of the Diemand et al. 2005; Ishiyama et al. 2010

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We'll assume a "generalized NFW profile:"

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{r_0}\right)^{\gamma} \left(1 + \frac{r}{r_0}\right)^{3-\gamma}}$$

 r_0 is set by the concentration ho_0 is set by the virial mass

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 $ho(r) \propto r^{-\gamma}$

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We'll assume a "generalized NFW profile:"

For $\gamma < 2$, the deflection angle decreases as the star approaches the subhalo center!

 $r \ll r_0$ $ho(r) \propto r^{-\gamma}$ $lpha \propto r^{2-\gamma}$ deflection angle

 $r \gg r_0$ $ho(r) \propto r^{-3}$ $lpha \propto r^{-1}$

deflection angle

Lensing with a General Profile



The steepness of the density profile determines the shape of the image's path across the sky.

- Steeper profiles give more vertical deflection as the subhalo passes under the star.
- Steeper profiles give more rapid image motion.

Lens virial mass: $5 \times 10^5 M_{\odot}$ Concentration: $R_{\rm vir}/r_{-2} = 99$ Impact parameter: I arcsecond



Lensing with a $\rho(r) \propto r^{-1.5}$ Profile



Can we detect microarcsecond astrometric changes?

High-Precision Astrometry: Gaia

Gaia is an ESO satellite scheduled to launch in Nov. 2012

- designed for all-sky astrometry
- broad search for astrometric variability
- two Im-class telescopes imaging onto same detectors
- astrometric precision per epoch: ~35 microarcseconds for its brightest targets
- covers ~5,000,000 stars at this precision
- 83 epochs on average

High Precision Astrometry: SIM

SIM PlanetQuest was the top space mission recommended by NASA's Exoplanet Task Force.

- designed to find Earth-mass planets
 - 6m-baseline interferometer
 - astrometric precision per epoch:
 - ► **micro**arcsecond for planetfinding
 - ▶ 4 *micro*arcseconds for general
 - high-efficiency astrometry
 - capable of observing faint stars
- targeted mission with adjustable number of visits per star University of Waterloo: September 15, 2010

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Astrometry from the Ground

Without SIM, our best hope is to detect astrometric microlensing from the ground. It'll be difficult, but techniques are being developed to make it possible!

The statistical error:

$$\sigma_x \propto \frac{\rm FWHM}{\rm SNR}$$





- Focal plane distortion; characterize using crowded fields
- Atmospheric refraction; work in narrow bands in the near IR
- Changes in the instrument; guard your telescope
- Atmospheric turbulence; use adaptive optics, and be clever

Astrometry with Adaptive Optics



Adaptive Optics

Adaptive optics makes astrometry much easier:

reduces FWHM

enhances SNR by concentrating photons relative to background

provides more reference stars

Astrometry with Adaptive Optics

Unfortunately, adaptive optics has its limits.



AO makes corrections based on the light from a guide star (or laser).
Other stars are seen though different turbulence.
The result: random (but correlated) motion between stars.

Optimizing AO Astrometric Precision

The correlations between the residual stellar jitters can be used to minimize their impact on astrometric measurements!

Cameron, Britton and Kulkarni (2009) I) Make a vector from the target star to each reference star.

2) Apply weights to each vector to sum to the target position: $\vec{p} = \mathbf{W}\tilde{\mathbf{d}}$

3) Optimize the weights to minimize the covariance matrix for the target star's position: $\Sigma_{\vec{p}} = \mathbf{W}^T \Sigma_{\vec{d}} \mathbf{W}$



High Precision Astrometry!

Observing the Proper Motion of an M-dwarf



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Give me a bigger telescope...

Statistical Astrometric Precision with Large Telescopes



Systematic errors currently limit Keck to ~100uas precision, but efforts are ongoing...

Our Detection Strategy



The typical subhalo lensing event has three stages:

- I. The image barely moves when the subhalo center is approaching.
- 2. The image rapidly shifts position as the subhalo center passes by.
- 3. The image is nearly fixed at its new position as the subhalo center moves away.

This image motion is easily distinguished from lensing by a point mass; point masses give closed image trajectories.

Our Detection Strategy

To detect this image motion, we propose a simple strategy:



- I. Observe stars for a calibration period (2 years).
- 2. Reject stars that accelerate during the calibration period (including binaries).
- 3. Measure each star's proper motion and parallax, and predict its future trajectory.
- 4. Observe the star during the detection run (4 years).
- 5. Measure deviations from the predicted trajectory.

Star's true position is at the origin.

Subhalo center passes star two years into the detection run.

The Astrometric Signal for Lensing

Lensing Signal

 $\sum_{i=1}^{N_{epochs}} (Xm_i - Xp_i)^2 + (Ym_i - Yp_i)^2$

1 Magnitude squared of trajectory residuals

- The signal S measures the total displacement of the star from its expected position.
- SNR is S divided by the astrometric uncertainty per ID datapoint (including both the intrinsic uncertainty and the uncertainty in the star's predicted position).
- S depends linearly on the deflection angle.



Calculating the Lensing Signal



To evaluate the lensing signal, we want to separate the lensing geometry from the characteristics of the lens.

