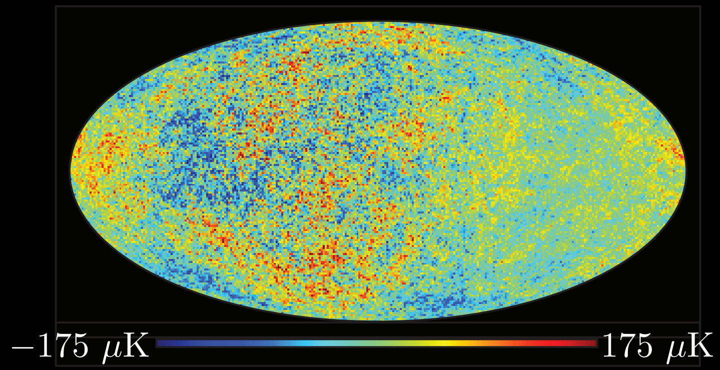
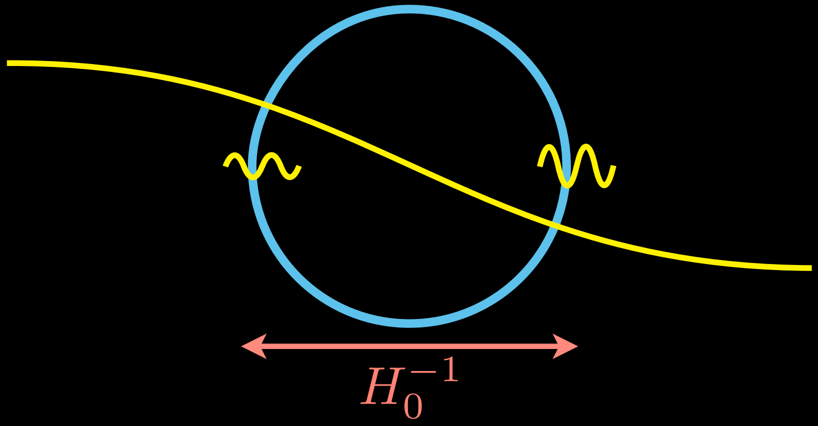
# Structure Beyond the Horizon:
## Inflationary Origins of the Cosmic Power Asymmetry





## Adrienne Erickcek
*California Institute of Technology*

*In collaboration with Sean Carroll and Marc Kamionkowski*


*"A Hemispherical Power Asymmetry from Inflation" Phys. Rev. D in press [arXiv:0806.0377]*

*"Superhorizon Perturbations and the CMB" Phys. Rev. D* **78** *083012 (2008) [arXiv:0808.1570]*

# Outline

## *I. Power Asymmetry from Superhorizon Structure*

- What power asymmety?
- How can we make one?

## *II. Superhorizon Perturbations and the CMB*

- If there were superhorizon structures, how would we know?
- Bad news...

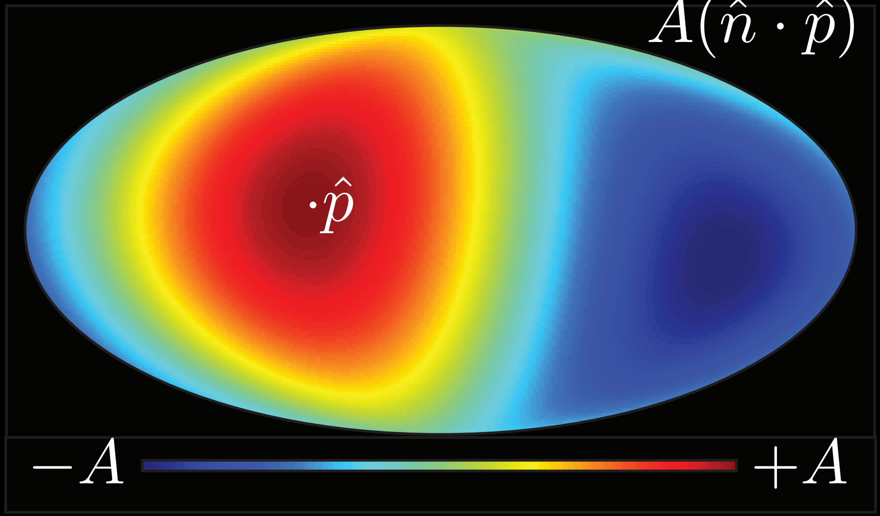
## *III. The Curvaton Alternative*

- What went wrong, and how do we fix it?
- What's a curvaton anyway?

## *IV. A Power Asymmetry from the Curvaton*

- How can we make a power asymmetry?
- Does it work?
- How do we test it?

# A Hemispherical Power Asymmety




$$\mathbf{T}(\hat{n}) = s(\hat{n})\left[1 + A(\hat{n}\cdot\hat{p})\right]$$

*CMB Temperature*

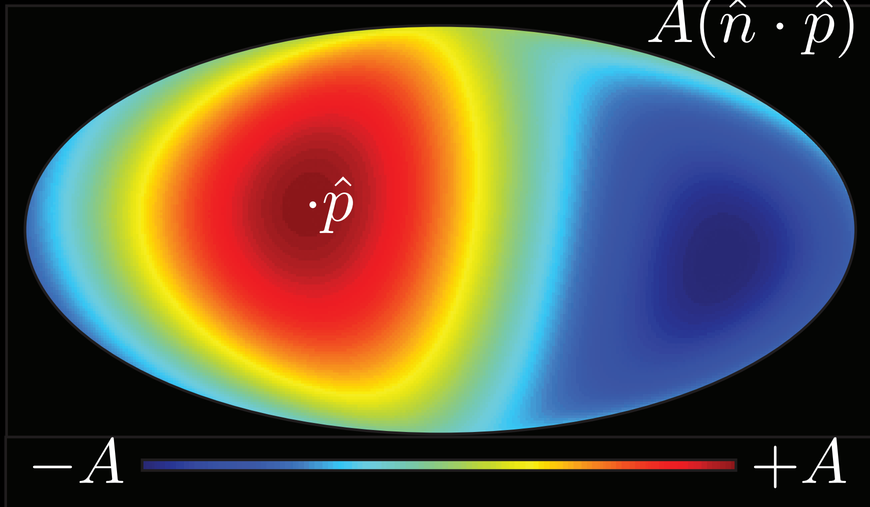
*Gaussian field with isotropic power*

*Modulation Amplitude*

*"North" pole of asymmetry*

*Simulated maps courtesy of H. K. Eriksen*

# A Hemispherical Power Asymmety




$$\mathbf{T}(\hat{n}) = s(\hat{n})\,[1 + A(\hat{n}\cdot\hat{p})]$$

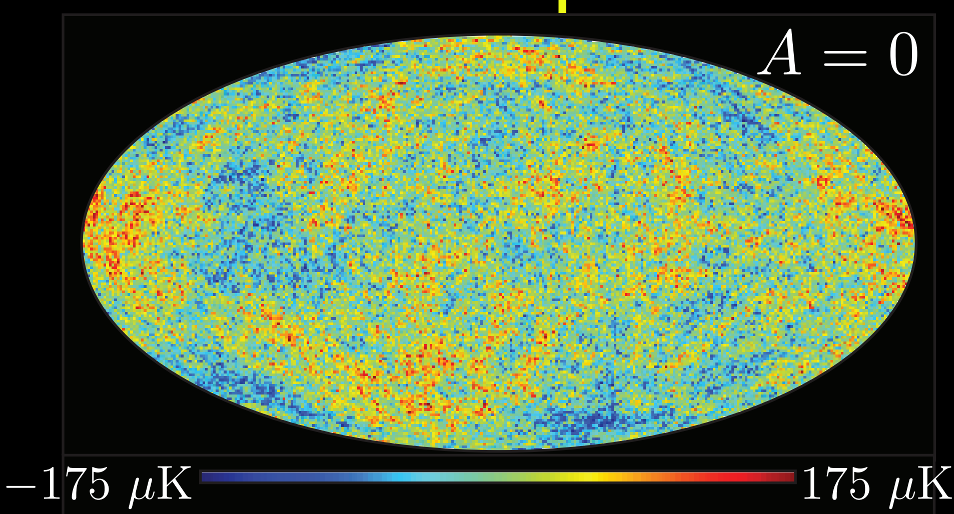
*CMB Temperature*

*Gaussian field with isotropic power*

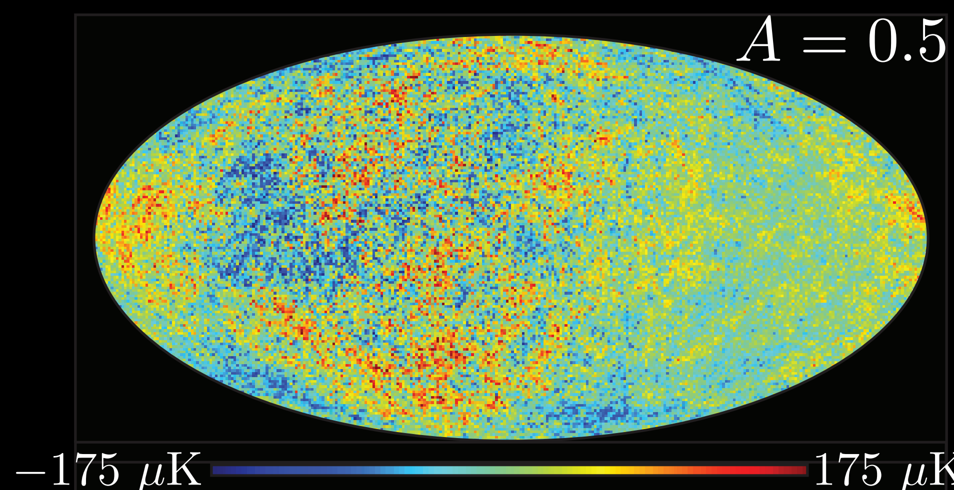
*Modulation Amplitude*

*"North" pole of asymmetry*

## Isotropic

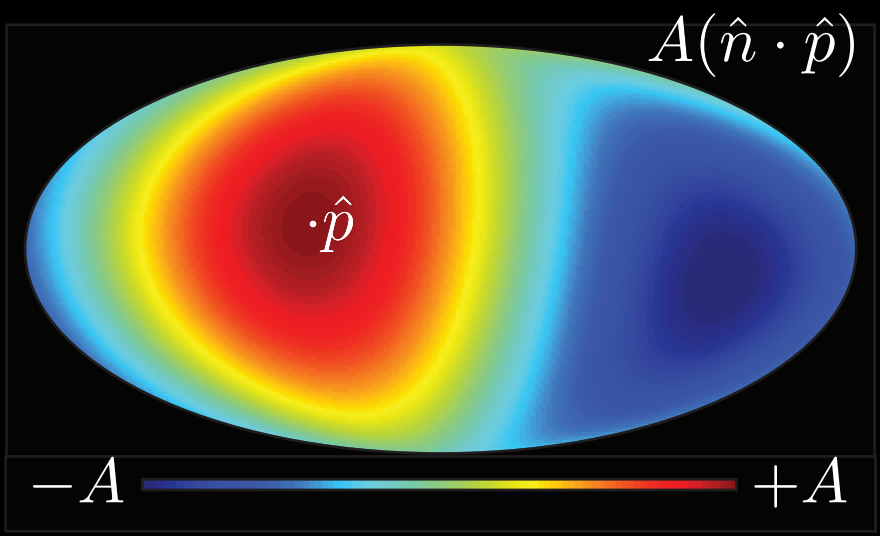



## Asymmetric




*Simulated maps courtesy of H. K. Eriksen*

# A Hemispherical Power Asymmety




$$\mathbf{T}(\hat{n}) = s(\hat{n})\left[1 + A(\hat{n}\cdot\hat{p})\right]$$

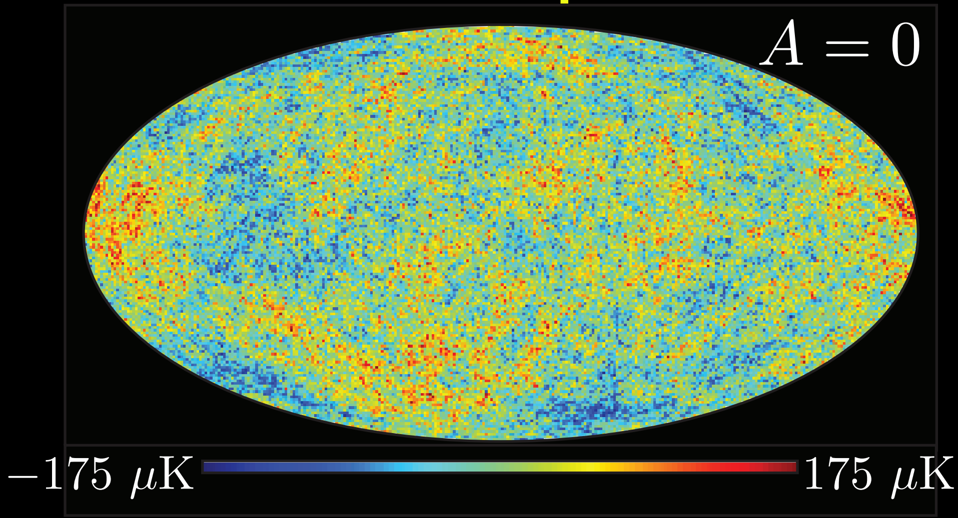
*CMB Temperature*

*Modulation Amplitude*

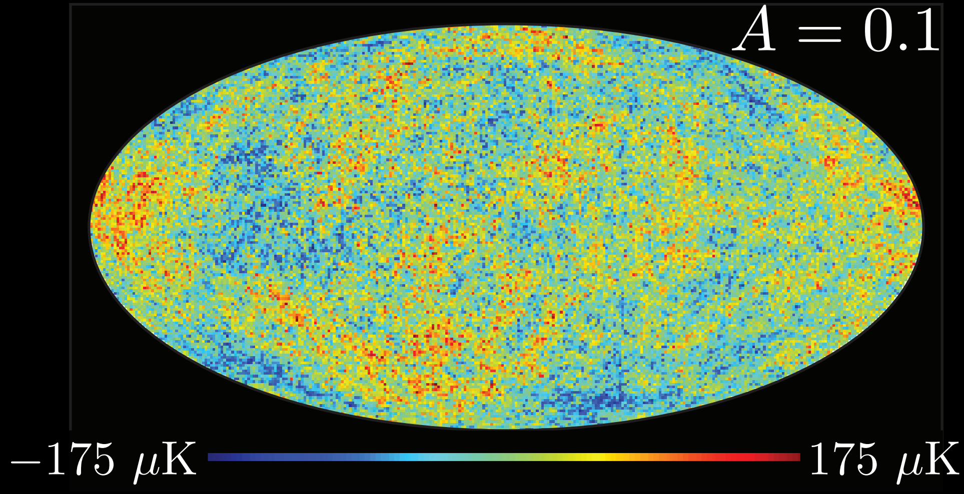
*Gaussian field with isotropic power*

*"North" pole of asymmetry*

## Isotropic



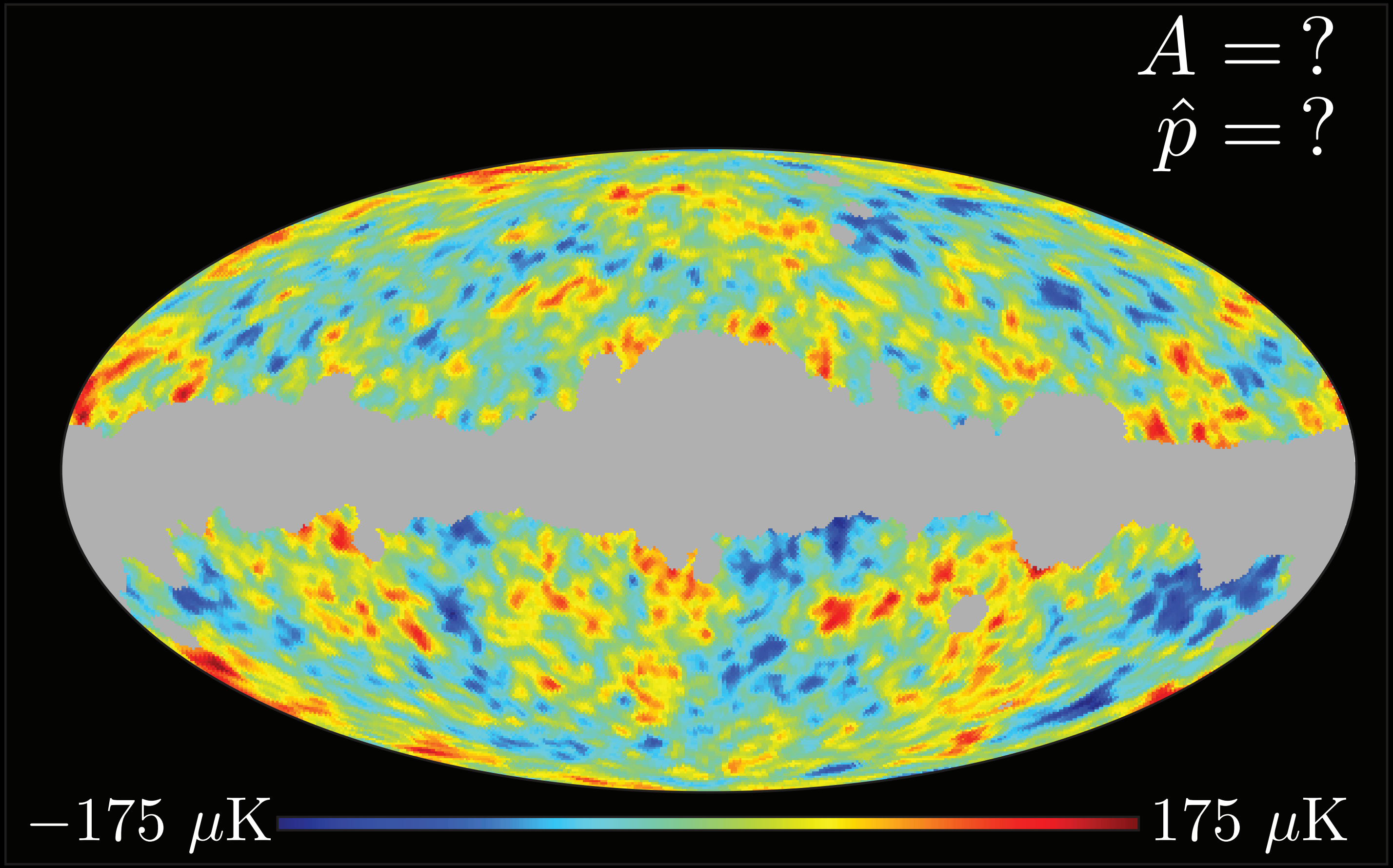

## Asymmetric




*Simulated maps courtesy of H. K. Eriksen*

# A Power Asymmetry?

## Isotropic or Asymmetric?




WMAP First Year Low-Resolution Map

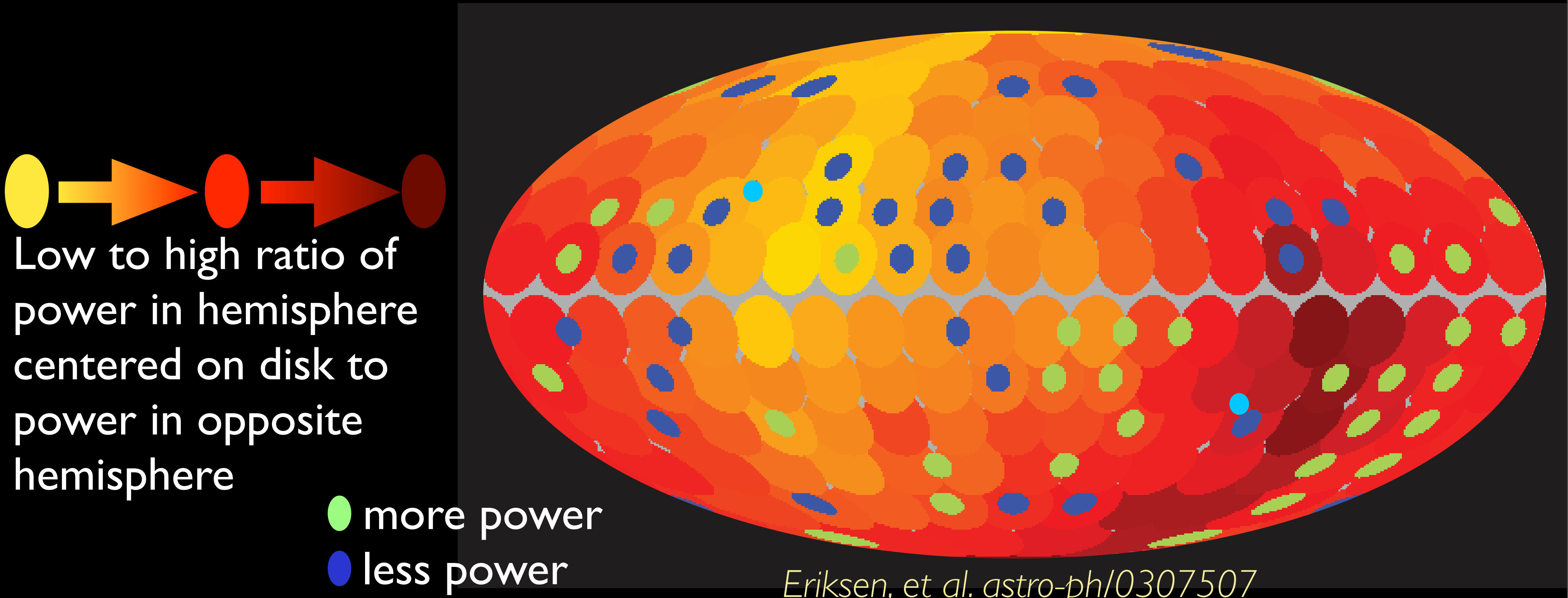
*Image from Eriksen, et al. astro-ph/0307507*

# An Asymmetric Universe!

There is a power asymmetry on large angular scales in the WMAP 1st year data.

*Eriksen, Hansen, Banday, Gorski, Lilje 2004*

$$\ell = 5 - 40$$



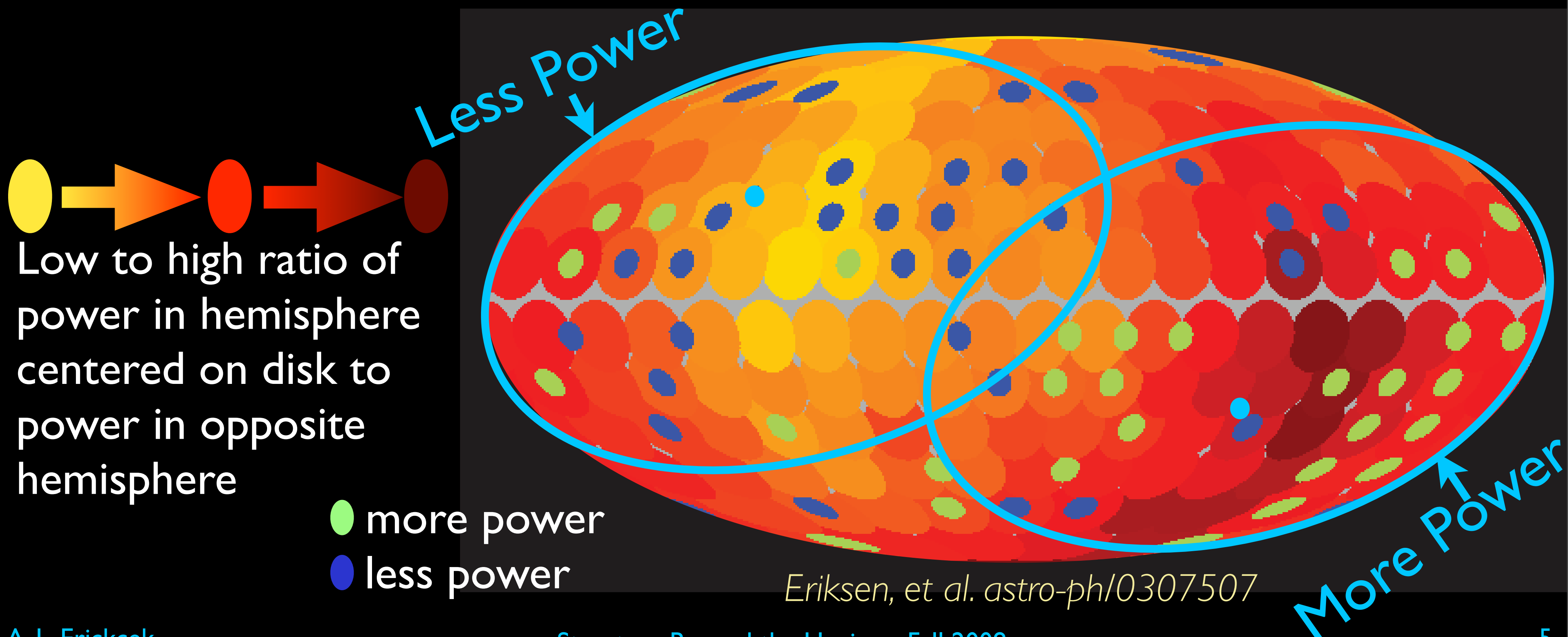

*Eriksen, et al. astro-ph/0307507*

# An Asymmetric Universe!

There is a power asymmetry on large angular scales in the WMAP 1st year data.