Geometric Coordinates:

$$\varphi \equiv \frac{\beta_{x,0}}{\theta_{\rm sub}}$$

impact parameter



Factorize the Deflection Angle:

$$\vec{\alpha}(t) \equiv \mathcal{F}(\gamma, M_{\text{vir}}, c, v_T, d_{\text{L}}, d_{\text{S}}, t_{\text{obs}}) \times \vec{\eta}(\gamma, \varphi, \tilde{\beta}, t/t_{\text{obs}}, \gamma)$$
depends on subhalo density profile, velocity, only depends on geometry distances to lens and source observation time and γ in $\rho(r) \propto r^{-\gamma}$

Calculating the Lensing Signal

Factorize the Deflection Angle:

 $\vec{\alpha}(t) \equiv \mathcal{F}(\gamma, M_{\text{vir}}, c, v_T, d_L, d_S, t_{\text{obs}}) \times \vec{\eta}(\gamma, \varphi, \tilde{\beta}, t/t_{\text{obs}},)$ Calculate the signal using $\vec{\alpha}$ $\vec{\eta}$ instead of $\vec{\alpha}$, call this the "geometric signal" S_g . Since the signal depends linearly on the deflection angle, $S = \mathcal{F} \times S_g$.



Two-year calibration run, four-year detection run.

Calculating Lensing Cross Sections

We define a lensing cross-section based on a minimum value for the lensing signal; all stars within this area will produce $S > S_{\min}$.





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10 Geometrical Area 0.1 $\gamma = 2.0$ $\gamma = 1.8$ $\gamma = 1.5$ $\gamma = 1.2$ 0.01 0.01 10 0.1 Minimum Geometrical Signal

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 $S_g > S_{g,\min}$

Calculating Lensing Cross Sections

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Lensing Cross Sections



Lens distance: 50 pc; Lens velocity: 200 km/s; Source Distance: 5 kpc

- The mass enclosed within 0.1 pc and the density profile completely determine the lensing cross section.
- For a given value for the minimum signal, there is a minimum subhalo mass that is capable of generating that signal.

Lensing Cross Sections



Lens distance: 50 pc; Lens velocity: 200 km/s; Source Distance: 5 kpc
We assume that the subhalo loses 99.9% of its virial mass due to tidal stripping.

- We use a concentration-mass relation for subhalos derived from the findings of the Aquarius simulations.
- Steeper dependence on γ : for a given virial mass, subhalos with shallower density profiles have less mass within 0.1 pc.

Lensing Event Rates Calculation

- We can combine the lensing cross sections with a subhalo mass function to calculate the fraction of the sky that is detectably lensed ($S > S_{\min}$) by a subhalo. • We assume that all the source stars are a fixed distance away: $A_{tot} \propto d_S$
- We assume that the subhalos are isotropically distributed.
- We can interpret this sky fraction as the probability that a given star is detectably lensed.

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Three candidate subhalo mass functions:

- All subhalos have the same mass, and a fraction f of the halo mass is contained within 0.1 pc of a subhalo center.
- All subhalos had the same virial mass, and all the dark matter was once in these subhalos.
- We use a local subhalo mass function derived from the Aquarius simulations, with $\frac{\mathrm{d}N}{\mathrm{d}M_{\mathrm{sub}}} \propto M_{\mathrm{sub}}^{-1.9}$ and $M_{\mathrm{sub}} = 0.01 M_{\mathrm{vir}}$.

Mono-Mass Lensing Event Rates

All subhalos have the same mass, and a fraction *f* of the halo mass is contained within 0.1 pc of a subhalo center



Lens velocity: 200 km/s; Source Distance: 2 kpc

- The number density of subhalos goes as $f imes M_{
 m sub}^{-1}$.
- The event rate peaks when the subhalos are just large enough to produce a sufficiently large lensing signal.
- Assuming that all dark matter was once in subhalos with the same viral mass is nearly the same as taking $f \simeq 10^{-4}$.

Event Rate from Aquarius



Springel et al. 2008

We derived a local subhalo mass function from the results of the Aquarius simulations.

- Aquarius gives a radial dependence for the number density of subhalos.
- We normalize our local mass function using Aquarius's subhalo mass function.

$$M_{\rm sub} = 0.01 M_{\rm vir}$$

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- We normalize our local mass function using Aquarius's subhalo

The minimum detectable signal depends on the instrument and the desired signal-to-noise ratio.

- SNR is the signal divided by the astrometric uncertainty per epoch.
- The total astrometric uncertainty includes the uncertainty in the predicted position of the star: $\sigma = 1.47\sigma_{\rm inst}$ for a 2 yr + 4 yr observation.
- The SNR should be sufficiently large to make false positives unlikely.

Instrument	$\sigma_{ m inst}$	# Stars	SNR	S_{\min}

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Subhalo Finder 2100	$0.5\mu{ m as}$	10^{11}	7	$5\mu{ m as}$

Detection with Targeted Observations

Finding a subhalo through astrometric microlensing is unlikely, but what if you know where to look?



Fermi may detect emission from dark matter annihilation in subhalos and could localize the center of emission down to a few sq. arcminutes.

Kuhlen et al. 2009



Lens distance: 50 pc; Lens velocity: 200 km/s; Source Distance: 5 kpc University of Waterloo: September 15, 2010

Summary

Local subhalos deflect the light from background stars, producing a unique astrometric microlensing signature.

- only the innermost 0.1 pc of the subhalo can produce a signal
- the star's apparent motion depends on the subhalo density profile
- the image deflection is measured in microarcseconds -- we can do that!

We can find these events by observing stars, predicting their motions, and then waiting for deviations.

we can calculate cross sections for lensing with a minimum signal
with a subhalo mass function, we can predict event rates

To see a subhalo lensing event, we'd have to get lucky!

- nearly impossible to find a subhalo through lensing, unless subhalos are more numerous and/or more concentrated than expected
- if Fermi points the way, high-precision astrometry can follow; we can detect subhalos within 100 pc of us with (stripped) masses $\gtrsim 1000 M_{\odot}$.