$\ell = 5 - 40$

*Eriksen, Hansen, Banday, Gorski, Lilje 2004*




Low to high ratio of power in hemisphere centered on disk to power in opposite hemisphere

more power

less power

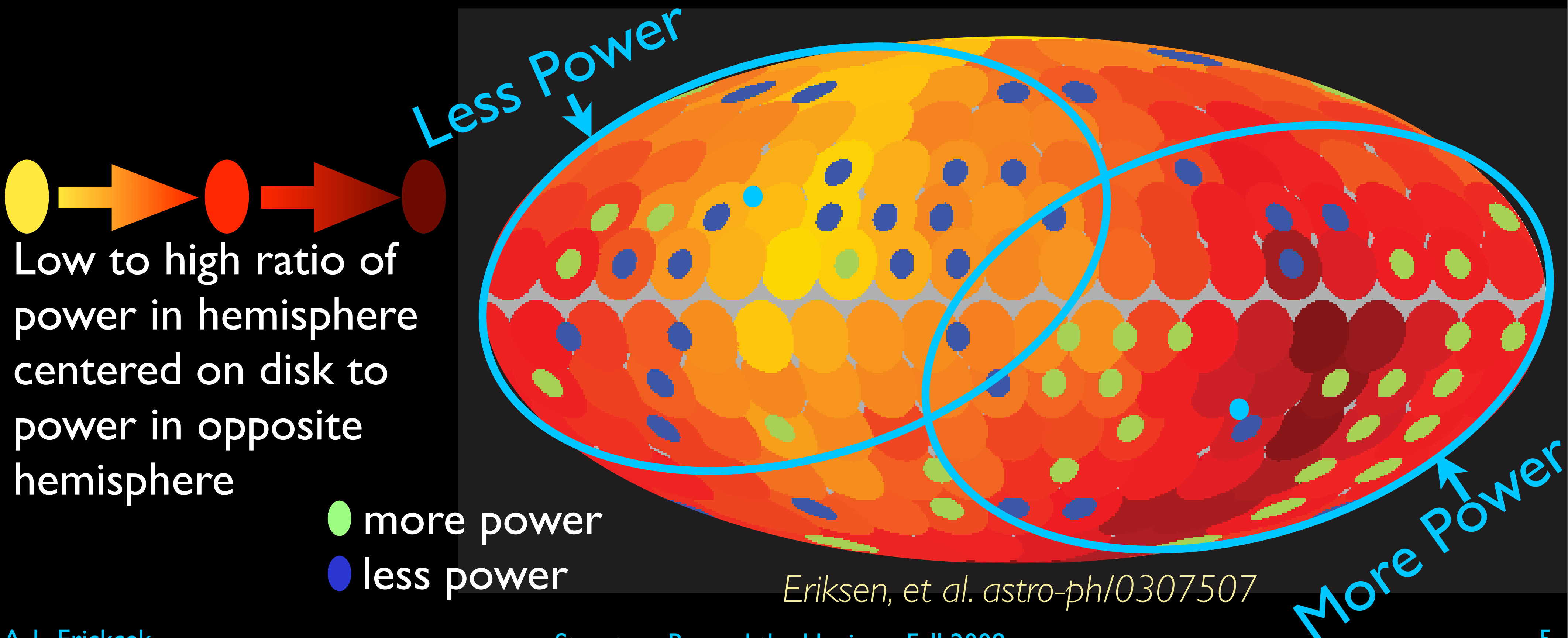
*Eriksen, et al. astro-ph/0307507*

# An Asymmetric Universe!

There is a power asymmetry on large angular scales in the WMAP 1st year data.

*Eriksen, Hansen, Banday, Gorski, Lilje 2004*

- Power asymmetry is maximized when the "equatorial" plane is tilted with respect to the Galactic plane: "north" pole at $(\ell, b) = (237°, -10°)$ .
- Only 0.7% of simulated symmetric maps contain this much asymmetry.




Low to high ratio of power in hemisphere centered on disk to power in opposite hemisphere

more power
less power

*Eriksen, et al. astro-ph/0307507*

# An Asymmetric Universe!

The asymmetry persists in the WMAP3 data.

*Eriksen, Banday, Gorski, Hansen, Lilje 2007*

$$\mathbf{T}(\hat{n}) = s(\hat{n})\,[1 + A(\hat{n}\cdot\hat{p})] + \mathbf{N}(\hat{n})$$

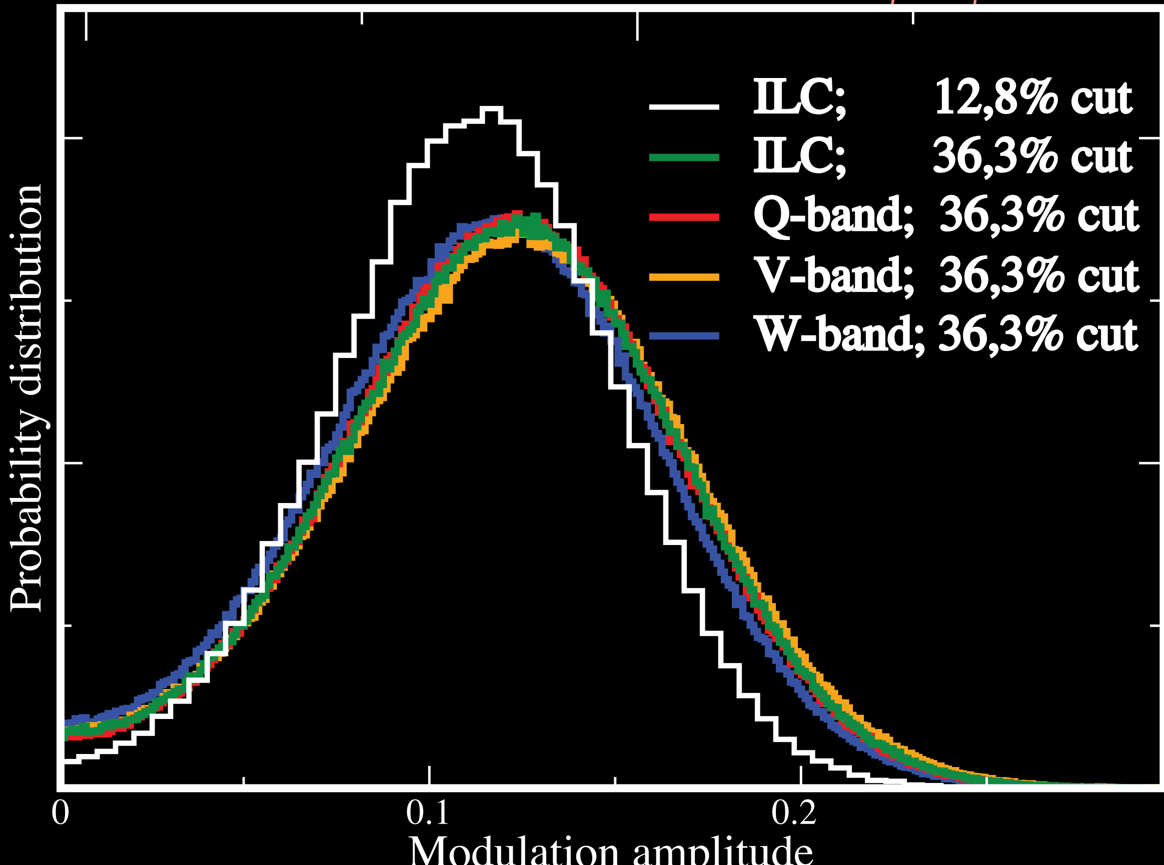
*Observed CMB Temperature*

*Gaussian field with isotropic power*

*Modulation Amplitude*

*"North" pole of asymmetry*

*Noise*




*Eriksen, et al. astro-ph/070108*

Bayesian analysis: $A \simeq 0.12$

"north" pole: $(\ell, b) \simeq (210°, -27°)$

The probability of measuring this amplitude or larger given an isotropic field is $0.01$.

# An Asymmetric Universe!

The asymmetry persists in the WMAP3 data.

*Eriksen, Banday, Gorski, Hansen, Lilje 2007*

$$\mathbf{T}(\hat{n}) = s(\hat{n})\,[1 + A(\hat{n}\cdot\hat{p})] + \mathbf{N}(\hat{n})$$

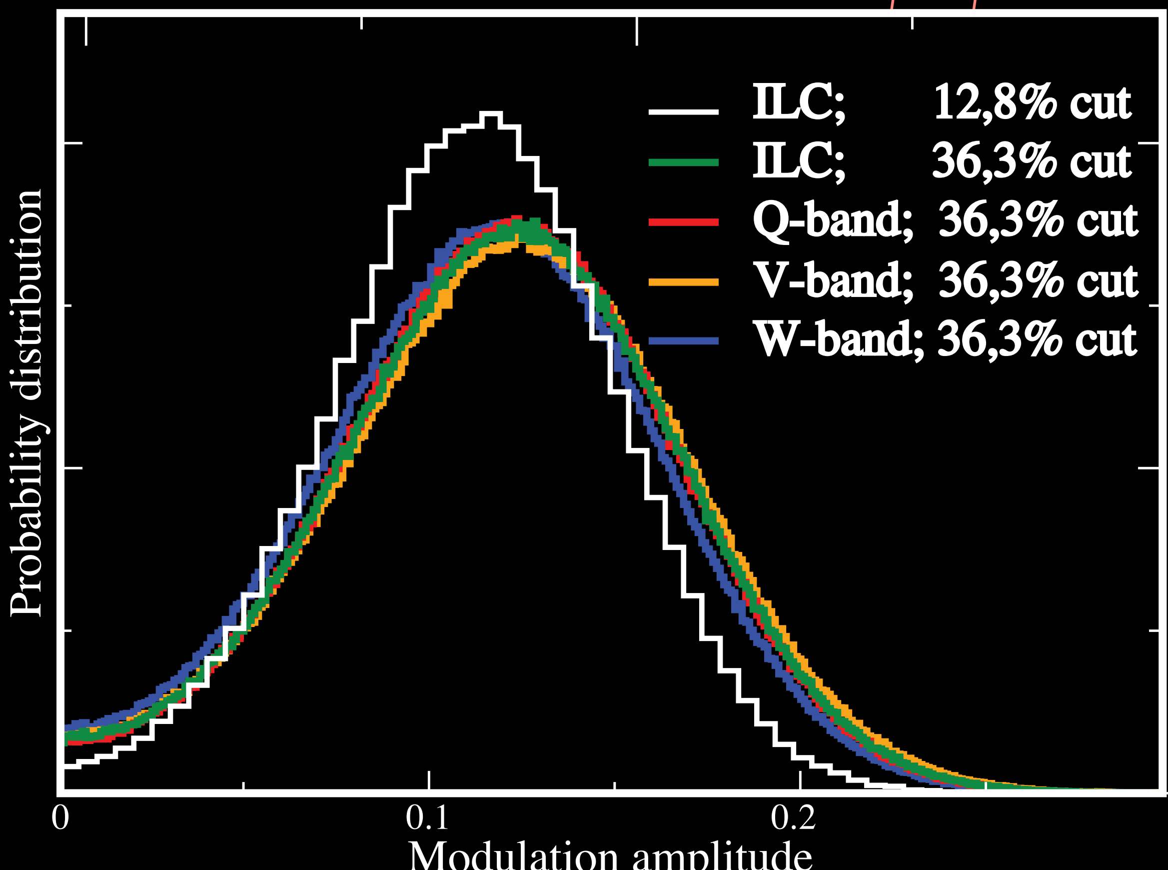
*Observed CMB Temperature*

*Gaussian field with isotropic power*

*Modulation Amplitude*

*"North" pole of asymmetry*

*Noise*




*Eriksen, et al. astro-ph/070108*

The asymmetry is difficult to explain with foregrounds:

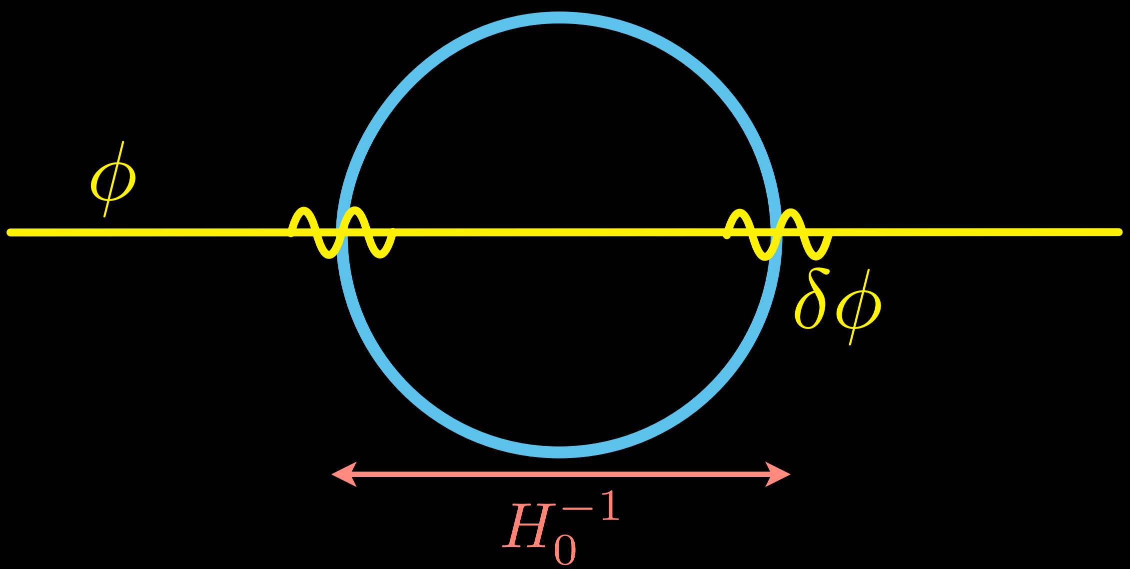
- present in all colors
- not aligned with the Galaxy

The asymmetry is difficult to explain with systematics:

- also detected by COBE

*Hansen, et al. 2004, Eriksen, et al. 2004*

# Asymmetry from a "Supermode"



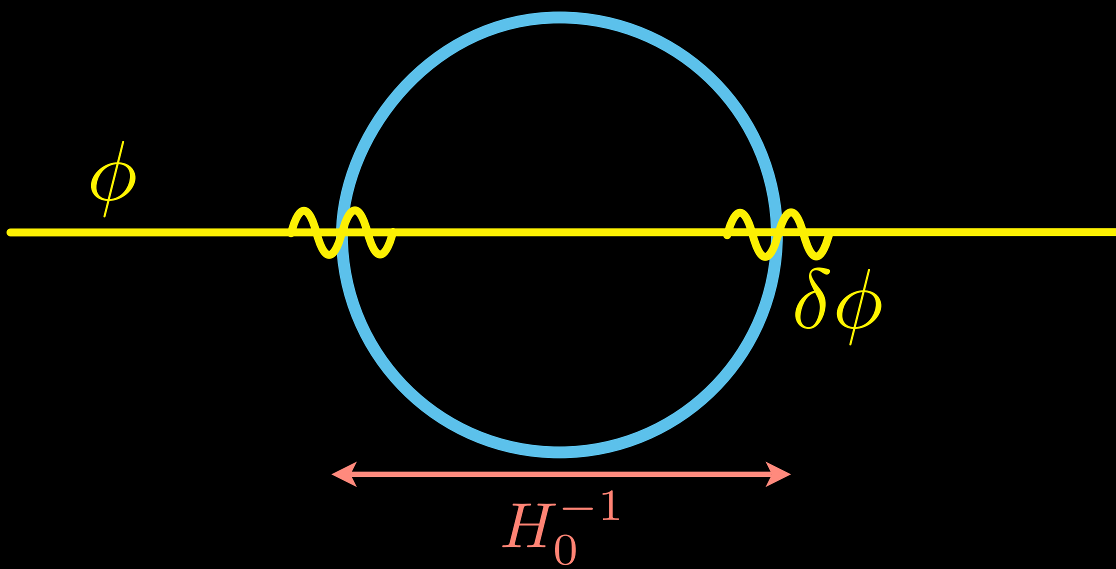

The amplitude of quantum fluctuations depends on the background value of the inflaton field.

$$P_\Psi = \frac{2}{9k^3}\left[\frac{H(\phi)^2}{\dot{\phi}}\right]^2\Bigg|_{k=aH}$$

*Power Spectrum of Potential Fluctuations*

$$\mathrm{d}s^2 = -(1+2\Psi)\mathrm{d}t^2 + a^2(t)\delta_{ij}(1-2\Psi)\mathrm{d}x^i\mathrm{d}x^j$$

# Asymmetry from a "Supermode"

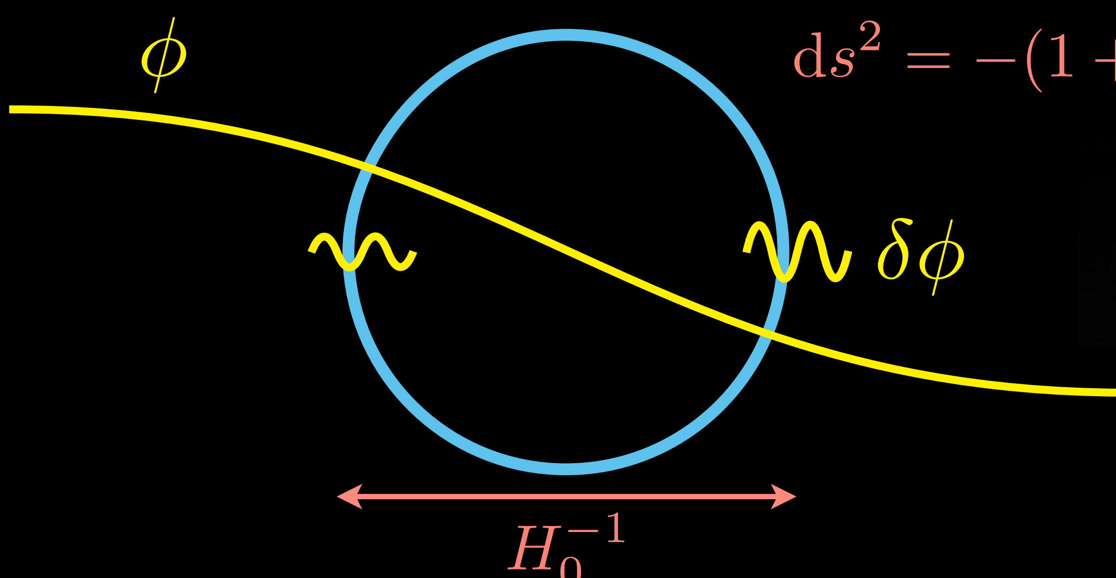


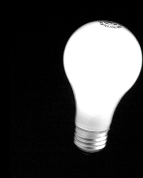

The amplitude of quantum fluctuations depends on the background value of the inflaton field.

$$P_\Psi = \frac{2}{9k^3}\left[\frac{H(\phi)^2}{\dot{\phi}}\right]^2\Bigg|_{k=aH}$$

*Power Spectrum of Potential Fluctuations*

$$\mathrm{d}s^2 = -(1+2\Psi)\mathrm{d}t^2 + a^2(t)\delta_{ij}(1-2\Psi)\mathrm{d}x^i\mathrm{d}x^j$$



*Create asymmetry by adding a large-amplitude superhorizon fluctuation: a "supermode."*

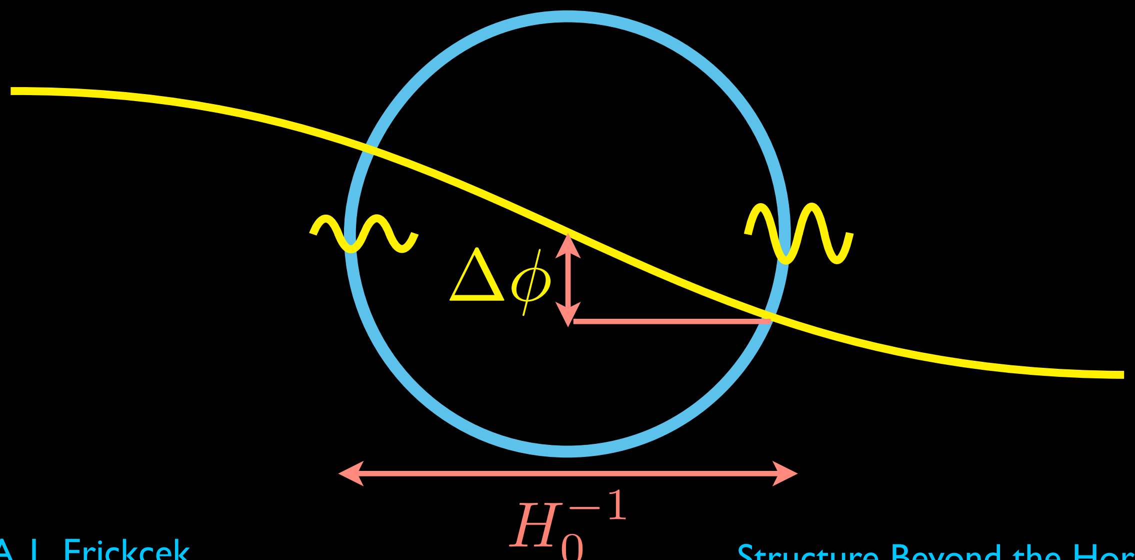
# Asymmetry from a "Supermode"

A modulation amplitude $A \simeq 0.12 \Longrightarrow \dfrac{\Delta P_\Psi(k)}{P_\Psi(k)_{360^\circ}} \simeq \pm 0.20$

Generating this much asymmetry requires a BIG supermode.

- Perturbations with different wavelengths are very weakly coupled.
- The fluctuation power is not very sensitive to $\phi \Longleftrightarrow n_s \simeq 1$ .

$$\frac{\Delta P_\Psi}{P_\Psi} = -2\sqrt{\frac{\pi}{\epsilon}}(1-n_s)\frac{\Delta\phi}{m_{\rm Pl}}$$



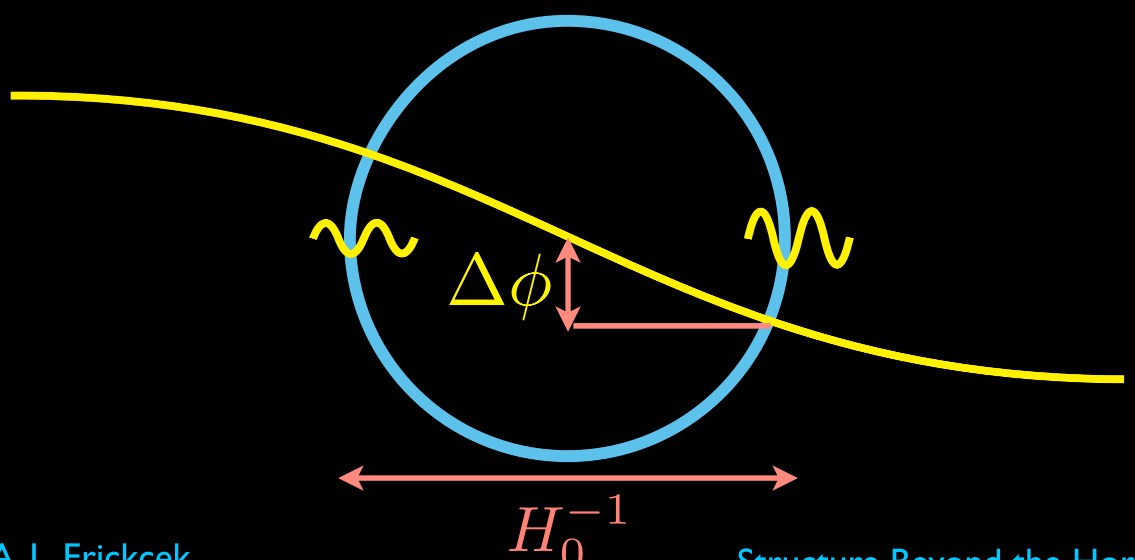
# Asymmetry from a "Supermode"

A modulation amplitude $A \simeq 0.12 \Longrightarrow \dfrac{\Delta P_\Psi(k)}{P_\Psi(k)_{360^\circ}} \simeq \pm 0.20$

Generating this much asymmetry requires a BIG supermode.

- Perturbations with different wavelengths are very weakly coupled.
- The fluctuation power is not very sensitive to $\phi \Longleftrightarrow n_s \simeq 1$ .

$$\frac{\Delta P_\Psi}{P_\Psi} = -2\sqrt{\frac{\pi}{\epsilon}}(1-n_s)\frac{\Delta\phi}{m_{\rm Pl}}$$



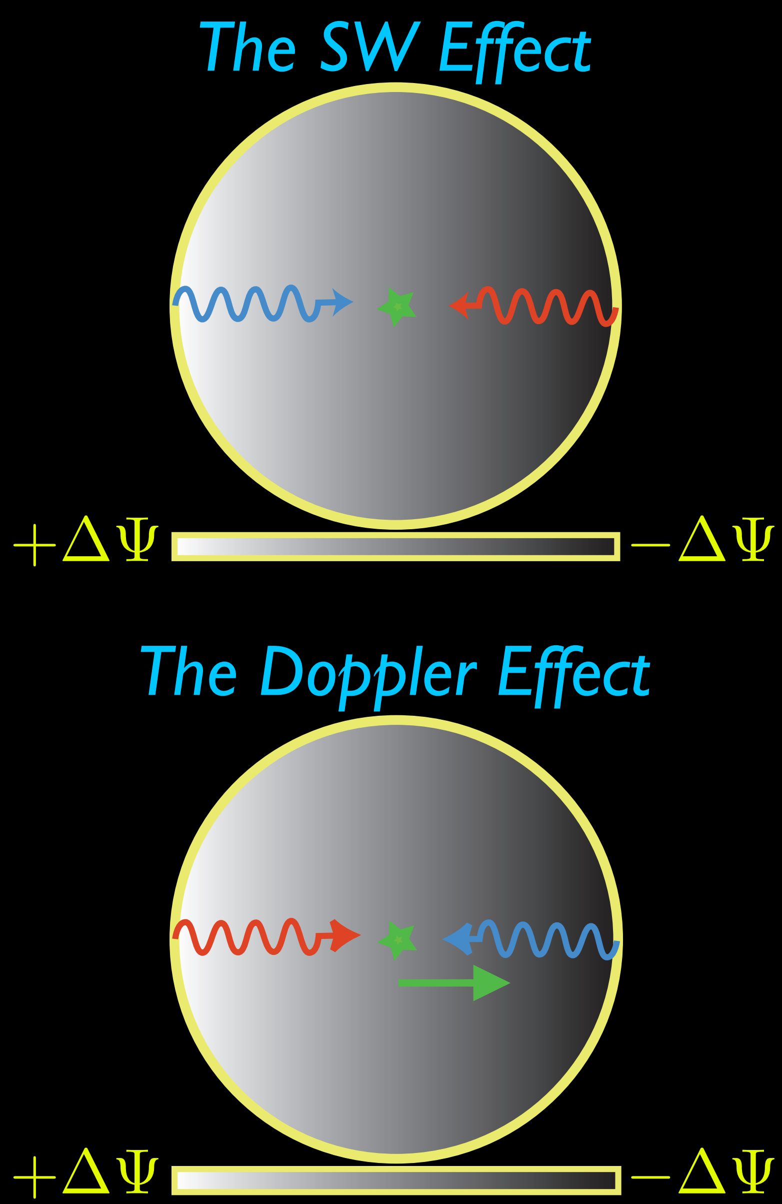

$$\Delta\phi \Longrightarrow \Delta\Psi \Longrightarrow \Delta T$$

*Surely the resulting temperature dipole would be far too large?*

# *Part II*

## Superhorizon Perturbations
## and the
## Cosmic Microwave Background

# The Dipole Sometimes Cancels...

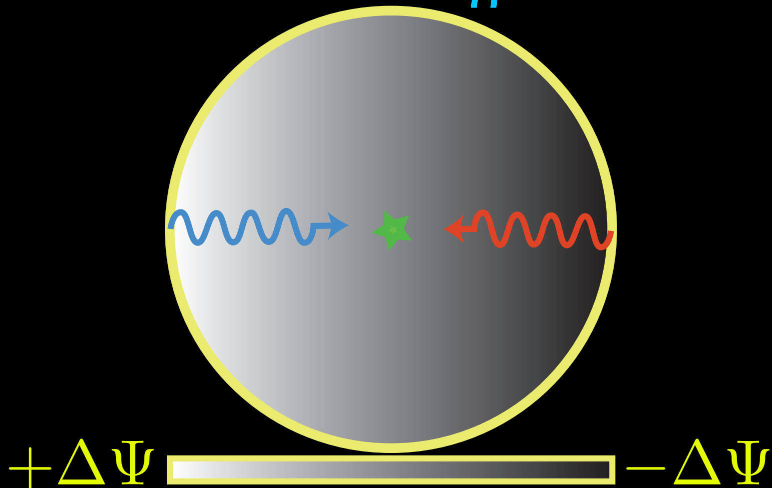



In an Einstein - deSitter Universe, a superhorizon perturbation induces no CMB dipole. *Grishchuk, Zel'dovich 1978*

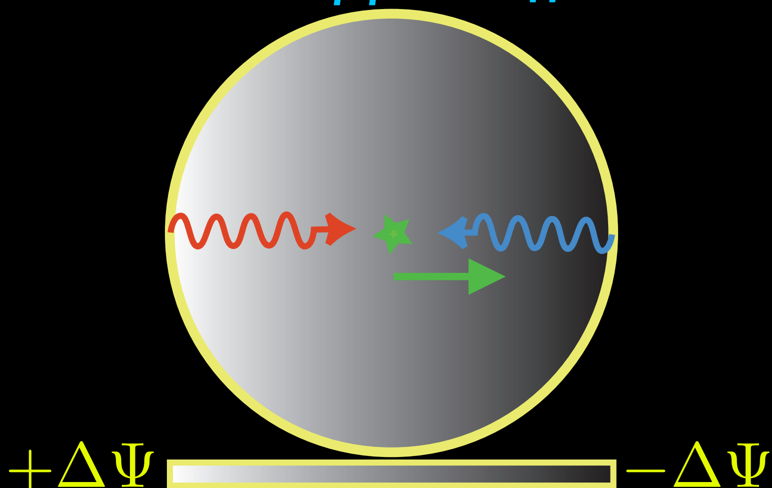
- A superhorizon mode: $\Psi(\vec{x}) \simeq \Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}\right]$
- The SW dipole: $\dfrac{\Delta T}{T} = \dfrac{1}{3}\Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}_{\rm dec}\right]$
- The Doppler dipole:

# The Dipole Sometimes Cancels...

*The SW Effect*




*The Doppler Effect*



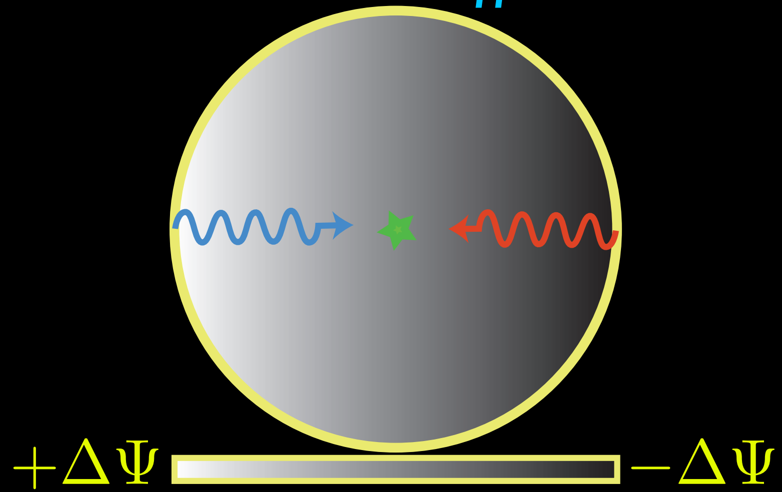

In an **Einstein - deSitter** Universe, a superhorizon perturbation induces **no CMB dipole.** *Grishchuk, Zel'dovich 1978*

- A superhorizon mode: $\Psi(\vec{x}) \simeq \Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}\right]$
- The SW dipole: $\dfrac{\Delta T}{T} = \dfrac{1}{3}\Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}_{\rm dec}\right]$
- The Doppler dipole:

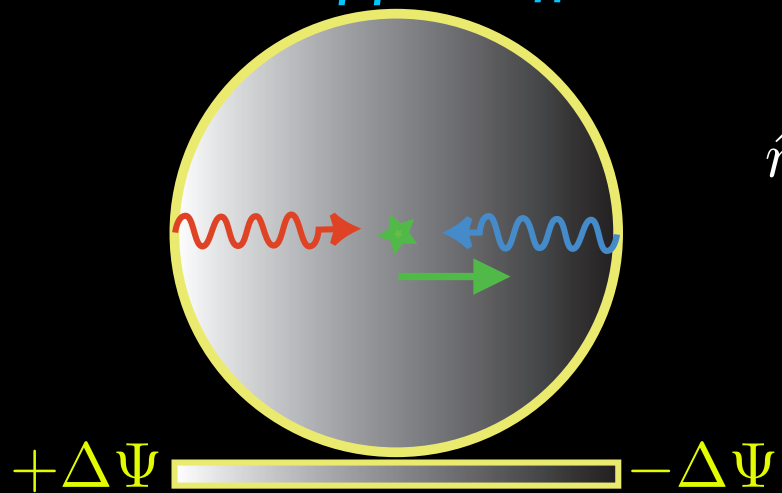
$$\frac{\Delta T}{T} = \hat{n}\cdot\left[\vec{v}(t_0,\vec{0}) - \vec{v}(t_{\rm dec},\vec{x}_{\rm dec})\right]$$

# The Dipole Sometimes Cancels...

*The SW Effect*




*The Doppler Effect*




In an Einstein - deSitter Universe, a superhorizon perturbation induces no CMB dipole. *Grishchuk, Zel'dovich 1978*

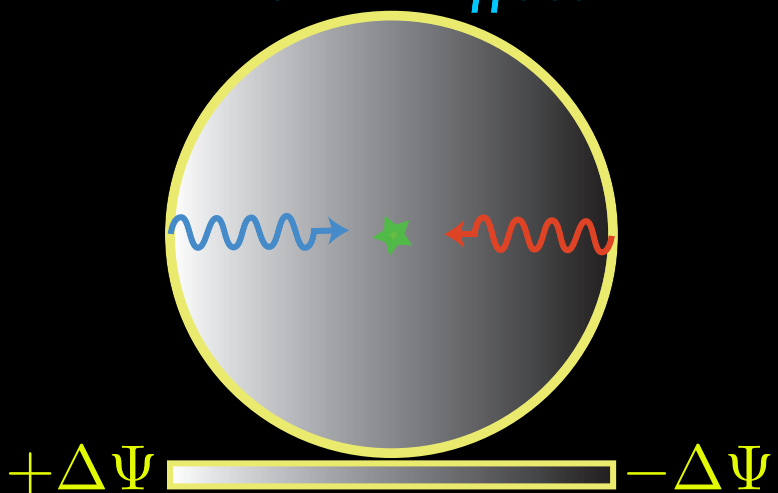
- A superhorizon mode: $\Psi(\vec{x}) \simeq \Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}\right]$
- The SW dipole: $\dfrac{\Delta T}{T} = \dfrac{1}{3}\Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}_{\rm dec}\right]$
- The Doppler dipole:

$$\frac{\Delta T}{T} = \hat{n}\cdot\left[\vec{v}(t_0,\vec{0}) - \vec{v}(t_{\rm dec},\vec{x}_{\rm dec})\right]$$

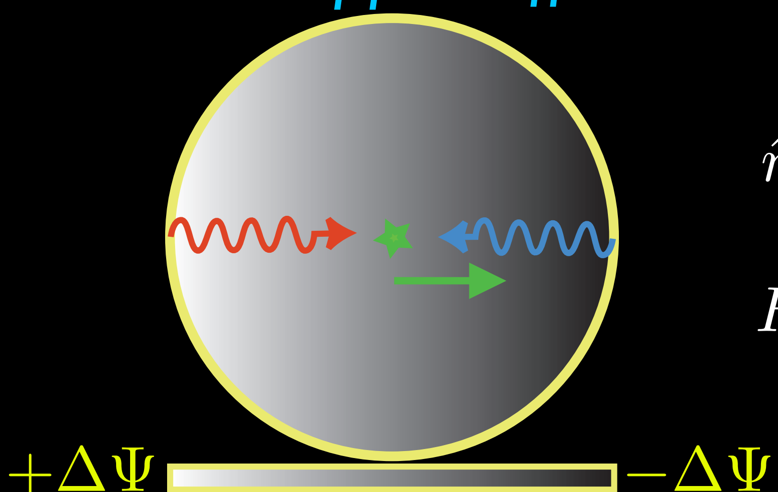
$$\hat{n}\cdot\vec{v}(t) = -\frac{2}{3}\frac{\sqrt{a(t)}}{H_0 x_{\rm dec}}\left[\vec{x}_{\rm dec}\cdot\vec{\nabla}\Psi\right]$$

# The Dipole Sometimes Cancels...

*The SW Effect*




*The Doppler Effect*




In an **Einstein - deSitter** Universe, a superhorizon perturbation induces **no CMB dipole.** *Grishchuk, Zel'dovich 1978*

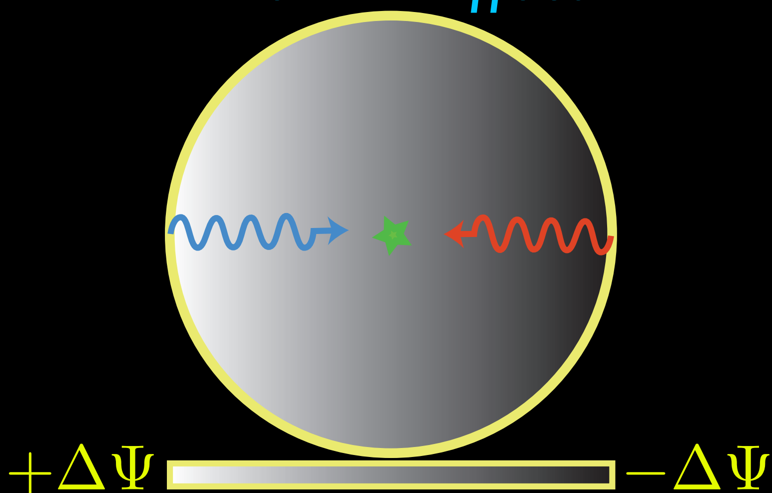
- A superhorizon mode: $\Psi(\vec{x}) \simeq \Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}\right]$
- The SW dipole: $\dfrac{\Delta T}{T} = \dfrac{1}{3}\Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}_{\rm dec}\right]$
- The Doppler dipole:

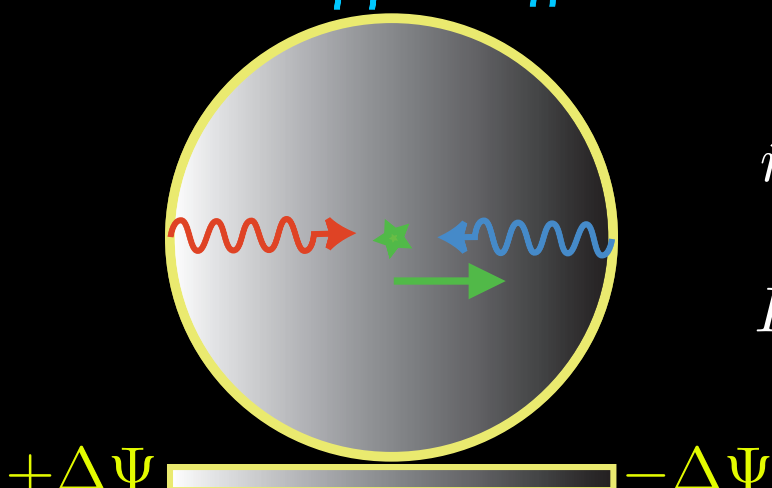
$$\frac{\Delta T}{T} = \hat{n}\cdot\left[\vec{v}(t_0,\vec{0}) - \vec{v}(t_{\rm dec},\vec{x}_{\rm dec})\right]$$

$$\hat{n}\cdot\vec{v}(t) = -\frac{2}{3}\frac{\sqrt{a(t)}}{H_0 x_{\rm dec}}\left[\vec{x}_{\rm dec}\cdot\vec{\nabla}\Psi\right]$$

$$H_0 x_{\rm dec} = 2\left[1-\sqrt{a(t_{\rm dec})}\right]$$

# The Dipole Sometimes Cancels...




In an Einstein - deSitter Universe, a superhorizon perturbation induces no CMB dipole. *Grishchuk, Zel'dovich 1978*

- A superhorizon mode: $\Psi(\vec{x}) \simeq \Psi_{\mathrm{SM}}\left[\vec{k}\cdot\vec{x}\right]$
- The SW dipole: $\dfrac{\Delta T}{T} = \dfrac{1}{3}\Psi_{\mathrm{SM}}\left[\vec{k}\cdot\vec{x}_{\mathrm{dec}}\right]$
- The Doppler dipole:

$$\frac{\Delta T}{T} = \hat{n}\cdot\left[\vec{v}(t_0,\vec{0}) - \vec{v}(t_{\mathrm{dec}},\vec{x}_{\mathrm{dec}})\right]$$

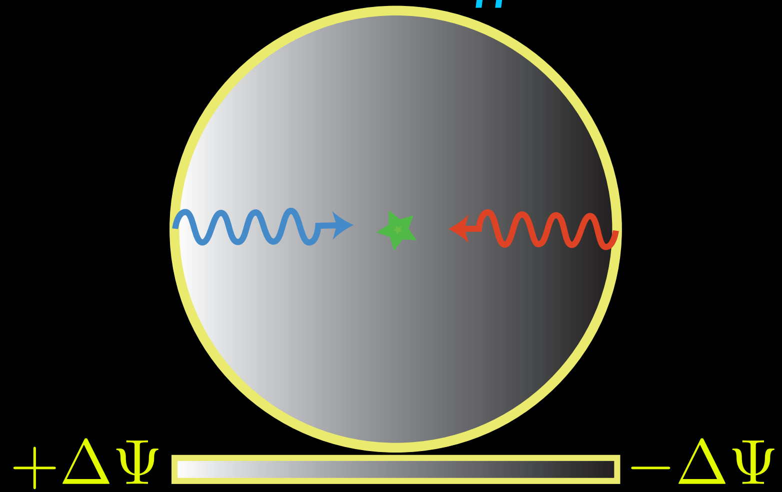
$$\hat{n}\cdot\vec{v}(t) = -\frac{2}{3}\frac{\sqrt{a(t)}}{H_0 x_{\mathrm{dec}}}\left[\vec{x}_{\mathrm{dec}}\cdot\vec{\nabla}\Psi\right]$$

$$H_0 x_{\mathrm{dec}} = 2\left[1-\sqrt{a(t_{\mathrm{dec}})}\right]$$

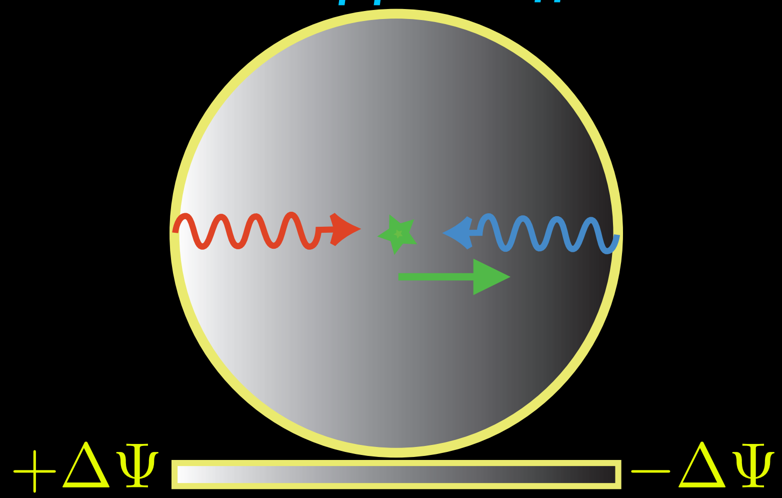
$$\frac{\Delta T}{T} = \frac{-(2/3)\left[1-\sqrt{a(t_{\mathrm{dec}})}\right]}{2\left[1-\sqrt{a(t_{\mathrm{dec}})}\right]}\Psi_{\mathrm{SM}}\left[\vec{k}\cdot\vec{x}_{\mathrm{dec}}\right]$$

# The Dipole Sometimes Cancels...

*The SW Effect*



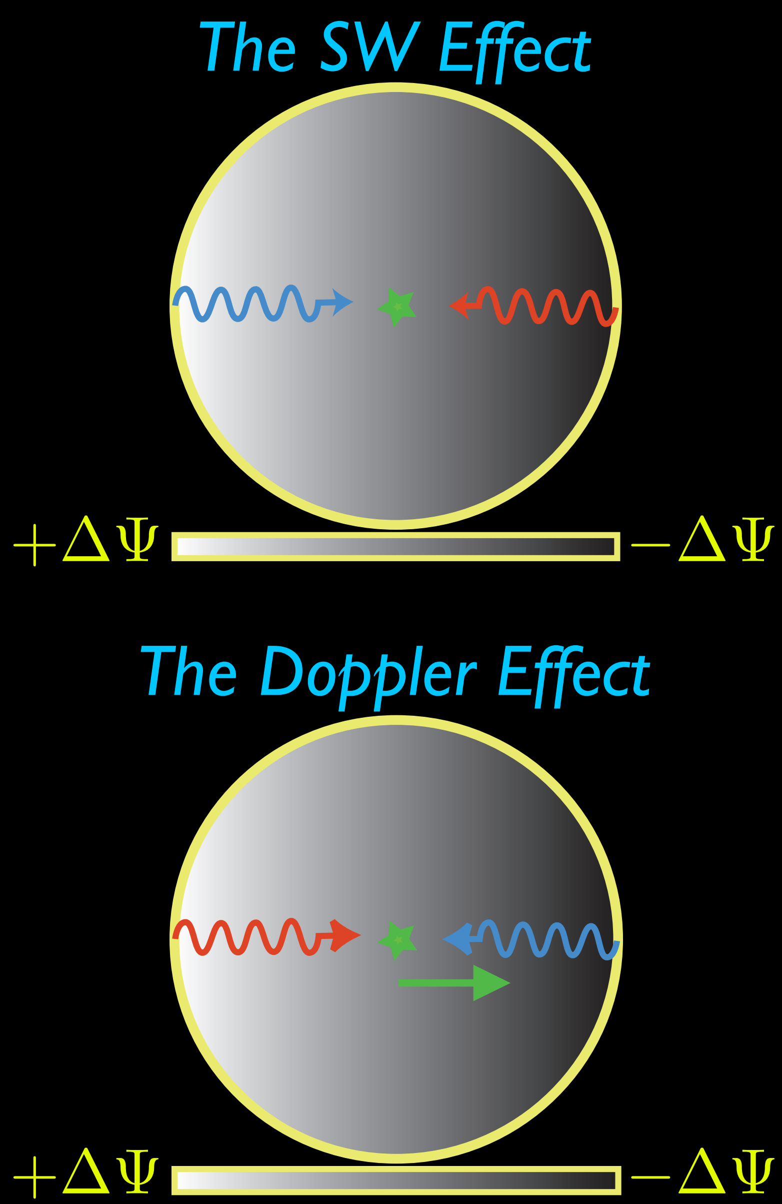

*The Doppler Effect*




In an Einstein - deSitter Universe, a superhorizon perturbation induces no CMB dipole. *Grishchuk, Zel'dovich 1978*

- A superhorizon mode: $\Psi(\vec{x}) \simeq \Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}\right]$
- The SW dipole: $\dfrac{\Delta T}{T} = \dfrac{1}{3}\Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}_{\rm dec}\right]$
- The Doppler dipole: $\dfrac{\Delta T}{T} = -\dfrac{1}{3}\Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}_{\rm dec}\right]$

# The Dipole Sometimes Cancels...

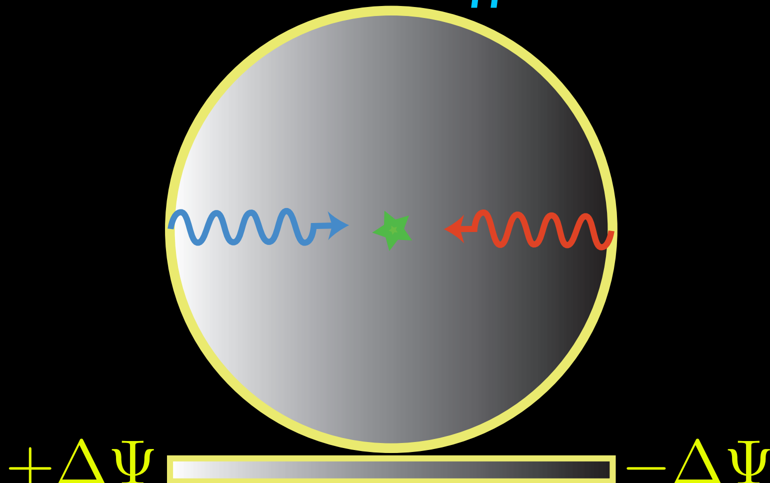



In an Einstein - deSitter Universe, a superhorizon perturbation induces no CMB dipole. *Grishchuk, Zel'dovich 1978*

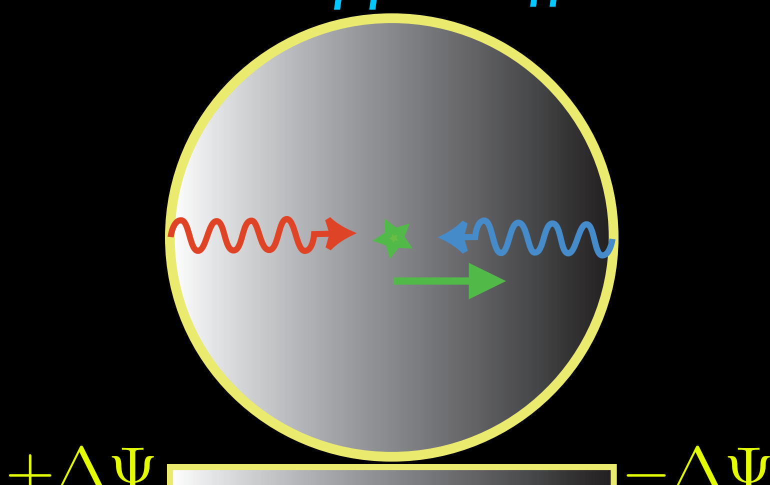
- A superhorizon mode: $\Psi(\vec{x}) \simeq \Psi_{\mathrm{SM}}\left[\vec{k}\cdot\vec{x}\right]$
- The SW dipole: $\dfrac{\Delta T}{T} = \dfrac{1}{3}\Psi_{\mathrm{SM}}\left[\vec{k}\cdot\vec{x}_{\mathrm{dec}}\right]$
- The Doppler dipole: $\dfrac{\Delta T}{T} = -\dfrac{1}{3}\Psi_{\mathrm{SM}}\left[\vec{k}\cdot\vec{x}_{\mathrm{dec}}\right]$
- Since $\Psi$ is constant, there is no ISW effect.

# The Dipole Sometimes Cancels...

*The SW Effect*




*The Doppler Effect*



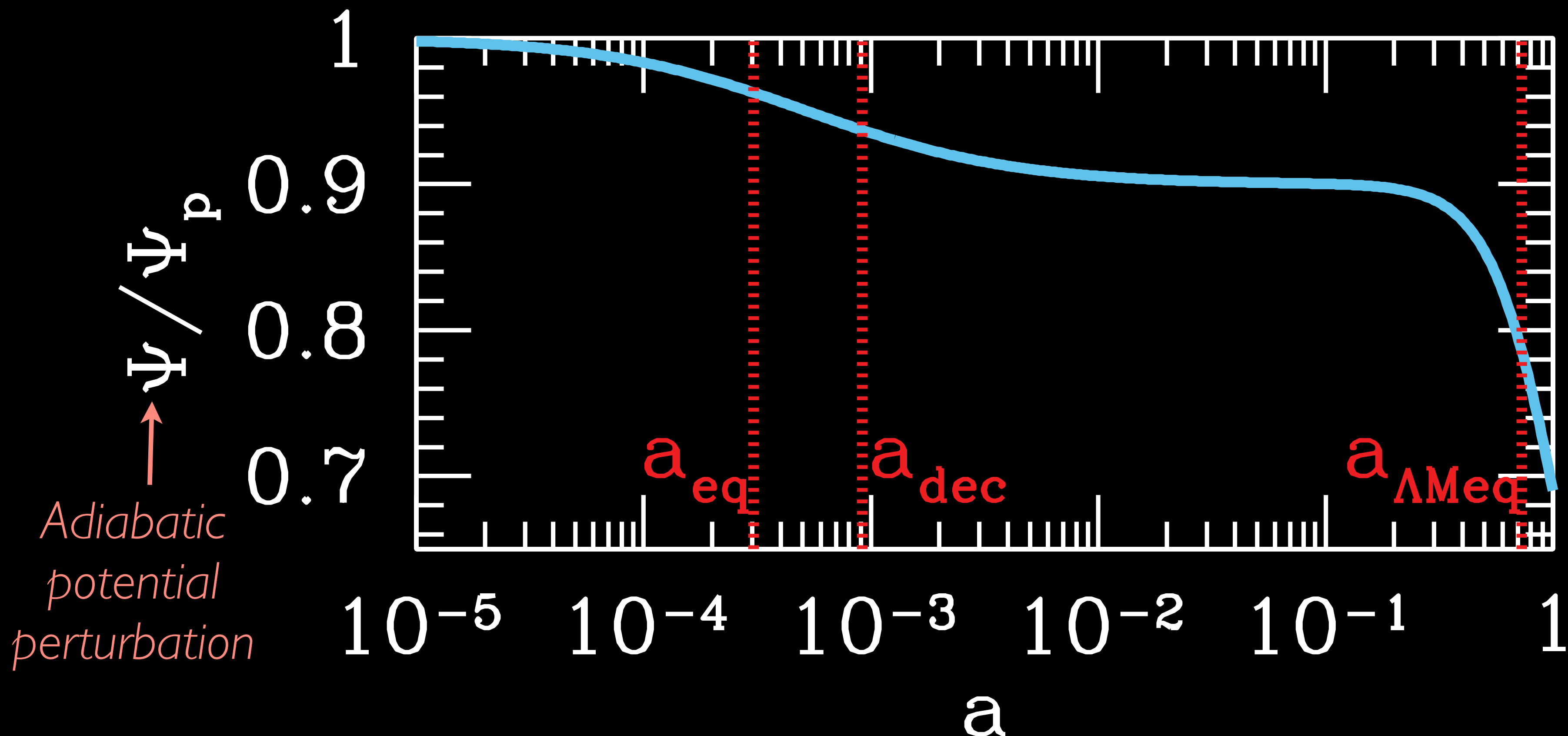

In an **Einstein - deSitter** Universe, a superhorizon perturbation induces **no CMB dipole.** *Grishchuk, Zel'dovich 1978*

- A superhorizon mode: $\Psi(\vec{x}) \simeq \Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}\right]$
- The SW dipole: $\dfrac{\Delta T}{T} = \dfrac{1}{3}\Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}_{\rm dec}\right]$
- The Doppler dipole: $\dfrac{\Delta T}{T} = -\dfrac{1}{3}\Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}_{\rm dec}\right]$
- Since $\Psi$ is constant, there is **no ISW effect.**

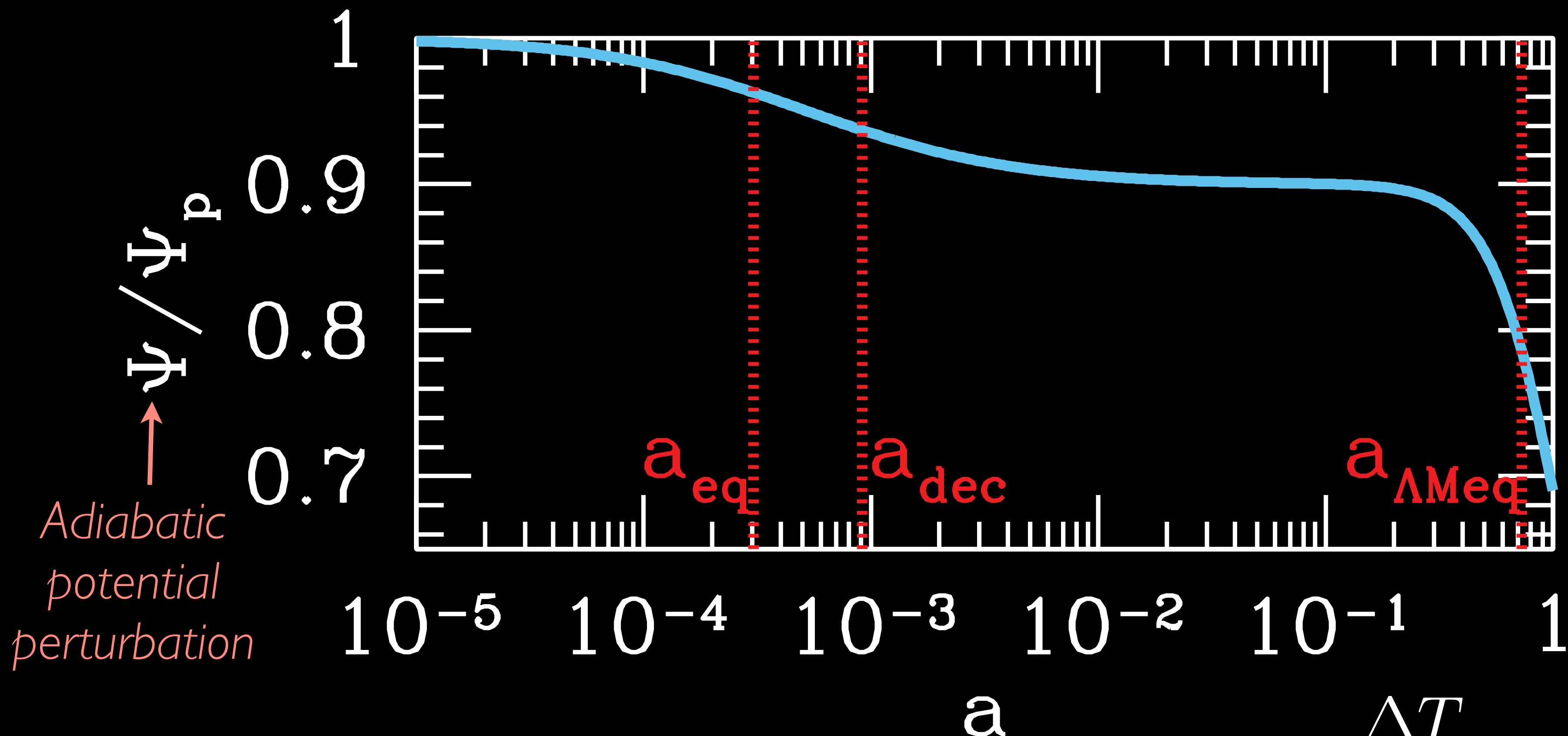
*Well that's cute.*

*But the situation is much more complicated in a Universe like ours!*

# The Evolving Potential in ΛCDM



# The Evolving Potential in $\Lambda$CDM




- Radiation at decoupling increases SW effect: $\frac{\Delta T}{T} = 0.4\Psi$
- $\Lambda$ increases $x_{\mathrm{dec}}$ and reduces the Doppler dipole.
- Evolution of $\Psi$ leads to ISW effect that will partially cancel the SW anisotropy: $\frac{\Delta T}{T} = 2\int_{t_{\mathrm{dec}}}^{t_0} \frac{\mathrm{d}\Psi}{\mathrm{d}t}[t, \vec{x}(t)]\mathrm{d}t$

# The Dipole Cancels!

Adiabatic superhorizon perturbation:

$$\Psi(\vec{x}) = \Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}\right]$$

$$kH_0^{-1} \ll 1$$

Temperature anisotropy:

$$\frac{\Delta T}{T}(\hat{n}) = \delta_1 \Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}_{\rm dec}\right]$$

*includes SW, Doppler and ISW anisotropies*

# The Dipole Cancels!

Adiabatic superhorizon perturbation:

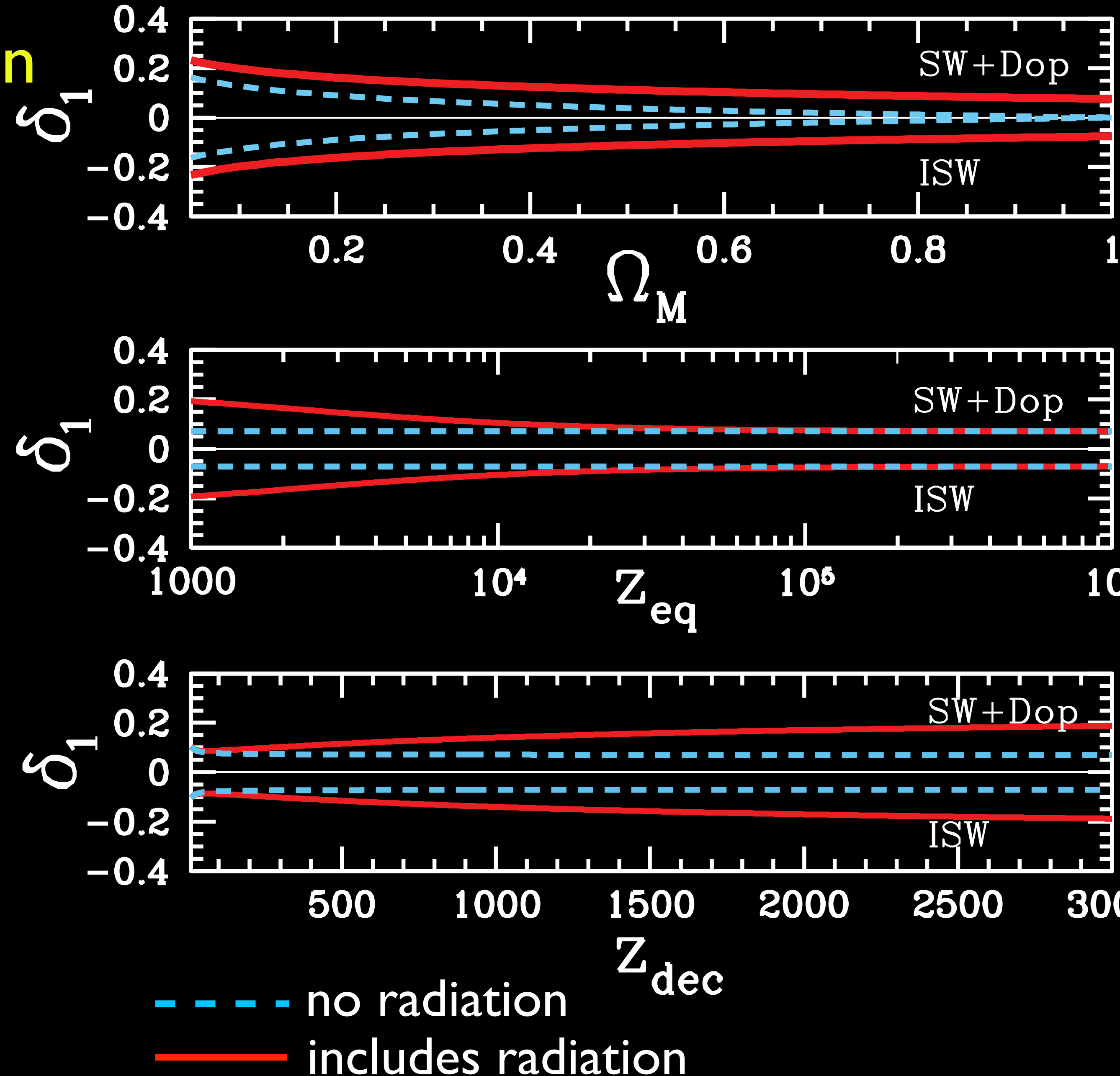
$$\Psi(\vec{x}) = \Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}\right]$$

$$kH_0^{-1} \ll 1$$

Temperature anisotropy:

$$\frac{\Delta T}{T}(\hat{n}) = \delta_1 \Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}_{\rm dec}\right]$$

*includes SW, Doppler and ISW anisotropies*



# The Dipole Cancels!

Adiabatic superhorizon perturbation:

$$\Psi(\vec{x}) = \Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}\right]$$

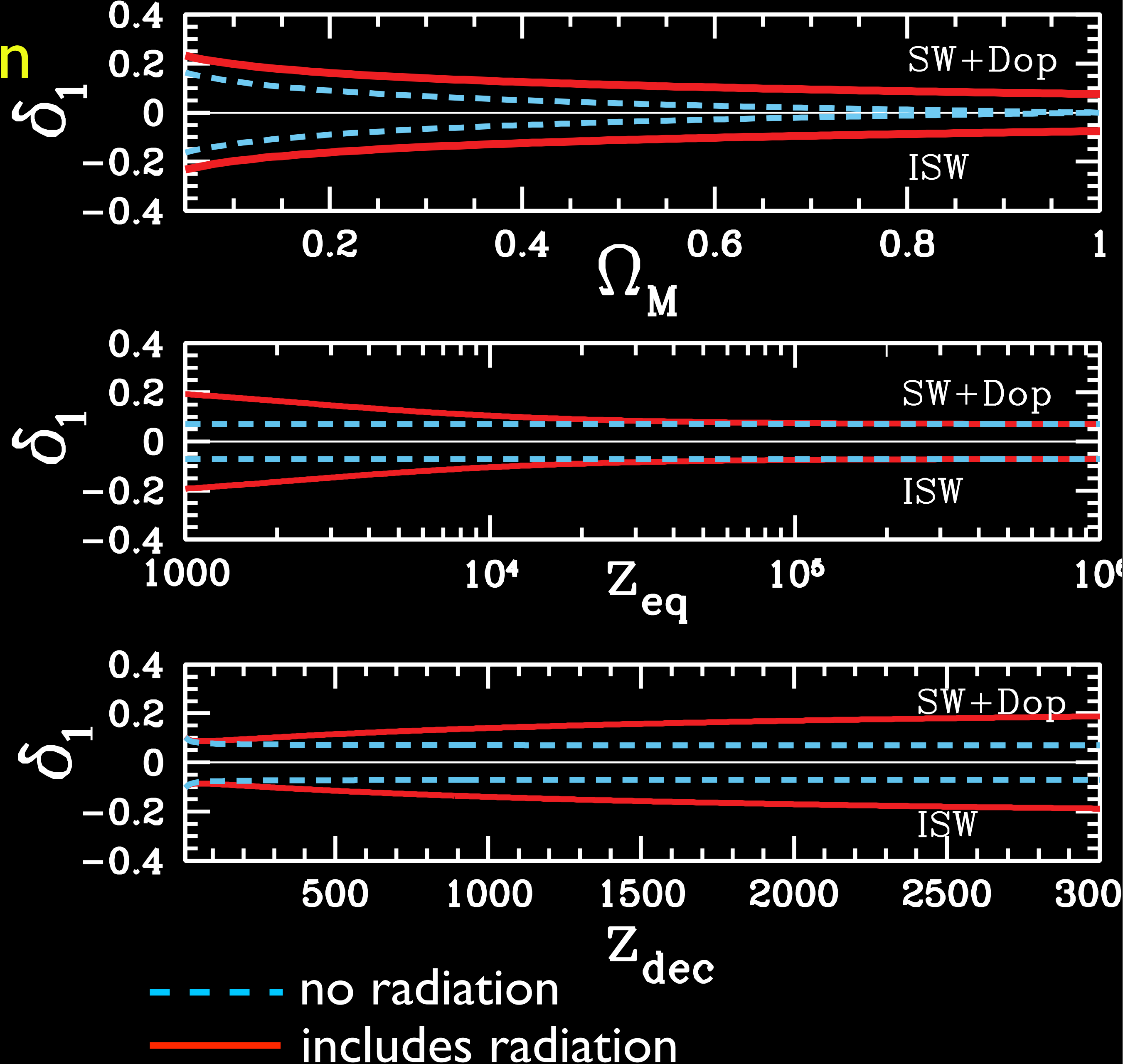
$$kH_0^{-1} \ll 1$$

Temperature anisotropy:

$$\frac{\Delta T}{T}(\hat{n}) = \delta_1 \Psi_{\rm SM}\left[\vec{k}\cdot\vec{x}_{\rm dec}\right]$$

*includes SW, Doppler and ISW anisotropies*

***The dipole cancels for all flat*** $\Lambda\mathrm{CDM}$ ***universes, even if radiation is included.***



# Matter and radiation aren't special...

The $\mathcal{O}(kx_{\rm dec})$ terms in $\Delta T$ for adiabatic perturbations cancel in flat universes that contain

- matter
- radiation
- cosmological constant

What if there's something else?

# Matter and radiation aren't special...

The $\mathcal{O}(kx_{\rm dec})$ terms in $\Delta T$ for adiabatic perturbations cancel in flat universes that contain

- matter
- radiation
- cosmological constant

*exotic fluid*
$w \geq 1/3$
*dominates early universe*

What if there's something else? $H^2(a) = H_0^2 \left[ \frac{\Omega_X}{a^{3(1+w)}} + \Omega_\Lambda \right]$

*cosmological constant*

# Matter and radiation aren't special...

The $\mathcal{O}(kx_{\rm dec})$ terms in $\Delta T$ for adiabatic perturbations cancel in flat universes that contain

- matter
- radiation
- cosmological constant

*exotic fluid*
$w \geq 1/3$
*dominates early universe*

What if there's something else?

$$H^2(a) = H_0^2 \left[ \frac{\Omega_X}{a^{3(1+w)}} + \Omega_\Lambda \right]$$

*cosmological constant*

*The dipole terms still cancel for adiabatic perturbations!*

Is there a physical reason for dipole cancellation in flat universes with superhorizon adiabatic perturbations?

- special synchronous gauge: metric is FRW + $\mathcal{O}(k^2 H_0^{-2})$
  *Hirata and Seljak 2005*
- galaxies have no peculiar velocity in synchronous gauge
- no $\mathcal{O}(kx_{\rm dec})$ temperature anisotropies

# Beyond the Dipole

A single superhorizon mode: $\Psi(\vec{x},t) = \Psi_{\mathrm{SM}}(t)\sin[\vec{k}\cdot\vec{x} + \varpi]$

$kH_0^{-1} \ll 1$

*phase of our location*

# Beyond the Dipole

A single superhorizon mode: $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x}+\varpi]$

$kH_0^{-1} \ll 1$

distance to last scattering surface

phase of our location

Temperature anisotropy:

Expansion in powers of $\vec{k}\cdot\vec{x}_{\rm d}$

$$\frac{\Delta T}{T}(\hat{n}) = \Psi_{\rm SM}\left[(\vec{k}\cdot\vec{x}_{\rm d})\delta_1\cos\varpi - (\vec{k}\cdot\vec{x}_{\rm d})^2\delta_2\frac{\sin\varpi}{2} - (\vec{k}\cdot\vec{x}_{\rm d})^3\delta_3\frac{\cos\varpi}{6}\right]$$

Observed CMB Temperature

Dipole

Quadrupole

Octupole

# Beyond the Dipole

A single superhorizon mode: $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x}+\varpi]$

$kH_0^{-1} \ll 1$

distance to last scattering surface

phase of our location

Temperature anisotropy: *Expansion in powers of* $\vec{k}\cdot\vec{x}_{\rm d}$

$$\frac{\Delta T}{T}(\hat{n}) = \Psi_{\rm SM}\left[\cancel{(\vec{k}\cdot\vec{x}_{\rm d})\delta_1\cos\varpi} - (\vec{k}\cdot\vec{x}_{\rm d})^2\delta_2\frac{\sin\varpi}{2} - (\vec{k}\cdot\vec{x}_{\rm d})^3\delta_3\frac{\cos\varpi}{6}\right]$$

*Observed CMB Temperature* *Dipole* *Quadrupole* *Octupole*

# Beyond the Dipole

A single superhorizon mode: $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x}+\varpi]$

$kH_0^{-1} \ll 1$

*distance to last scattering surface*

*phase of our location*

Temperature anisotropy: *Expansion in powers of $\vec{k}\cdot\vec{x}_{\rm d}$*

$$\frac{\Delta T}{T}(\hat{n}) = \Psi_{\rm SM}\left[\cancel{(\vec{k}\cdot\vec{x}_{\rm d})\delta_1\cos\varpi} - (\vec{k}\cdot\vec{x}_{\rm d})^2\delta_2\frac{\sin\varpi}{2} - (\vec{k}\cdot\vec{x}_{\rm d})^3\delta_3\frac{\cos\varpi}{6}\right]$$

*Observed CMB Temperature* *Dipole* *Quadrupole* *Octupole*

Multipole moments: $\frac{\Delta T}{T}(\hat{n}) = \sum_{\ell,m} a_{\ell m}Y_{\ell m}(\hat{n})$ $\longleftarrow \hat{k} = \hat{z}$

# Beyond the Dipole

**A single superhorizon mode:** $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x}+\varpi]$

$kH_0^{-1} \ll 1$

distance to last scattering surface

phase of our location

**Temperature anisotropy:** Expansion in powers of $\vec{k}\cdot\vec{x}_{\rm d}$

$$\frac{\Delta T}{T}(\hat{n}) = \Psi_{\rm SM}\left[\cancel{(\vec{k}\cdot\vec{x}_{\rm d})\delta_1\cos\varpi} - (\vec{k}\cdot\vec{x}_{\rm d})^2\delta_2\frac{\sin\varpi}{2} - (\vec{k}\cdot\vec{x}_{\rm d})^3\delta_3\frac{\cos\varpi}{6}\right]$$

Observed CMB Temperature | Dipole | Quadrupole | Octupole

**Multipole moments:** $$\frac{\Delta T}{T}(\hat{n}) = \sum_{\ell,m} a_{\ell m}Y_{\ell m}(\hat{n}) \longleftarrow \hat{k} = \hat{z}$$

$$a_{10} = -\sqrt{\frac{4\pi}{3}}(kx_{\rm d})^3\delta_3\frac{\cos\varpi}{10}\Psi_{\rm SM}(t_{\rm d})$$

- residual dipole moment
- comparable to octupole moment
- less restrictive constraint due to our proper motion

# Beyond the Dipole

A single superhorizon mode: $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x} + \varpi]$

$kH_0^{-1} \ll 1$

*distance to last scattering surface*

*phase of our location*

Temperature anisotropy: *Expansion in powers of $\vec{k}\cdot\vec{x}_{\rm d}$*

$$\frac{\Delta T}{T}(\hat{n}) = \Psi_{\rm SM}\left[\cancel{(\vec{k}\cdot\vec{x}_{\rm d})\delta_1\cos\varpi} - (\vec{k}\cdot\vec{x}_{\rm d})^2\delta_2\frac{\sin\varpi}{2} - (\vec{k}\cdot\vec{x}_{\rm d})^3\delta_3\frac{\cos\varpi}{6}\right]$$

*Observed CMB Temperature* *Dipole* *Quadrupole* *Octupole*

Multipole moments: $$\frac{\Delta T}{T}(\hat{n}) = \sum_{\ell,m} a_{\ell m}Y_{\ell m}(\hat{n}) \longleftarrow \hat{k} = \hat{z}$$

$$a_{10} = -\sqrt{\frac{4\pi}{3}}(kx_{\rm d})^3\delta_3\frac{\cos\varpi}{10}\Psi_{\rm SM}(t_{\rm d})$$

- residual dipole moment
- comparable to octupole moment
- less restrictive constraint due to our proper motion

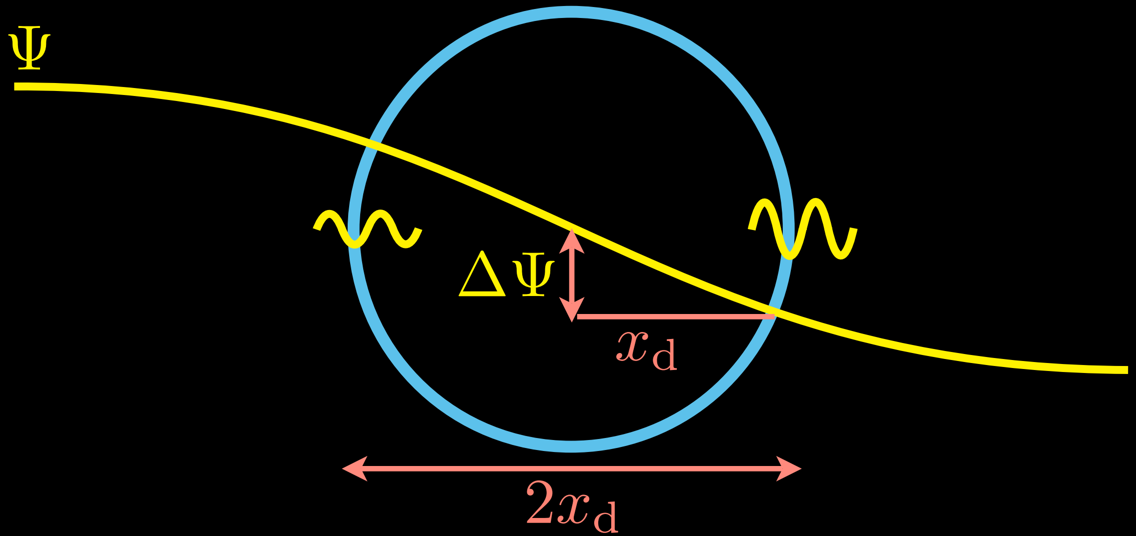
$$\boxed{\begin{aligned} a_{20} &= -\sqrt{\frac{4\pi}{5}}(kx_{\rm d})^2\delta_2\frac{\sin\varpi}{3}\Psi_{\rm SM}(t_{\rm d}) \\ a_{30} &= -\sqrt{\frac{4\pi}{7}}(kx_{\rm d})^3\delta_3\frac{\cos\varpi}{15}\Psi_{\rm SM}(t_{\rm d}) \end{aligned}}$$

# The Quadrupole Constraint

**Supermode:** $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x} + \varpi]$ ← *phase of our location*

Recall the motivation: $\Delta\phi \Longrightarrow$ power asymmetry

$$\Delta\phi \Longrightarrow \Delta\Psi \Longrightarrow \Delta T$$




$$\Delta\Psi \simeq (kx_{\rm d})\Psi_{\rm SM}|\cos\varpi|$$

*distance to last scattering surface*

# The Quadrupole Constraint

**Supermode:** $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x}+\varpi]$ ← *phase of our location*

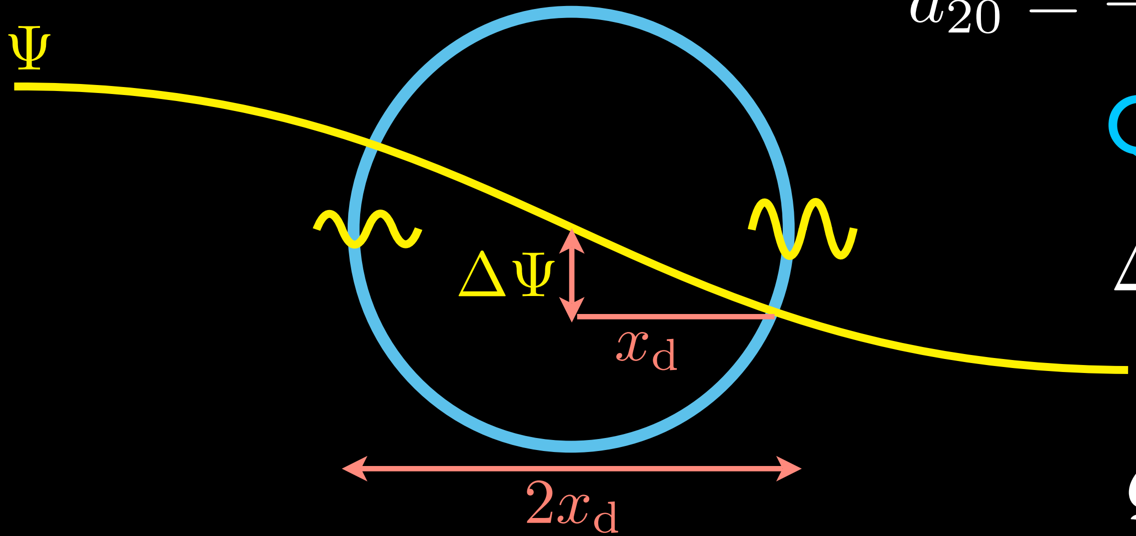
Recall the motivation: $\Delta\phi \Longrightarrow$ power asymmetry

$$\Delta\phi \Longrightarrow \Delta\Psi \Longrightarrow \Delta T$$

The supermode induces a CMB quadrupole:

$$a_{20} = -\sqrt{\frac{4\pi}{5}}(kx_{\rm d})^2\delta_2\frac{\sin\varpi}{3}\Psi_{\rm SM}(t_{\rm d})$$

$\delta_2 = 0.33$




$$\Delta\Psi \simeq (kx_{\rm d})\Psi_{\rm SM}|\cos\varpi|$$

*distance to last scattering surface*

**Quadrupole Constraint:**

$$\Delta\Psi(kx_{\rm d})|\tan\varpi| \lesssim 5.8\mathcal{Q}$$

*maximum allowed* $|a_{20}|$

$$\mathcal{Q} \lesssim 3\sqrt{C_2} \simeq 1.8\times10^{-5}$$

# The Quadrupole Constraint

Supermode: $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x}+\varpi]$ *phase of our location*

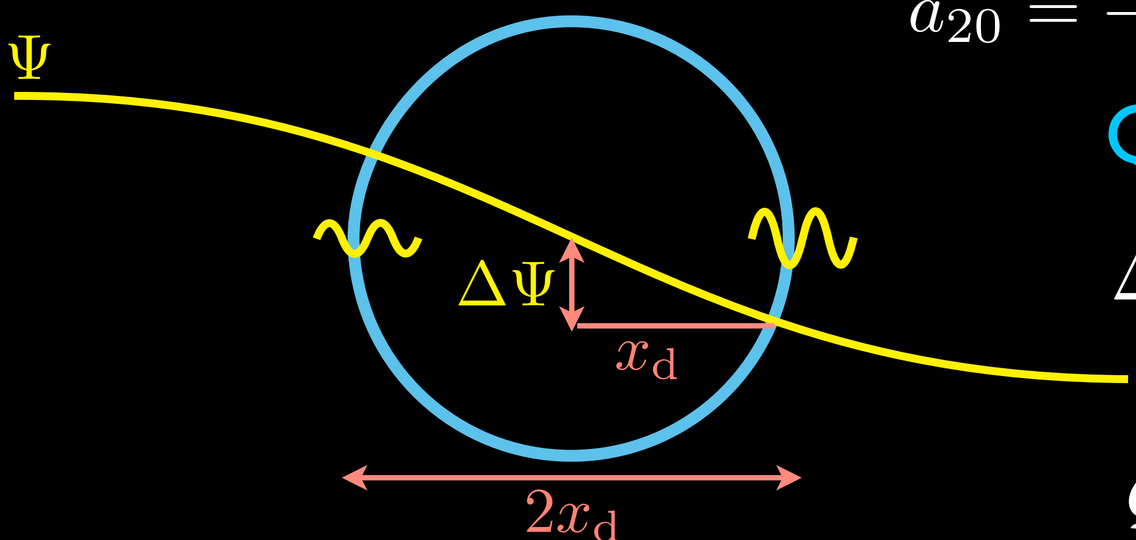
Recall the motivation: $\Delta\phi \Longrightarrow$ power asymmetry

$$\Delta\phi \Longrightarrow \Delta\Psi \Longrightarrow \Delta T$$

The supermode induces a CMB quadrupole:

$\delta_2 = 0.33$

$$a_{20} = -\sqrt{\frac{4\pi}{5}}(kx_{\rm d})^2\delta_2\frac{\sin\varpi}{3}\Psi_{\rm SM}(t_{\rm d})$$




$$\Delta\Psi \simeq (kx_{\rm d})\Psi_{\rm SM}|\cos\varpi|$$

*distance to last scattering surface*

Quadrupole Constraint:

$$\Delta\Psi(kx_{\rm d})|\tan\varpi| \lesssim 5.8\mathcal{Q}$$

*maximum allowed* $|a_{20}|$

$$\mathcal{Q} \lesssim 3\sqrt{C_2} \simeq 1.8\times 10^{-5}$$

*Quadrupole vanishes if* $\varpi = 0$.

# The Octupole Constraint

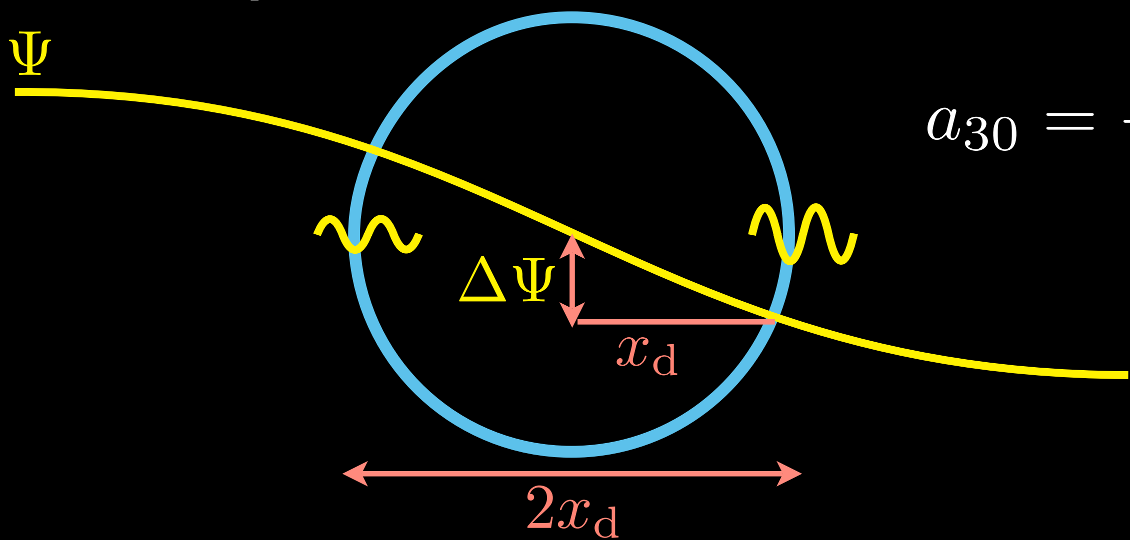
**Supermode:** $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x}+\varpi]$

*phase of our location*

The supermode induces a CMB **octupole**:

*$\delta_3 = 0.35$*

$$a_{30} = -\sqrt{\frac{4\pi}{7}}(kx_{\rm d})^3\delta_3\frac{\cos\varpi}{15}\Psi_{\rm SM}(t_{\rm d})$$




## Octupole Constraint:

$$\Delta\Psi(kx_{\rm d})^2 \lesssim 32\mathcal{O} \longleftarrow |a_{30}|$$

$$\mathcal{O} \lesssim 3\sqrt{C_3} \simeq 2.7\times 10^{-5}$$

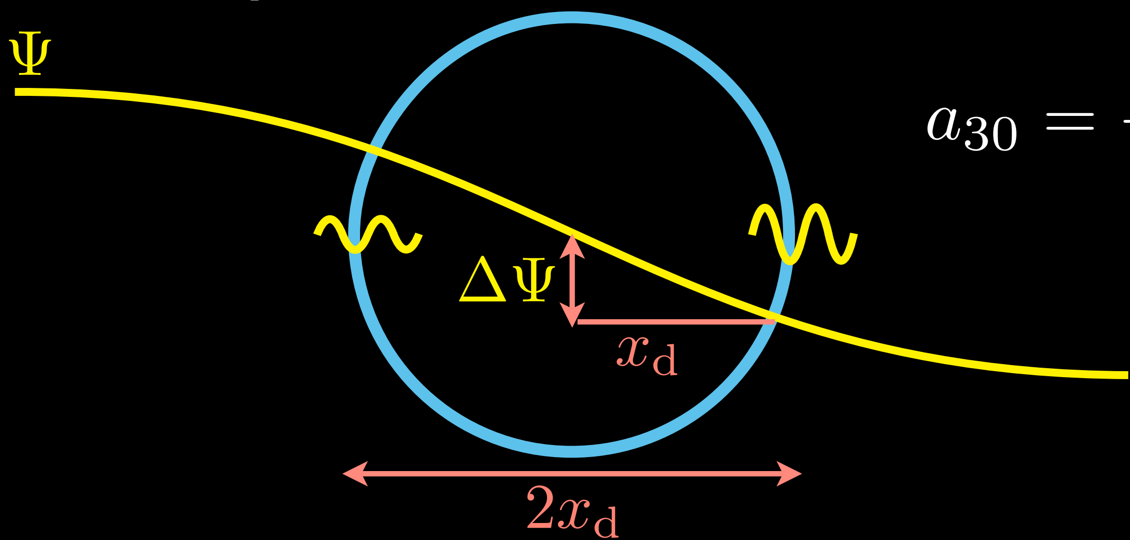
$$\Delta\Psi \simeq (kx_{\rm d})\Psi_{\rm SM}|\cos\varpi|$$

*distance to last scattering surface*

# The Octupole Constraint

**Supermode:** $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x}+\varpi]$ ← *phase of our location*

The supermode induces a CMB **octupole:**




$\delta_3 = 0.35$

$$a_{30} = -\sqrt{\frac{4\pi}{7}}(kx_{\rm d})^3\delta_3\frac{\cos\varpi}{15}\Psi_{\rm SM}(t_{\rm d})$$

$$\Delta\Psi \simeq (kx_{\rm d})\Psi_{\rm SM}|\cos\varpi|$$

*distance to last scattering surface*

## Octupole Constraint:

$$\Delta\Psi(kx_{\rm d})^2 \lesssim 32\mathcal{O} \leftarrow |a_{30}|$$

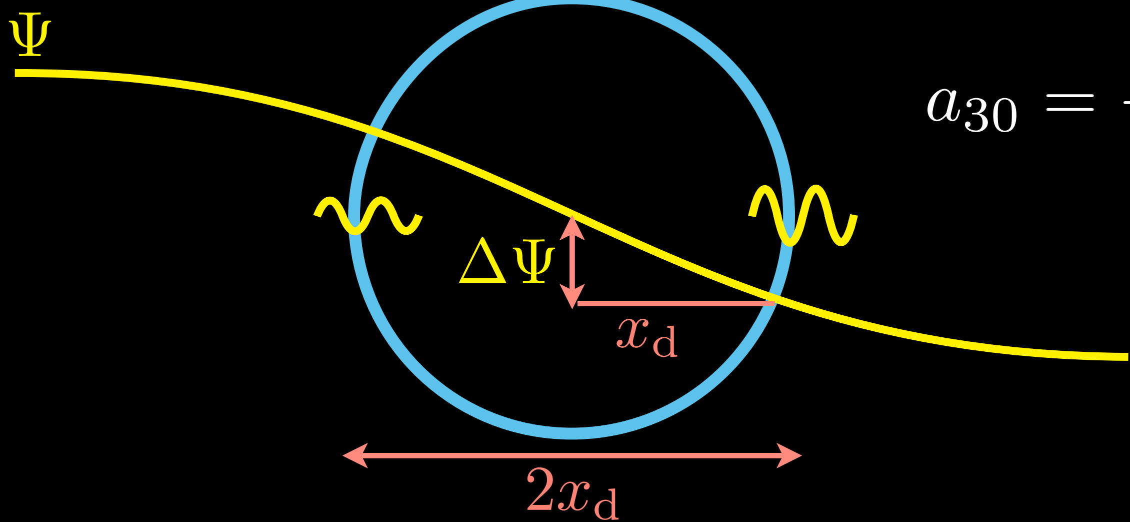
$$\mathcal{O} \lesssim 3\sqrt{C_3} \simeq 2.7\times10^{-5}$$

***Constraint is phase-independent.***

# The Octupole Constraint

**Supermode:** $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x}+\varpi]$ ← *phase of our location*

The supermode induces a CMB **octupole**:




$\delta_3 = 0.35$

$$a_{30} = -\sqrt{\frac{4\pi}{7}}(kx_{\rm d})^3\delta_3\frac{\cos\varpi}{15}\Psi_{\rm SM}(t_{\rm d})$$

$$\Delta\Psi \simeq (kx_{\rm d})\Psi_{\rm SM}|\cos\varpi|$$

*distance to last scattering surface*

## Octupole Constraint:

$$\Delta\Psi(kx_{\rm d})^2 \lesssim 32\mathcal{O} \longleftarrow |a_{30}|$$

$$\mathcal{O} \lesssim 3\sqrt{C_3} \simeq 2.7\times 10^{-5}$$

*Constraint is phase-independent.*

Evade constraint by decreasing $kx_{\rm d}$?

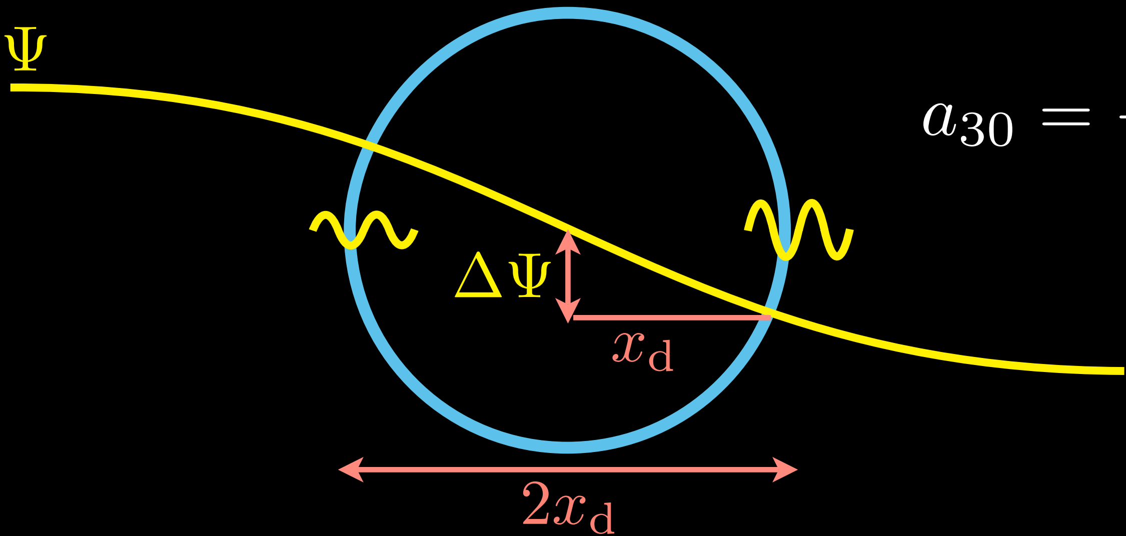
Not if we want **linearity beyond horizon**!

$$|\Psi| < 1 \implies \Delta\Psi \lesssim kx_{\rm d}$$

# The Octupole Constraint

**Supermode:** $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x} + \varpi]$ *phase of our location*

The supermode induces a CMB **octupole:**




$\delta_3 = 0.35$

$$a_{30} = -\sqrt{\frac{4\pi}{7}}(kx_{\rm d})^3\delta_3\frac{\cos\varpi}{15}\Psi_{\rm SM}(t_{\rm d})$$

$$\Delta\Psi \simeq (kx_{\rm d})\Psi_{\rm SM}|\cos\varpi|$$

*distance to last scattering surface*

## Octupole Constraint:

$$\Delta\Psi(kx_{\rm d})^2 \lesssim 32\mathcal{O} \longleftarrow |a_{30}|$$

$$\mathcal{O} \lesssim 3\sqrt{C_3} \simeq 2.7\times 10^{-5}$$

$$\Delta\Psi \lesssim [32\mathcal{O}]^{1/3} = 0.095$$

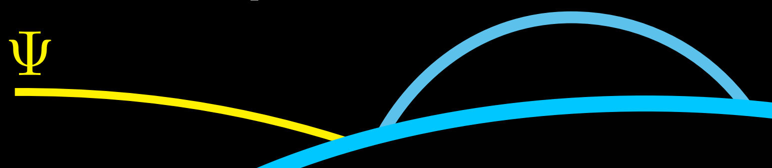
**Recall:** $\dfrac{\Delta P_\Psi}{P_\Psi} \propto \Delta\phi \propto \Delta\Psi$

$$\boxed{\frac{\Delta P_\Psi}{P_\Psi} \lesssim 0.01}$$

# The Octupole Constraint

**Supermode:** $\Psi(\vec{x},t) = \Psi_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x} + \varpi]$ ← phase of our location

The supermode induces a CMB **octupole:**




$\delta_3 = 0.35$

$$a_{30} = -\sqrt{\frac{4\pi}{7}}(kx_{\rm d})^3\delta_3\frac{\cos\varpi}{15}\Psi_{\rm SM}(t_{\rm d})$$

Octupole Constraint:

$$\Psi(kx_{\rm d})^2 \lesssim 32\mathcal{O} \longleftarrow |a_{30}|$$

$$\lesssim 3\sqrt{C_3} \simeq 2.7\times 10^{-5}$$

$$\Psi \lesssim [32\mathcal{O}]^{1/3} = 0.095$$

ll: $\frac{\Delta P_\Psi}{P_\Psi} \propto \Delta\phi \propto \Delta\Psi$

$$\boxed{\frac{\Delta P_\Psi}{P_\Psi} \lesssim 0.01}$$

Observed: $\frac{\Delta P_\Psi}{P_\Psi} \simeq 0.2$

*Way too big!*

# Part III

## The Curvaton Alternative

*Mollerach 1990; Linde, Mukhanov 1997; Lyth, Wands 2002; Moroi, Takahashi 2001; and others...*

# The Curvaton to the Rescue!

The problem with the inflaton model is two-fold:

- The fluctuation power is only weakly dependent on the background value.
  - $\Delta P \propto (1 - n_s)\Delta\phi$
  - A small power asymmetry requires a large fluctuation in $\phi$ .
- The inflaton dominates the energy density of the universe, so a "supermode" in the inflaton field generates a huge potential perturbation.
  - CMB octupole places upper bound on $\Delta\Psi$.
  - $\Delta P \propto \Delta\phi \propto \Delta\Psi$ with no wiggle room.

# The Curvaton to the Rescue!

The problem with the inflaton model is two-fold:

- The fluctuation power is only weakly dependent on the background value.
  - $\Delta P \propto (1 - n_s)\Delta\phi$
  - A small power asymmetry requires a large fluctuation in $\phi$ .
- The inflaton dominates the energy density of the universe, so a "supermode" in the inflaton field generates a huge potential perturbation.
  - CMB octupole places upper bound on $\Delta\Psi$.
  - $\Delta P \propto \Delta\phi \propto \Delta\Psi$ with no wiggle room.

The solution: the primordial fluctuations could be generated by a subdominant scalar field, the curvaton.

- The fluctuation power depends strongly on the background curvaton value.
- The CMB constraints on $\Delta\Psi$ do not directly constrain $\Delta P$. There is a new free parameter: the fraction of energy in the curvaton.

# The Curvaton during Inflation

- The inflaton still dominates the energy density and drives inflation.
- The curvaton $(\sigma)$ is a subdominant light scalar field during inflation.

$$\underbrace{V(\sigma)}_{\text{potential}} = \frac{1}{2}m_\sigma^2\sigma^2 \quad \text{with} \quad \underbrace{m_\sigma \ll H_{\inf}(\phi)}_{\text{light scalar field}} \quad \text{and} \quad \underbrace{\rho_\sigma \ll \rho_\phi}_{\text{subdominant}}$$

# The Curvaton during Inflation

- The inflaton still dominates the energy density and drives inflation.
- The curvaton $(\sigma)$ is a subdominant light scalar field during inflation.

$$V(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 \quad \text{with} \quad m_\sigma \ll H_{\rm inf}(\phi) \quad \text{and} \quad \rho_\sigma \ll \rho_\phi$$

*potential* *light scalar field* *subdominant*

- There are quantum fluctuations in both the inflaton and curvaton.

$$(\delta\phi)_{\rm rms} = (\delta\sigma)_{\rm rms} = \frac{H_{\rm inf}}{2\pi} \ll \bar{\sigma} \leftarrow$$

*quantum fluctuations* *homogeneous background value*

# The Curvaton during Inflation

- The inflaton still dominates the energy density and drives inflation.
- The curvaton $(\sigma)$ is a subdominant light scalar field during inflation.

$$\underbrace{V(\sigma)}_{\text{potential}} = \frac{1}{2} m_\sigma^2 \sigma^2 \quad \text{with} \quad \underbrace{m_\sigma \ll H_{\inf}(\phi)}_{\text{light scalar field}} \quad \text{and} \quad \underbrace{\rho_\sigma \ll \rho_\phi}_{\text{subdominant}}$$

- There are quantum fluctuations in both the inflaton and curvaton.

$$\underbrace{(\delta\phi)_{\rm rms} = (\delta\sigma)_{\rm rms}}_{\text{quantum fluctuations}} = \frac{H_{\inf}}{2\pi} \ll \bar{\sigma}$$

← homogeneous background value

- Outside the horizon, $\delta\sigma$ and $\bar{\sigma}$ obey the same equation of motion:

$$\ddot{\bar{\sigma}} + 3H\dot{\bar{\sigma}} + V'(\bar{\sigma}) = 0$$

$$\delta\ddot{\sigma} + 3H\delta\dot{\sigma} + \left[\frac{k^2}{a^2} + V''(\bar{\sigma})\right]\delta\sigma = 0$$

# The Curvaton during Inflation

- The **inflaton** still dominates the energy density and **drives inflation.**
- The **curvaton** $(\sigma)$ is a **subdominant light scalar field** during inflation.

$$\underbrace{V(\sigma)}_{\text{potential}} = \frac{1}{2}m_\sigma^2\sigma^2 \quad \text{with} \quad \underbrace{m_\sigma \ll H_{\text{inf}}(\phi)}_{\text{light scalar field}} \quad \text{and} \quad \underbrace{\rho_\sigma \ll \rho_\phi}_{\text{subdominant}}$$

- There are **quantum fluctuations** in both the inflaton and curvaton.

$$\underbrace{(\delta\phi)_{\text{rms}} = (\delta\sigma)_{\text{rms}}}_{\text{quantum fluctuations}} = \frac{H_{\text{inf}}}{2\pi} \ll \bar{\sigma} \leftarrow \text{homogeneous background value}$$

- Outside the horizon, $\delta\sigma$ and $\bar{\sigma}$ obey the same equation of motion:

$$\ddot{\bar{\sigma}} + 3H\dot{\bar{\sigma}} + \underbrace{V'(\bar{\sigma})}_{m_\sigma^2\bar{\sigma}} = 0$$

$$\delta\ddot{\sigma} + 3H\delta\dot{\sigma} + \left[\cancel{\frac{k^2}{a^2}} + \overbrace{V''(\bar{\sigma})}^{m_\sigma^2\delta\sigma}\right]\delta\sigma = 0$$

*For superhorizon perturbations,* $\dfrac{\delta\sigma}{\bar{\sigma}}$ *is conserved both during and after inflation.*

# The Curvaton after Inflation

The curvaton equation of motion: $\ddot{\sigma} + 3H\dot{\sigma} + m_\sigma^2\sigma^2 = 0$

- As long as $m_\sigma \ll H$, the curvaton is frozen: $\dot{\sigma} = 0$
- When $m_\sigma \simeq H$, the curvaton **oscillates**: $\langle\dot{\sigma}^2\rangle = \langle m_\sigma^2\sigma^2\rangle$

$$p = \frac{1}{2}\dot{\sigma}^2 - \frac{1}{2}m_\sigma^2\sigma^2 \implies \langle p\rangle = 0$$

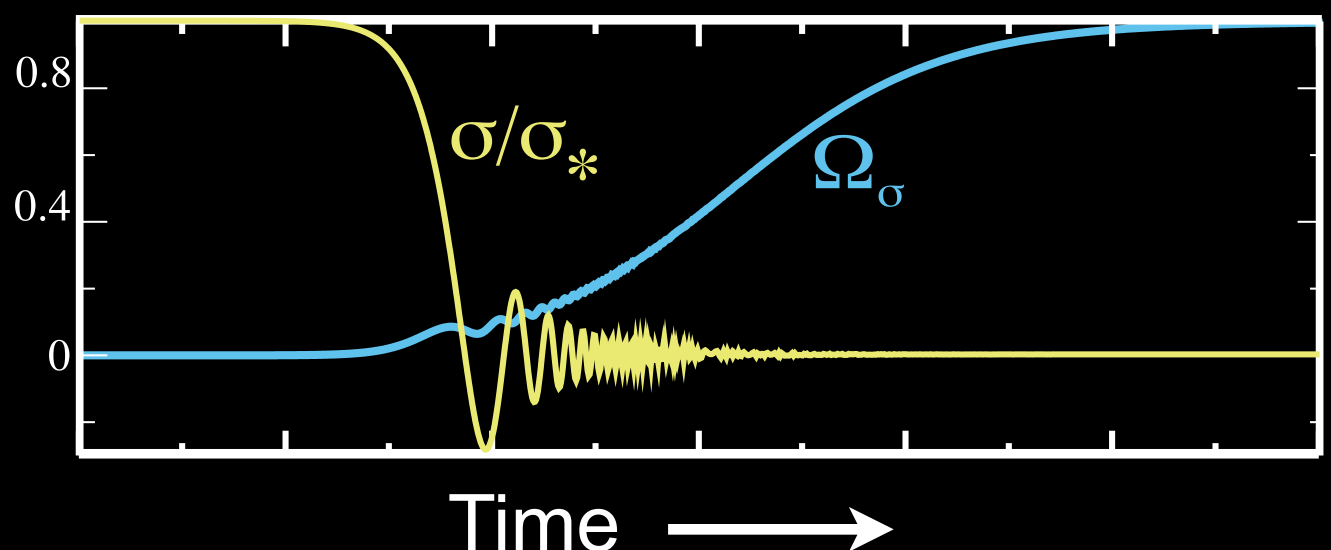
# The Curvaton after Inflation

The curvaton equation of motion: $\ddot{\sigma} + 3H\dot{\sigma} + m_\sigma^2\sigma^2 = 0$

- As long as $m_\sigma \ll H$, the curvaton is frozen: $\dot{\sigma} = 0$
- When $m_\sigma \simeq H$, the curvaton **oscillates**: $\langle\dot{\sigma}^2\rangle = \langle m_\sigma^2\sigma^2\rangle$

$$p = \frac{1}{2}\dot{\sigma}^2 - \frac{1}{2}m_\sigma^2\sigma^2 \implies \langle p\rangle = 0$$

While the curvaton oscillates, it **behaves as matter**: $\rho_\sigma \propto a^{-3}$
Meanwhile, $\rho_r \propto a^{-4}$, so $\rho_\sigma/\rho_r$ **increases.**




*Langlois and Vernizzi, PRD 70 063522 (2004).*

# Growth of a Curvature Perturbation

Curvature perturbation: $\zeta = -\Psi - H\dfrac{\delta\rho}{\dot{\rho}}$

Superhorizon $\zeta$ is not conserved due to curvaton isocurvature fluctuation, but $\zeta_i = -\Psi - H\dfrac{\delta\rho_i}{\dot{\rho}_i}$ is constant.

$$\zeta = \frac{4\rho_r\zeta_r + 3\rho_\sigma\zeta_\sigma}{4\rho_r + 3\rho_\sigma}$$

As $\rho_\sigma/\rho_r$ increases, $\zeta$ evolves.

# Growth of a Curvature Perturbation

Curvature perturbation: $\zeta = -\Psi - H\dfrac{\delta\rho}{\dot\rho}$

Superhorizon $\zeta$ is not conserved due to curvaton isocurvature fluctuation, but $\zeta_i = -\Psi - H\dfrac{\delta\rho_i}{\dot\rho_i}$ is constant.

$$\zeta = \frac{4\rho_r\zeta_r + 3\rho_\sigma\zeta_\sigma}{4\rho_r + 3\rho_\sigma}$$

As $\rho_\sigma/\rho_r$ increases, $\zeta$ evolves.

In the very early universe, the curvaton decays into radiation.

- decay at $\Gamma \simeq H$
- residual curvature perturbation:

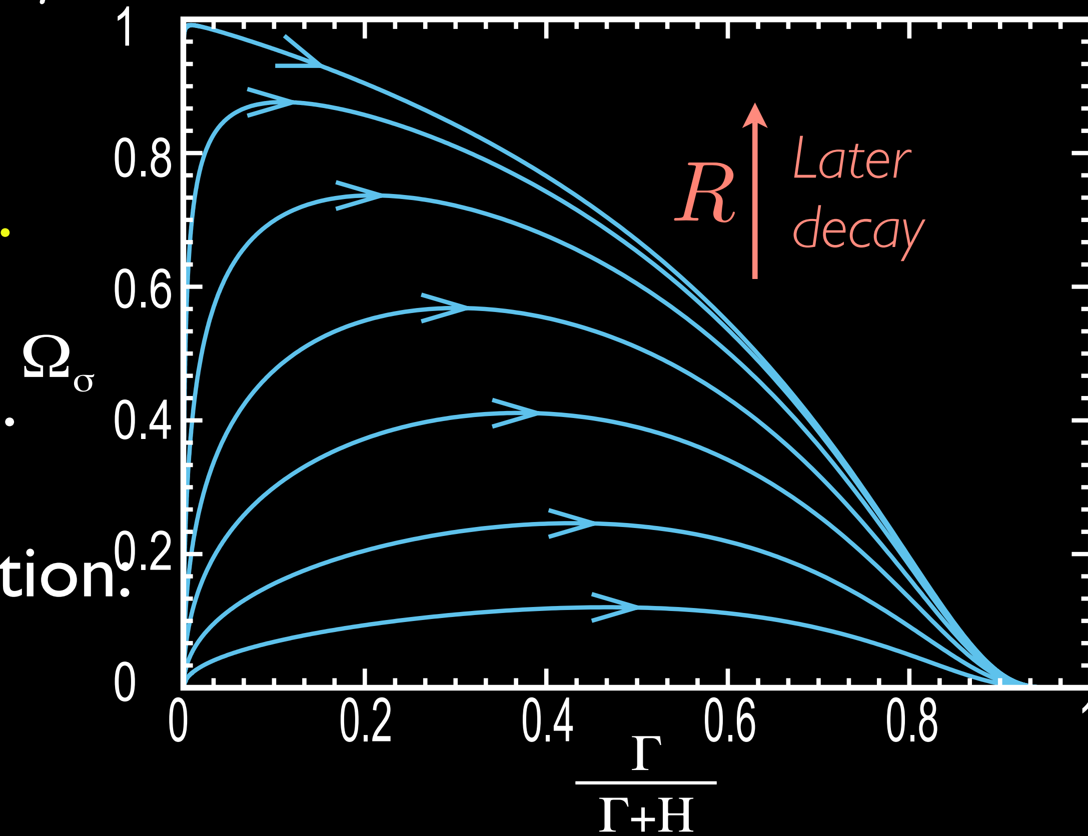
$\zeta = R\zeta_\sigma$

curvature perturbation from curvaton

new parameter

$$R \simeq \frac{3}{4}\left.\frac{\rho_\sigma}{\rho_r}\right|_{\Gamma=H}$$

$R \ll 1$




Malik, Wands and Ungarelli. PRD 67 063516 (2003)

# Power Spectrum from the Curvaton

Fluctuations in the curvaton field become curvature perturbations.

$$\zeta = R\zeta_\sigma = \frac{R}{3}\frac{\delta\rho_\sigma}{\rho_\sigma} \quad \text{where} \quad R \simeq \frac{3}{4}\left.\frac{\rho_\sigma}{\rho_r}\right|_{\Gamma=H} \quad \text{and} \quad R \ll 1$$

*curvature perturbation from curvaton*

*evaluated just prior to curvaton decay*

*keep the curvaton subdominant*

# Power Spectrum from the Curvaton

Fluctuations in the curvaton field become curvature perturbations.

$$\zeta = R\zeta_\sigma = \frac{R}{3}\frac{\delta\rho_\sigma}{\rho_\sigma} \quad \text{where} \quad R \simeq \frac{3}{4}\left.\frac{\rho_\sigma}{\rho_r}\right|_{\Gamma=H} \quad \text{and} \quad R \ll 1$$

*curvature perturbation from curvaton*

*evaluated just prior to curvaton decay*

*keep the curvaton subdominant*

**Curvaton energy:** $\rho_\sigma = \frac{1}{2}m_\sigma^2\sigma^2 \Longrightarrow \frac{\delta\rho_\sigma}{\rho_\sigma} = 2\left(\frac{\delta\sigma}{\bar{\sigma}}\right) + \left(\frac{\delta\sigma}{\bar{\sigma}}\right)^2$

*conserved outside horizon*

**Quantum fluctuations:** $(\delta\sigma)_{\mathrm{rms}} = \frac{H_{\mathrm{inf}}}{2\pi} \ll \bar{\sigma}$

During matter domination, $\Psi = -\frac{3}{5}\zeta.$

*potential perturbation at decoupling*

$$\boxed{P_{\Psi,\sigma} \propto R^2\left(\frac{H_{\mathrm{inf}}}{\bar{\sigma}_*}\right)^2}$$

# Power Spectrum from the Curvaton

Fluctuations in the curvaton field become curvature perturbations.

$$\zeta = R\zeta_\sigma = \frac{R}{3}\frac{\delta\rho_\sigma}{\rho_\sigma} \quad \text{where} \quad R \simeq \frac{3}{4}\left.\frac{\rho_\sigma}{\rho_r}\right|_{\Gamma=H} \quad \text{and} \quad R \ll 1$$

*curvature perturbation from curvaton*

*evaluated just prior to curvaton decay*

*keep the curvaton subdominant*

**Curvaton energy:** $\rho_\sigma = \frac{1}{2}m_\sigma^2\sigma^2 \Longrightarrow \frac{\delta\rho_\sigma}{\rho_\sigma} = 2\left(\frac{\delta\sigma}{\bar{\sigma}}\right) + \left(\frac{\delta\sigma}{\bar{\sigma}}\right)^2$

*conserved outside horizon*

**Quantum fluctuations:** $(\delta\sigma)_{\mathrm{rms}} = \frac{H_{\mathrm{inf}}}{2\pi} \ll \bar{\sigma}$

During matter domination, $\Psi = -\frac{3}{5}\zeta.$

*potential perturbation at decoupling*

$$\boxed{P_{\Psi,\sigma} \propto (\bar{\sigma}_*)^4\left(\frac{H_{\mathrm{inf}}}{\bar{\sigma}_*}\right)^2}$$

# Power *Asymmetry* from the Curvaton

Fluctuations in the curvaton field become curvature perturbations.

$$\zeta = R\zeta_\sigma = \frac{R}{3}\frac{\delta\rho_\sigma}{\rho_\sigma} \quad \text{where} \quad R \simeq \frac{3}{4}\left.\frac{\rho_\sigma}{\rho_r}\right|_{\Gamma=H} \quad \text{and} \quad R \ll 1$$

*curvature perturbation from curvaton*

*evaluated just prior to curvaton decay*

*keep the curvaton subdominant*

**Curvaton energy:** $\rho_\sigma = \frac{1}{2}m_\sigma^2\sigma^2 \Longrightarrow \frac{\delta\rho_\sigma}{\rho_\sigma} = 2\left(\frac{\delta\sigma}{\bar{\sigma}}\right) + \left(\frac{\delta\sigma}{\bar{\sigma}}\right)^2$

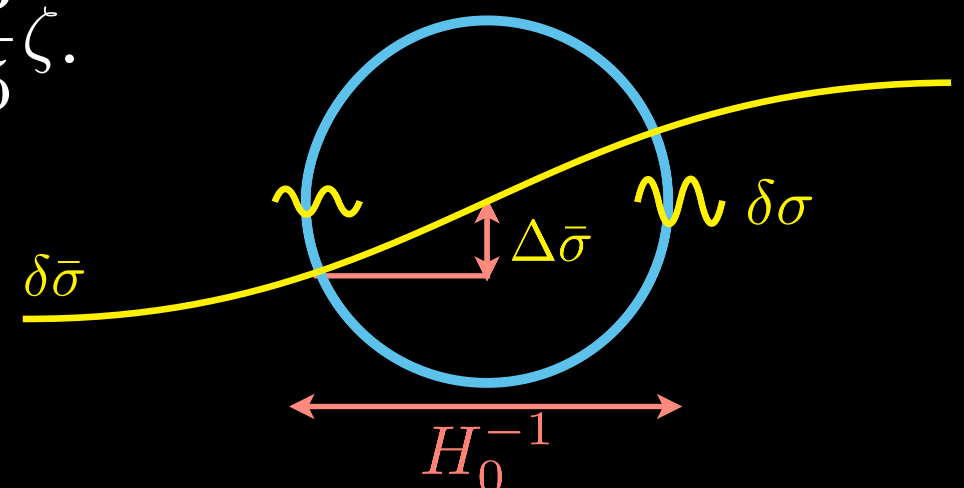
*conserved outside horizon*

**Quantum fluctuations:** $(\delta\sigma)_{\mathrm{rms}} = \frac{H_{\mathrm{inf}}}{2\pi} \ll \bar{\sigma}$

During matter domination, $\Psi = -\frac{3}{5}\zeta.$

*potential perturbation at decoupling*

$$\boxed{P_{\Psi,\sigma} \propto (\bar{\sigma}_*)^4\left(\frac{H_{\mathrm{inf}}}{\bar{\sigma}_*}\right)^2}$$



# Power *Asymmetry* from the Curvaton

Fluctuations in the curvaton field become **curvature perturbations.**

$$\zeta = R\zeta_\sigma = \frac{R}{3}\frac{\delta\rho_\sigma}{\rho_\sigma} \quad \text{where} \quad R \simeq \frac{3}{4}\left.\frac{\rho_\sigma}{\rho_r}\right|_{\Gamma=H} \quad \text{and} \quad R \ll 1$$

*curvature perturbation from curvaton*

*evaluated just prior to curvaton decay*

*keep the curvaton subdominant*

**Curvaton energy:** $\rho_\sigma = \frac{1}{2}m_\sigma^2\sigma^2 \Longrightarrow \frac{\delta\rho_\sigma}{\rho_\sigma} = 2\left(\frac{\delta\sigma}{\bar{\sigma}}\right) + \left(\frac{\delta\sigma}{\bar{\sigma}}\right)^2$

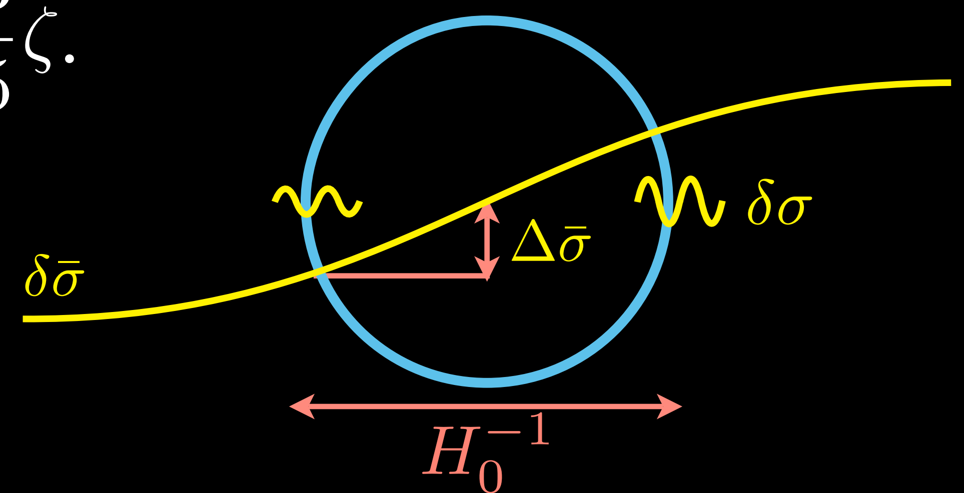
*conserved outside horizon*

**Quantum fluctuations:** $(\delta\sigma)_{\mathrm{rms}} = \frac{H_{\mathrm{inf}}}{2\pi} \ll \bar{\sigma}$

During matter domination, $\Psi = -\frac{3}{5}\zeta.$

*potential perturbation at decoupling*

$$\boxed{\frac{\Delta P_{\Psi,\sigma}}{P_{\Psi,\sigma}} = 2\frac{\Delta\bar{\sigma}}{\bar{\sigma}}}$$



# *Part IV*

## A Power Asymmetry from the Curvaton

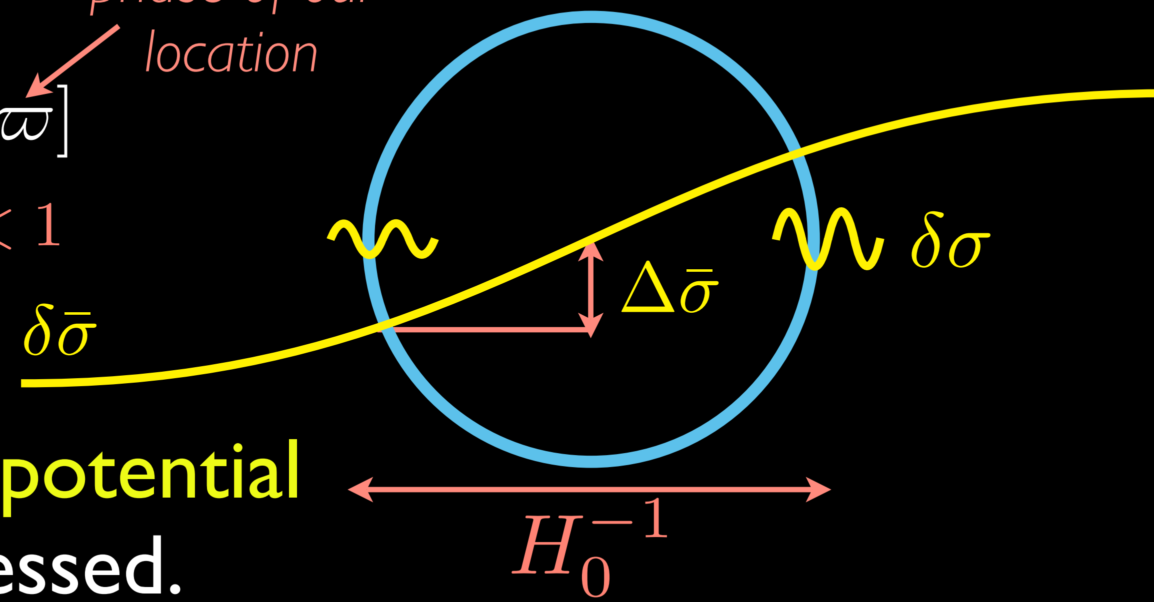
# Curvaton Supermodes in the CMB

Curvaton supermode:

$$\delta\bar{\sigma}(\vec{x},t) = \bar{\sigma}_{\mathrm{SM}}(t)\sin[\vec{k}\cdot\vec{x}+\varpi]$$

*phase of our location*

$$kH_0^{-1} \ll 1$$




The curvaton supermode generates a superhorizon potential fluctuation, but it is suppressed.

$$R \simeq \frac{3\rho_\sigma}{4\rho} \text{ just prior to decay}$$

$$\Psi = -\frac{R}{5}\left[2\left(\frac{\delta\bar{\sigma}}{\bar{\sigma}}\right)+\left(\frac{\delta\bar{\sigma}}{\bar{\sigma}}\right)^2\right] \longleftarrow \frac{\delta\rho_\sigma}{\rho}$$

The potential perturbation is not sinusoidal!

# Curvaton Supermodes in the CMB

**Curvaton supermode:**

$$\delta\bar{\sigma}(\vec{x},t) = \bar{\sigma}_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x}+\varpi]$$

$kH_0^{-1} \ll 1$

$$F_2(\varpi) = \sin\varpi - \left(\frac{\bar{\sigma}_{\rm SM}}{\bar{\sigma}}\right)\cos 2\varpi$$

$$F_3(\varpi) = \cos\varpi + 2\left(\frac{\bar{\sigma}_{\rm SM}}{\bar{\sigma}}\right)\sin 2\varpi$$

**Temperature anisotropy:**

$$\frac{\Delta T}{T}(\hat{n}) = \frac{2R}{5}\frac{\bar{\sigma}_{\rm SM}}{\bar{\sigma}}\left[\underbrace{(\vec{k}\cdot x_{\rm d})^2\delta_2\frac{F_2(\varpi)}{2}}_{\textit{Quadrupole}} + \underbrace{(\vec{k}\cdot x_{\rm d})^3\delta_3\frac{F_3(\varpi)}{6}}_{\textit{Octupole}}\right]$$

- The CMB quadrupole and octupole have complicated $\varpi$ dependencies.
- There is no phase that eliminates the quadrupole for all values of $\bar{\sigma}_{\rm SM}$.

# Curvaton Supermodes in the CMB

Curvaton supermode:

$$\delta\bar{\sigma}(\vec{x},t) = \bar{\sigma}_{\rm SM}(t)\sin[\vec{k}\cdot\vec{x}+\varpi]$$

$kH_0^{-1} \ll 1$

$$F_2(\varpi) = \sin\varpi - \left(\frac{\bar{\sigma}_{\rm SM}}{\bar{\sigma}}\right)\cos 2\varpi$$

$$F_3(\varpi) = \cos\varpi + 2\left(\frac{\bar{\sigma}_{\rm SM}}{\bar{\sigma}}\right)\sin 2\varpi$$

Temperature anisotropy:

$$\frac{\Delta T}{T}(\hat{n}) = \frac{2R}{5}\frac{\bar{\sigma}_{\rm SM}}{\bar{\sigma}}\left[\underbrace{(\vec{k}\cdot x_{\rm d})^2\delta_2\frac{F_2(\varpi)}{2}}_{\text{Quadrupole}} + \underbrace{(\vec{k}\cdot x_{\rm d})^3\delta_3\frac{F_3(\varpi)}{6}}_{\text{Octupole}}\right]$$

- The CMB quadrupole and octupole have complicated $\varpi$ dependencies.
- There is no phase that eliminates the quadrupole for all values of $\bar{\sigma}_{\rm SM}$.

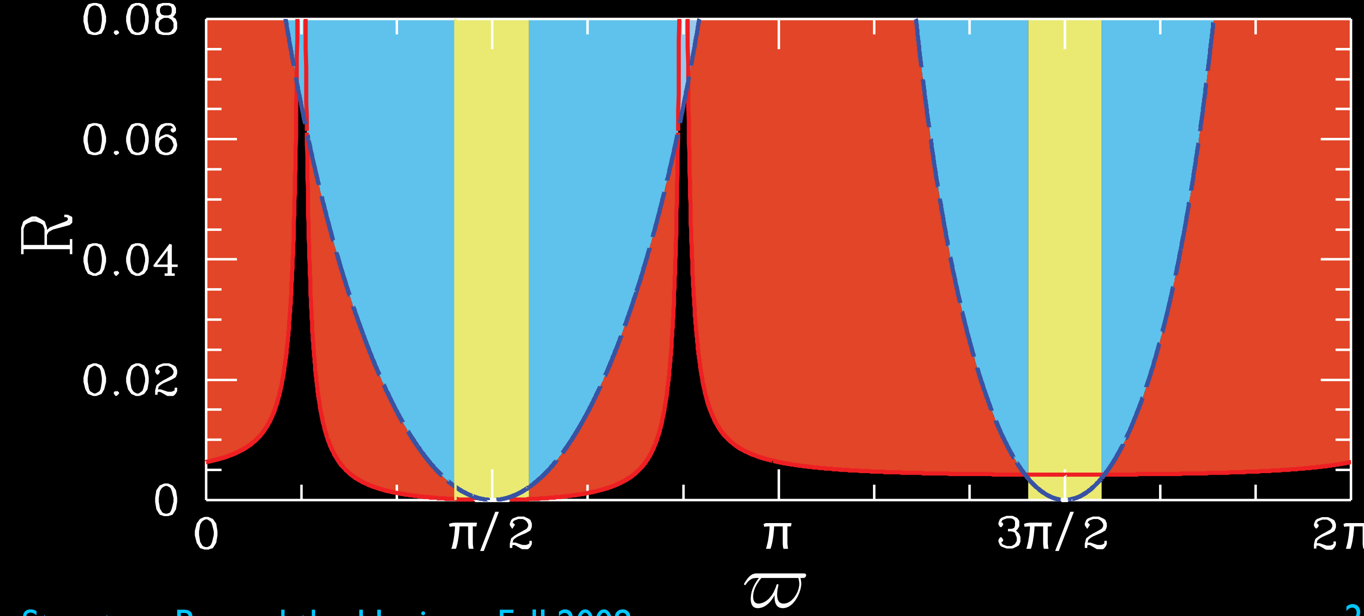
*Excluded by Quadrupole*

*Excluded by Octupole*

*Not superhorizon*

$$\bar{\sigma}_{\rm SM} = \bar{\sigma}$$

$$\Delta\bar{\sigma}/\bar{\sigma} = 0.2$$



# Curvaton Supermodes in the CMB

The CMB **quadrupole** implies an upper bound:

$$R\left(\frac{\Delta\bar{\sigma}}{\bar{\sigma}}\right)^2 \lesssim \frac{5}{2}(5.8Q) \quad \text{for} \quad \varpi = 0$$

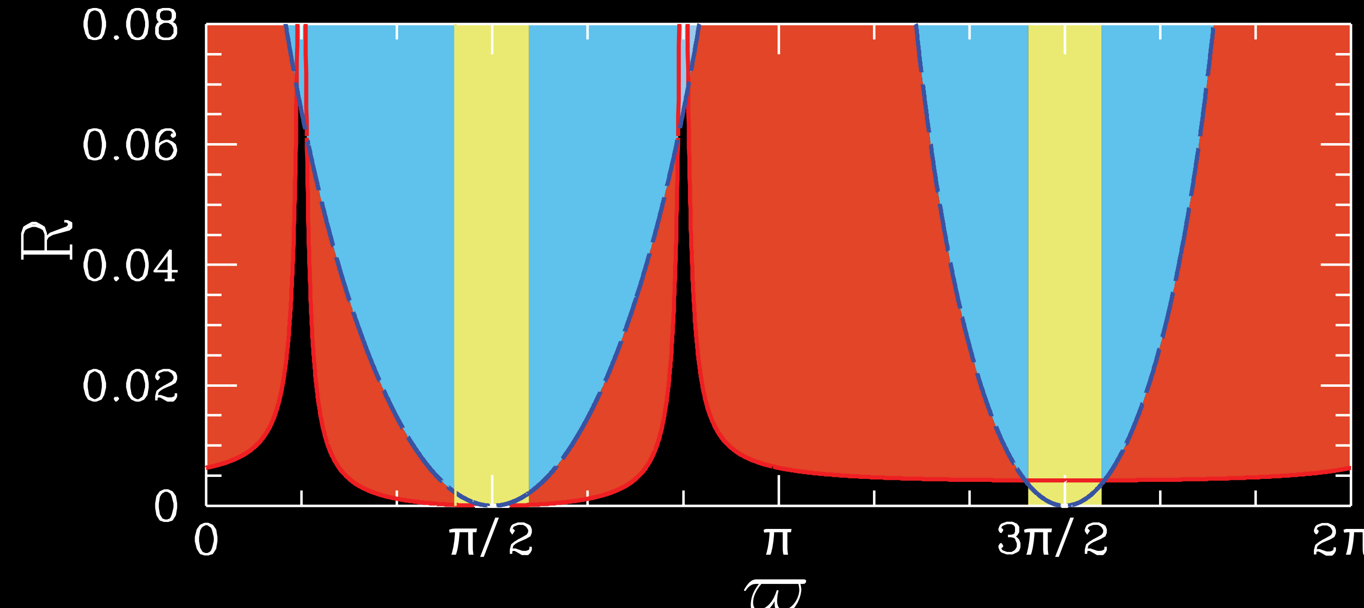
$$\left(\frac{\Delta P_\Psi}{P_\Psi}\right)$$

*Most other phases give similar bounds.*

*Excluded by Quadrupole*
*Excluded by Octupole*
*Not superhorizon*

$$\bar{\sigma}_{\rm SM} = \bar{\sigma}$$

$$\Delta\bar{\sigma}/\bar{\sigma} = 0.2$$



# Perturbation Mixture

Both the curvaton and the inflaton may contribute to $P_\Psi$.

$$\epsilon \equiv \frac{m_{\mathrm{Pl}}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]^2 \qquad (\delta\phi)_{\mathrm{rms}} = (\delta\sigma)_{\mathrm{rms}} = \frac{H_{\mathrm{inf}}}{2\pi}$$

*quantum fluctuations*

$$R \simeq \frac{3}{4}\left.\frac{\rho_\sigma}{\rho_r}\right|_{\Gamma=H}$$

$$P_{\Psi,\phi} = \left(\frac{9}{10}\right)^2 \frac{8\pi}{9\epsilon}\left(\frac{H_{\mathrm{inf}}^2}{k^3 m_{\mathrm{Pl}}^2}\right) \qquad P_{\Psi,\sigma} = \left(\frac{2R}{5}\right)^2 \frac{H_{\mathrm{inf}}^2}{2k^3\bar{\sigma}^2}$$

# Perturbation Mixture

Both the curvaton and the inflaton may contribute to $P_\Psi$.

$$\epsilon \equiv \frac{m_{\rm Pl}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]^2$$

$$(\delta\phi)_{\rm rms} = (\delta\sigma)_{\rm rms} = \frac{H_{\rm inf}}{2\pi}$$

*quantum fluctuations*

$$R \simeq \frac{3}{4}\left.\frac{\rho_\sigma}{\rho_r}\right|_{\Gamma=H}$$

$$P_{\Psi,\phi} = \left(\frac{9}{10}\right)^2 \frac{8\pi}{9\epsilon}\left(\frac{H_{\rm inf}^2}{k^3 m_{\rm Pl}^2}\right)$$

$$P_{\Psi,\sigma} = \left(\frac{2R}{5}\right)^2 \frac{H_{\rm inf}^2}{2k^3\bar{\sigma}^2}$$

Define a new parameter: $\xi \equiv \dfrac{P_{\Psi,\sigma}}{P_{\Psi,\sigma}+P_{\Psi,\phi}}$

$$\xi = \frac{\tilde{\epsilon}}{\tilde{\epsilon}+1} \qquad \tilde{\epsilon} \equiv \frac{1}{9\pi}\left(\frac{m_{\rm Pl}}{\bar{\sigma}}\right)^2 R^2\epsilon$$

$$\bar{\sigma} \ll m_{\rm Pl} \implies \xi \simeq 1$$
$$\bar{\sigma} \lesssim m_{\rm Pl} \implies \xi \ll 1$$

# Perturbation Mixture

Both the curvaton and the inflaton may contribute to $P_\Psi$.

$$\epsilon \equiv \frac{m_{\rm Pl}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]^2$$

$$(\delta\phi)_{\rm rms} = (\delta\sigma)_{\rm rms} = \frac{H_{\rm inf}}{2\pi}$$

*quantum fluctuations*

$$R \simeq \frac{3}{4}\left.\frac{\rho_\sigma}{\rho_r}\right|_{\Gamma=H}$$

$$P_{\Psi,\phi} = \left(\frac{9}{10}\right)^2 \frac{8\pi}{9\epsilon}\left(\frac{H_{\rm inf}^2}{k^3 m_{\rm Pl}^2}\right)$$

$$P_{\Psi,\sigma} = \left(\frac{2R}{5}\right)^2 \frac{H_{\rm inf}^2}{2k^3\bar{\sigma}^2}$$

Define a new parameter: $\xi \equiv \dfrac{P_{\Psi,\sigma}}{P_{\Psi,\sigma} + P_{\Psi,\phi}}$

$$\xi = \frac{\tilde{\epsilon}}{\tilde{\epsilon}+1} \qquad \tilde{\epsilon} \equiv \frac{1}{9\pi}\left(\frac{m_{\rm Pl}}{\bar{\sigma}}\right)^2 R^2 \epsilon$$

$$\bar{\sigma} \ll m_{\rm Pl} \implies \xi \simeq 1$$

$$\bar{\sigma} \lesssim m_{\rm Pl} \implies \xi \ll 1$$

**Tensor-Scalar Ratio:**

$$\boxed{r = 16\epsilon(1-\xi)}$$

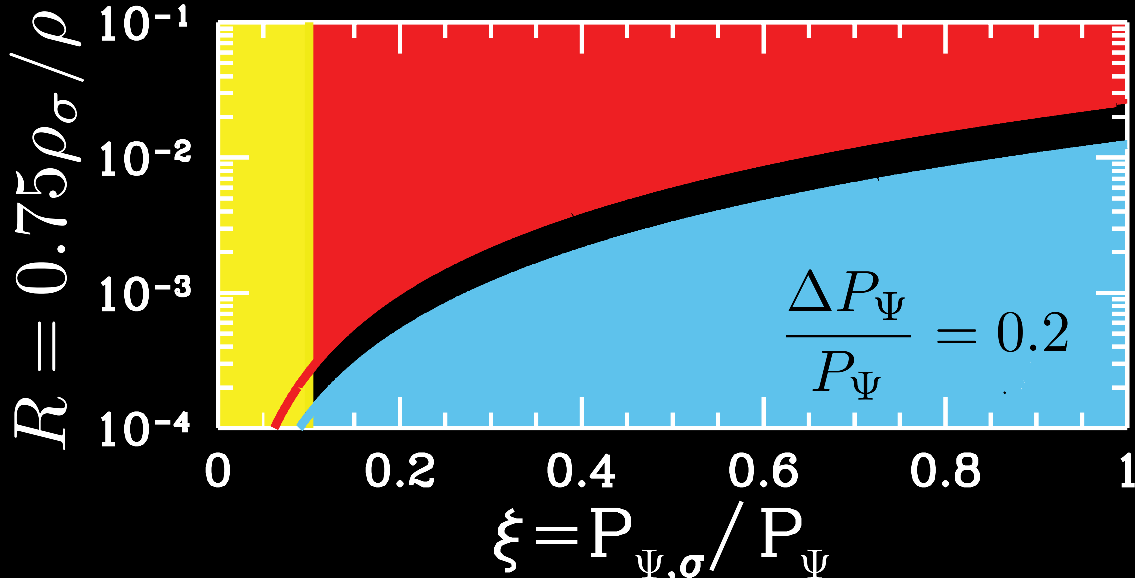
# Constraining the Curvaton Model

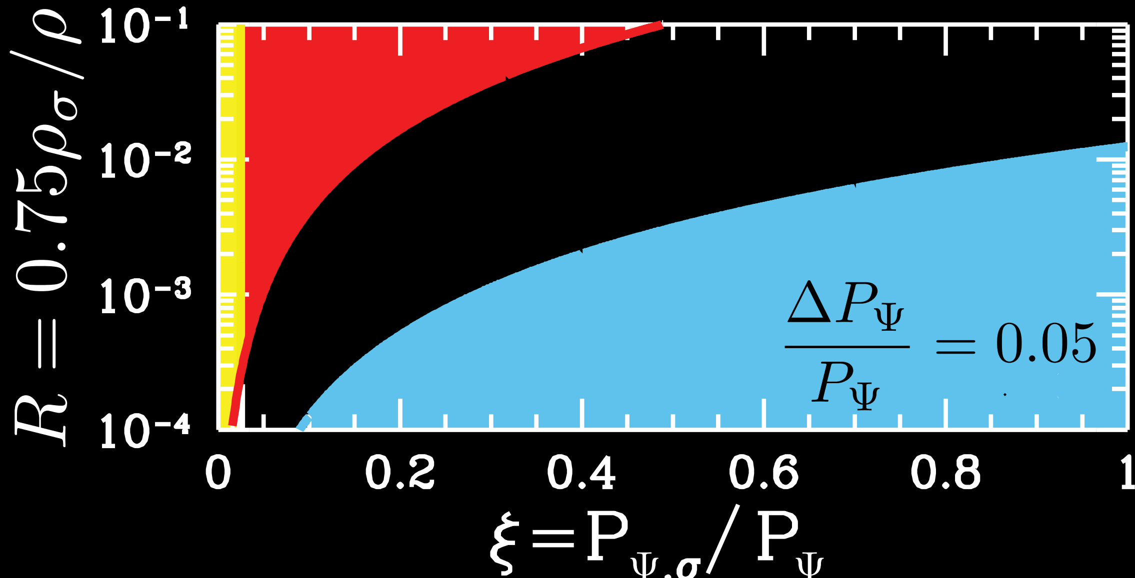
The curvaton and inflaton both contribute to $P_\Psi(k)$:

$$\xi \equiv \frac{P_{\Psi,\sigma}}{P_\Psi}$$ *fractional power from curvaton*

$$\frac{\Delta P_\Psi}{P_\Psi} = 2\xi \frac{\Delta\bar{\sigma}}{\bar{\sigma}}$$ *power asymmetry*

$$\frac{\Delta\bar{\sigma}}{\bar{\sigma}} \lesssim 1 \Longrightarrow \xi \gtrsim \frac{1}{2}\frac{\Delta P_\Psi}{P_\Psi}$$



# Constraining the Curvaton Model

The curvaton and inflaton both contribute to $P_\Psi(k)$:

$$\xi \equiv \frac{P_{\Psi,\sigma}}{P_\Psi}$$ *fractional power from curvaton*

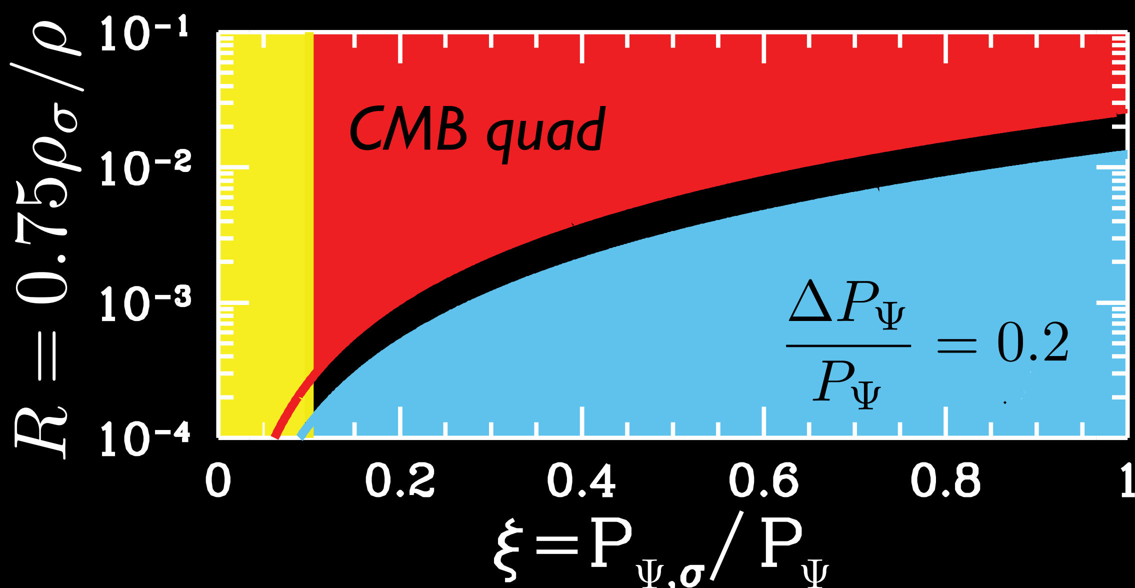
$$\frac{\Delta P_\Psi}{P_\Psi} = 2\xi \frac{\Delta\bar{\sigma}}{\bar{\sigma}}$$ *power asymmetry*

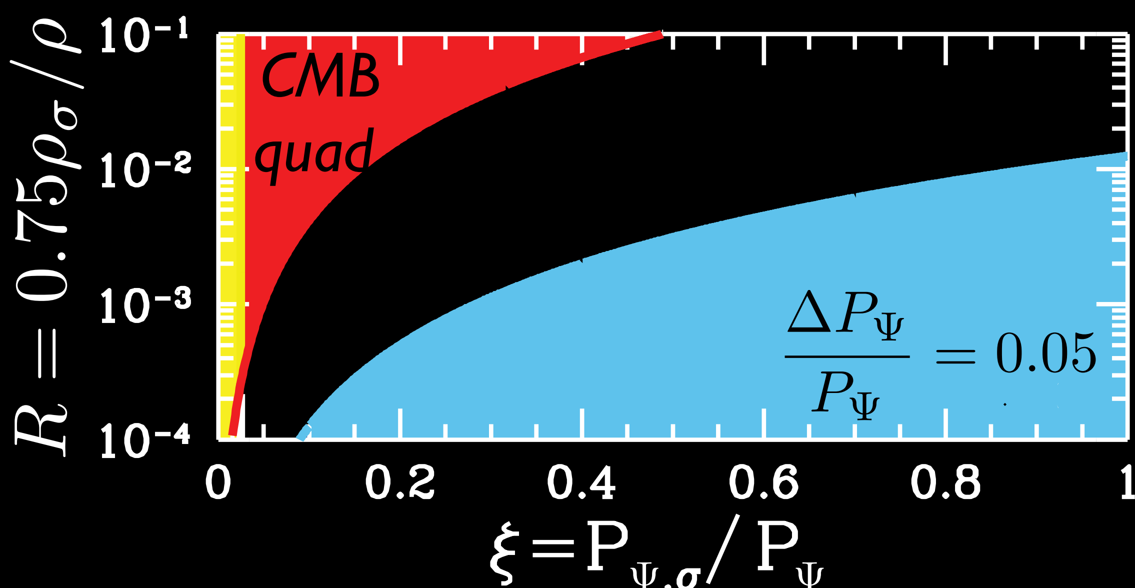
$$\frac{\Delta\bar{\sigma}}{\bar{\sigma}} \lesssim 1 \Longrightarrow \xi \gtrsim \frac{1}{2}\frac{\Delta P_\Psi}{P_\Psi}$$

## CMB Quadrupole:

$$R\left(\frac{\Delta\bar{\sigma}}{\bar{\sigma}}\right)^2 \lesssim \frac{5}{2}(5.8\mathcal{Q})$$

$$R \lesssim 58\mathcal{Q}\,\xi^2\left(\frac{\Delta P_\Psi}{P_\Psi}\right)^{-2}$$



# Constraining the Curvaton Model

## Non-Gaussianity Constraints

$$\Psi = -\frac{R}{5}\left[2\left(\frac{\delta\sigma}{\bar{\sigma}}\right) + \left(\frac{\delta\sigma}{\bar{\sigma}}\right)^2\right]$$

potential fluctuation — Gaussian fluctuation — Gaussian fluctuation squared

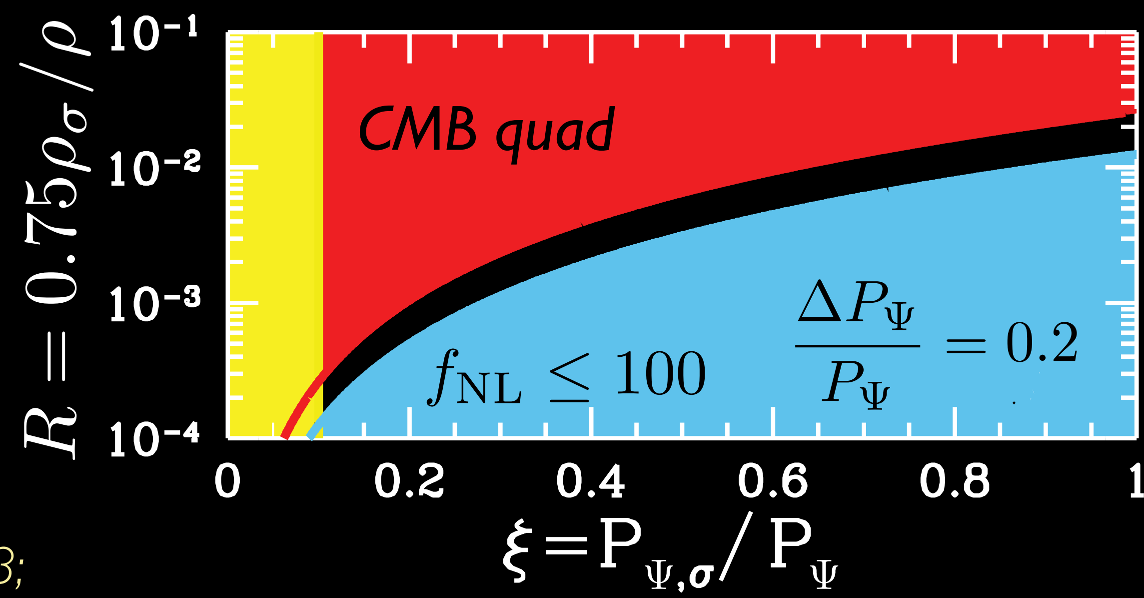
$$f_{\rm NL} \simeq \frac{5\xi^2}{4R}$$

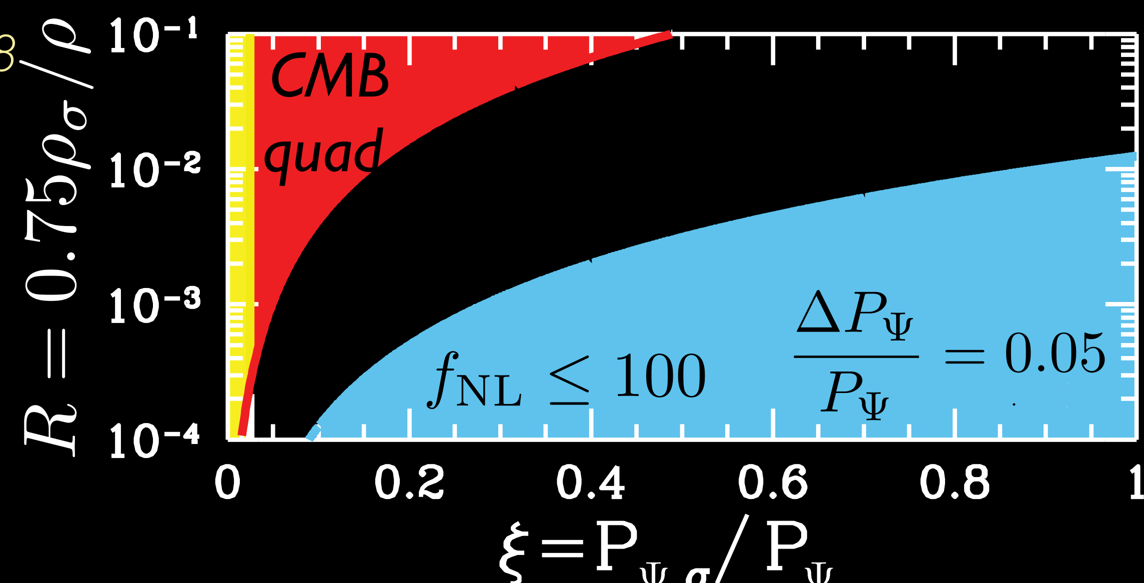
*Lyth, Ungarelli, Wands 2003; Ichikawa, Suyama, Takahashi, Yamaguchi 2008*

## Upperbound from WMAP:

$$f_{\rm NL} \lesssim 100$$

*Komatsu et al. 2008*
*Yadav, Wandelt 2008*



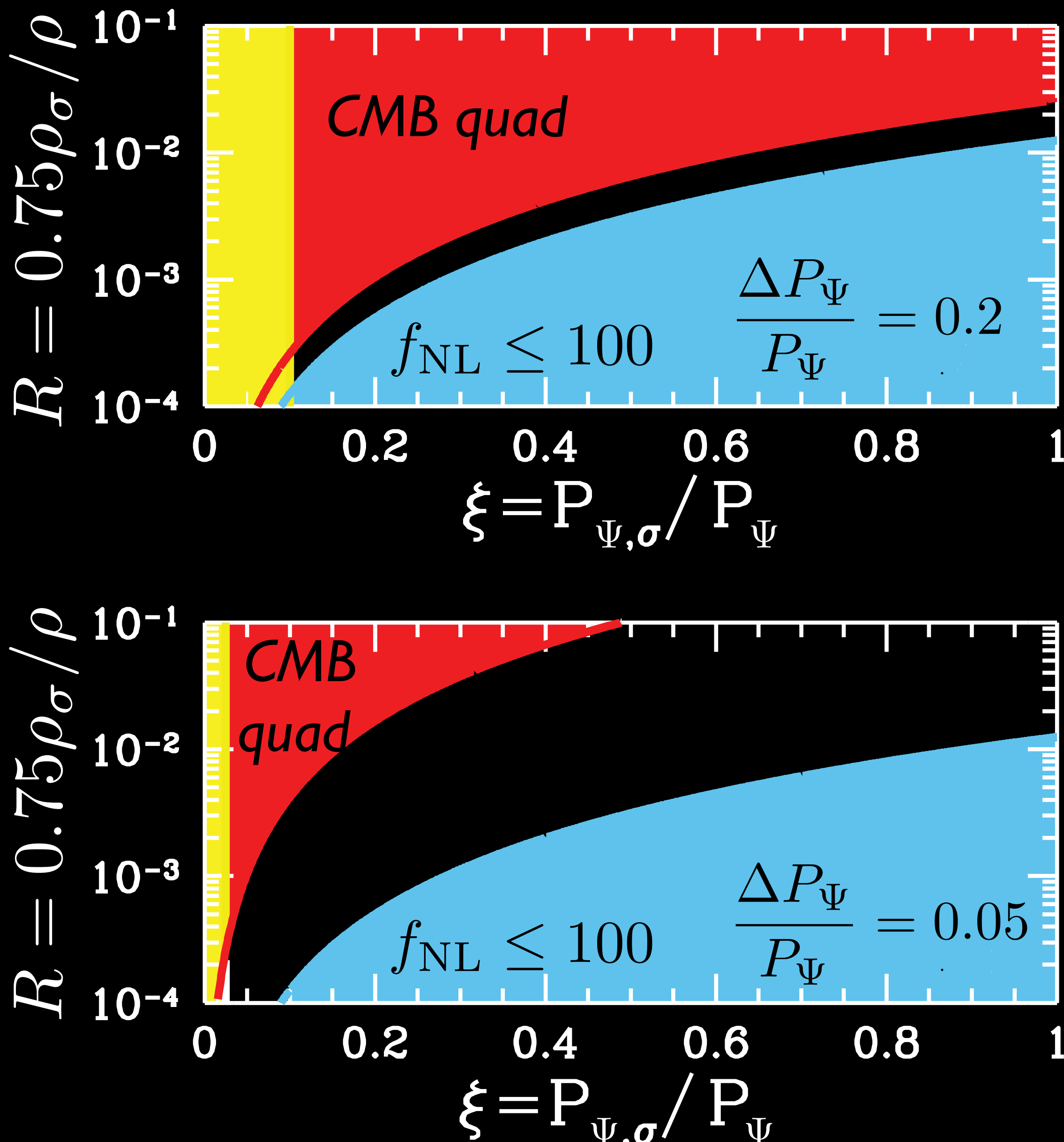
# Constraining the Curvaton Model

## The Allowed Region

$$\frac{5}{4\, f_{\rm NL,max}} \lesssim \frac{R}{\xi^2} \lesssim \frac{58\, Q}{(\Delta P_\Psi / P_\Psi)^2}$$

Non-Gaussianity    CMB Quadrupole

Allowed window



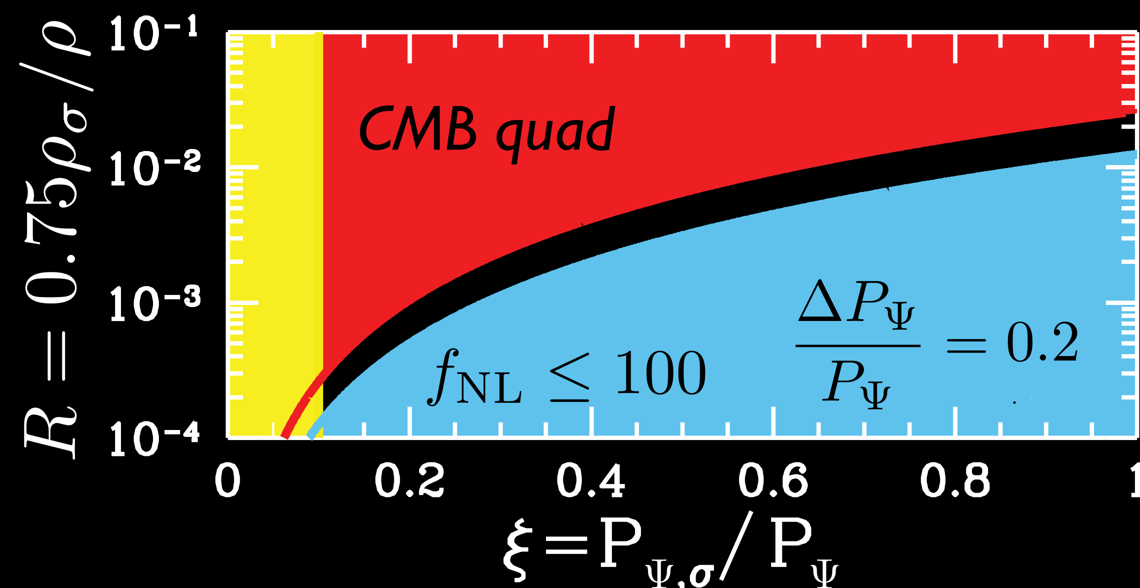
# Constraining the Curvaton Model

## The Allowed Region

$$\frac{5}{4\, f_{\mathrm{NL,max}}} \lesssim \frac{R}{\xi^2} \lesssim \frac{58\, Q}{(\Delta P_\Psi / P_\Psi)^2}$$

*Non-Gaussianity* *CMB Quadrupole*

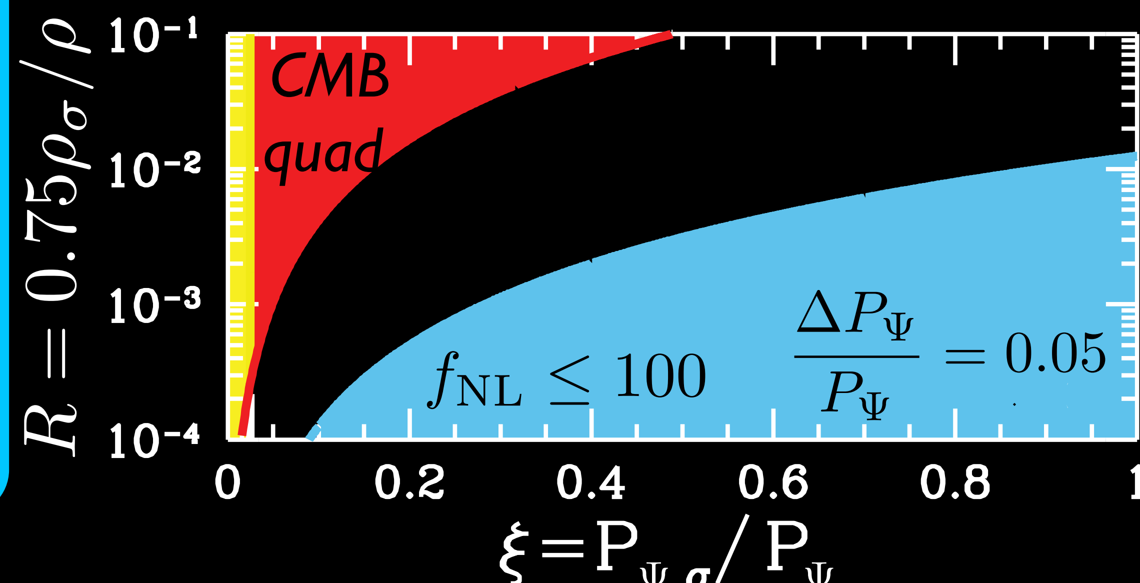
*Allowed window*




## The Dealbreaker

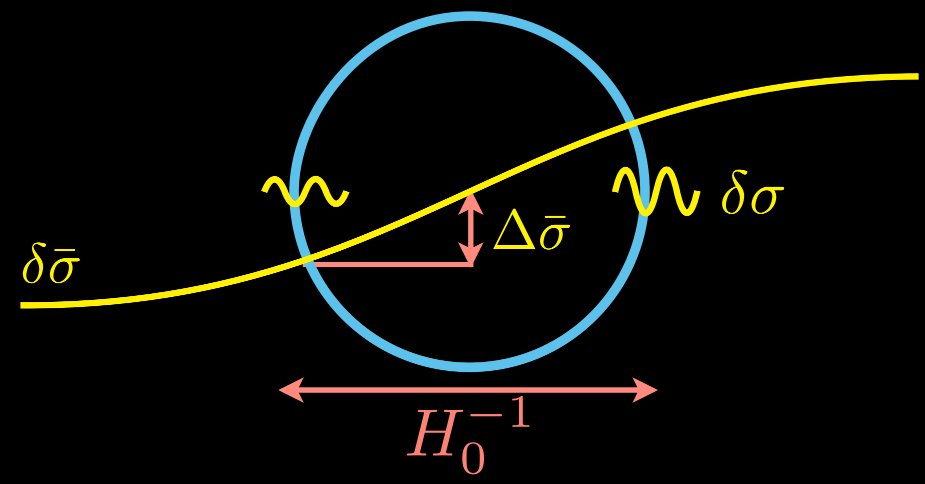
The window for $\dfrac{\Delta P_\Psi}{P_\Psi} = 0.2$

disappears if $f_{\mathrm{NL,max}} \lesssim 50$



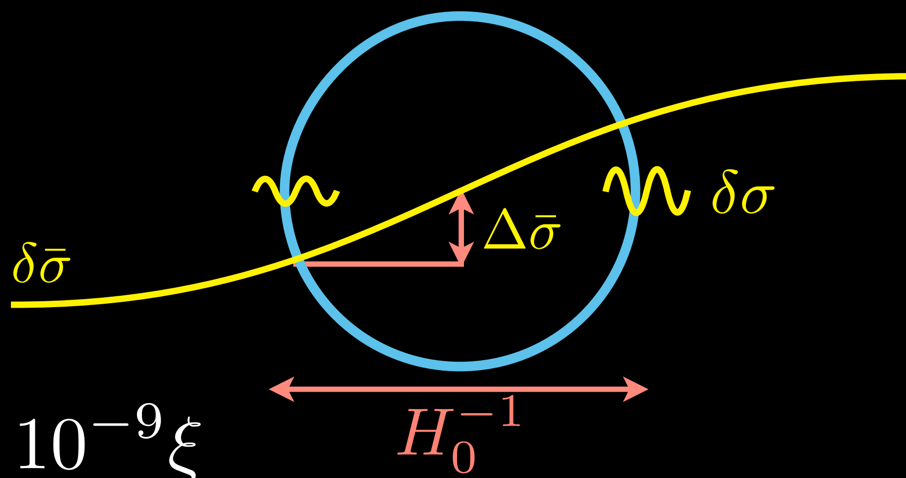
# Origins of the Supermode

Could the supermode be a
quantum fluctuation?



# Origins of the Supermode

Could the supermode be a
quantum fluctuation?



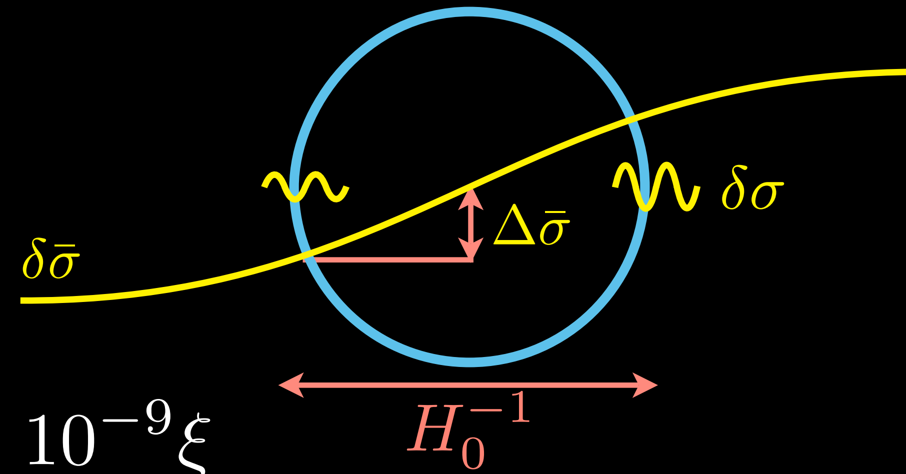

*Power spectrum from curvaton*

$$P_{\Psi,\sigma} = \left(\frac{2R}{5}\right)^2 \left\langle \left(\frac{\delta\sigma}{\bar{\sigma}}\right)^2 \right\rangle = \xi P_\Psi \simeq 10^{-9}\xi$$

*Observed power spectrum*

# Origins of the Supermode

Could the supermode be a quantum fluctuation?




*Power spectrum from curvaton*

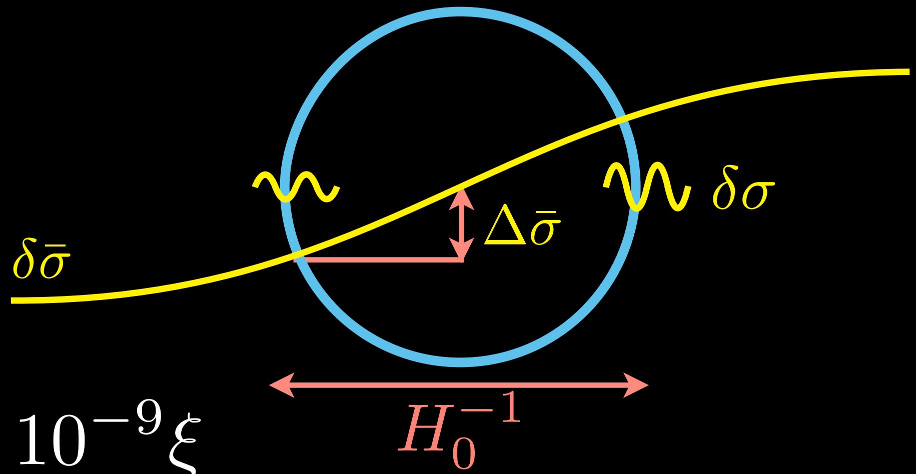
$$P_{\Psi,\sigma} = \left(\frac{2R}{5}\right)^2 \left\langle \left(\frac{\delta\sigma}{\bar{\sigma}}\right)^2 \right\rangle = \xi P_\Psi \simeq 10^{-9}\xi$$

*Observed power spectrum*

$$\frac{\bar{\sigma}}{\Delta\bar{\sigma}}\left(\frac{\delta\sigma}{\bar{\sigma}}\right)_{\text{rms}} = \frac{2\xi}{\Delta P_\Psi / P_\Psi}\left(8\times 10^{-5}\frac{\xi^{1/2}}{R}\right) \lesssim \frac{1}{5} \Longleftarrow \begin{cases} \dfrac{\Delta P_\Psi}{P_\Psi} \simeq 0.2 \\ f_{\text{NL}} \lesssim 100 \\ \xi \gtrsim \dfrac{1}{2}\dfrac{\Delta P_\Psi}{P_\Psi} \end{cases}$$

# Origins of the Supermode

Could the supermode be a
**quantum fluctuation?**




*Power spectrum from curvaton*

$$P_{\Psi,\sigma} = \left(\frac{2R}{5}\right)^2 \left\langle \left(\frac{\delta\sigma}{\bar{\sigma}}\right)^2 \right\rangle = \xi P_\Psi \simeq 10^{-9}\xi$$

*Observed power spectrum*

$$\frac{\bar{\sigma}}{\Delta\bar{\sigma}}\left(\frac{\delta\sigma}{\bar{\sigma}}\right)_{\rm rms} = \frac{2\xi}{\Delta P_\Psi / P_\Psi}\left(8 \times 10^{-5}\frac{\xi^{1/2}}{R}\right) \lesssim \frac{1}{5}$$

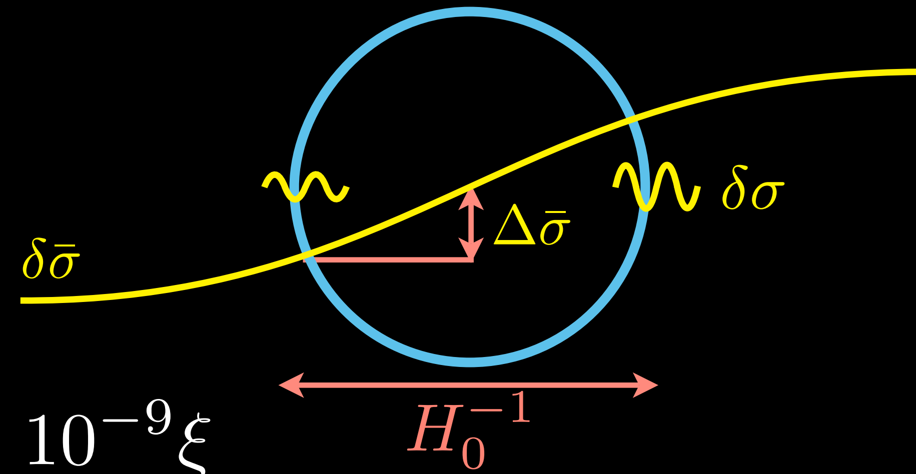
$$\begin{cases} \dfrac{\Delta P_\Psi}{P_\Psi} \simeq 0.2 \\ f_{\rm NL} \lesssim 100 \\ \xi \gtrsim \dfrac{1}{2}\dfrac{\Delta P_\Psi}{P_\Psi} \end{cases}$$

$$\bar{\sigma}_{\rm SM} > \Delta\bar{\sigma} > 5(\delta\sigma)_{\rm rms}$$

*The supermode would be at least a 5-sigma fluctuation: that's highly improbable!*

# Origins of the Supermode

Could the supermode be a quantum fluctuation?




*Power spectrum from curvaton*

$$P_{\Psi,\sigma} = \left(\frac{2R}{5}\right)^2 \left\langle \left(\frac{\delta\sigma}{\bar\sigma}\right)^2 \right\rangle = \xi P_\Psi \simeq 10^{-9}\xi$$

*Observed power spec*

$$\frac{\bar\sigma}{\Delta\bar\sigma}\left(\frac{\delta\sigma}{\bar\sigma}\right)_{\mathrm{rms}} = \frac{2\xi}{\Delta P_\Psi / P_\Psi}\left(8\times 10\right.$$

$0.2$

$0$

$\frac{P_\Psi}{P_\Psi}$

$$\bar\sigma_{\mathrm{SM}} > \Delta\bar\sigma > 5$$

Signature of

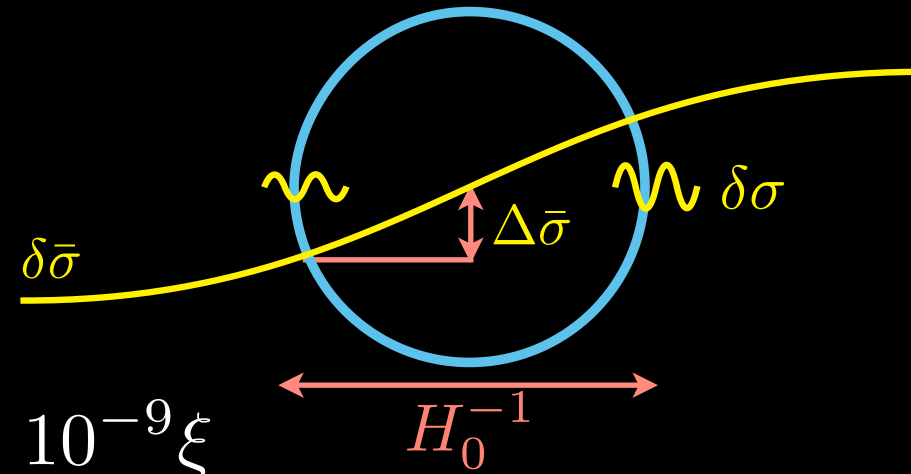
"*curvaton web?*"

*Linde and Mukhanov, 2006*

*The supermode would be at least a 5-sigma fluctuation: that's highly improbable!*

# Origins of the Supermode

Could the supermode be a
**quantum fluctuation?**




*Power s*

$\xi P_\Psi \simeq 10^{-9}\xi$

*served power spec*

*A* ***pre-inflationary*** *remnant?*

$\frac{\bar{\sigma}}{\Delta\bar{\sigma}}$ $\left(\frac{\quad}{\bar{\sigma}}\right)_{\mathrm{rms}}$ $\frac{\quad}{\Delta P_\Psi/P_\Psi}$ $\times 10$

$0.2$

$0$

$P_\Psi$

$P_\Psi$

*Signature of*
*"****curvaton web?****"*
*Linde and Mukhanov, 2006*

$\bar{\sigma}_{\mathrm{SM}} > \Delta\bar{\sigma} > 5$

*The supermode would be at least a 5-sigma fluctuation: that's highly improbable!*

# A Scale-Dependent Asymmetry?

There are indications that only large scales are asymmetric.

- Asymmetry detected for $\ell = 5 - 40$ .
- Some analyses see reduced asymmetry for $\ell \gtrsim 100$.

*Donoghue and Donoghue 2005; Lew 2008.*

*How could the asymmetry disappear at small scales?*

Only the perturbations from the curvaton are asymmetric; the inflaton perturbations are still statistically isotropic.

*Introduce scale dependence through* $\xi \equiv \dfrac{P_{\Psi,\sigma}}{P_{\Psi,\sigma} + P_{\Psi,\phi}}$ .

# A Scale-Dependent Asymmetry?

There are indications that only large scales are asymmetric.

- Asymmetry detected for $\ell = 5 - 40$.
- Some analyses see reduced asymmetry for $\ell \gtrsim 100$.

*Donoghue and Donoghue 2005; Lew 2008.*

*How could the asymmetry disappear at small scales?*

Only the perturbations from the curvaton are asymmetric; the inflaton perturbations are still statistically isotropic.

*Introduce scale dependence through* $\xi \equiv \dfrac{P_{\Psi,\sigma}}{P_{\Psi,\sigma} + P_{\Psi,\phi}}$.

- A feature in $V(\phi)$ *Gordon 2007*

$$\xi = \frac{\tilde{\epsilon}}{\tilde{\epsilon} + 1} \qquad \tilde{\epsilon} \equiv \frac{1}{9\pi}\left(\frac{m_{\rm Pl}}{\bar{\sigma}}\right)^2 R^2 \epsilon$$

# A Scale-Dependent Asymmetry?

There are indications that only large scales are asymmetric.

- Asymmetry detected for $\ell = 5 - 40$.
- Some analyses see reduced asymmetry for $\ell \gtrsim 100$.

*Donoghue and Donoghue 2005; Lew 2008.*

*How could the asymmetry disappear at small scales?*

Only the perturbations from the curvaton are asymmetric; the inflaton perturbations are still statistically isotropic.

*Introduce scale dependence through* $\xi \equiv \dfrac{P_{\Psi,\sigma}}{P_{\Psi,\sigma} + P_{\Psi,\phi}}$.

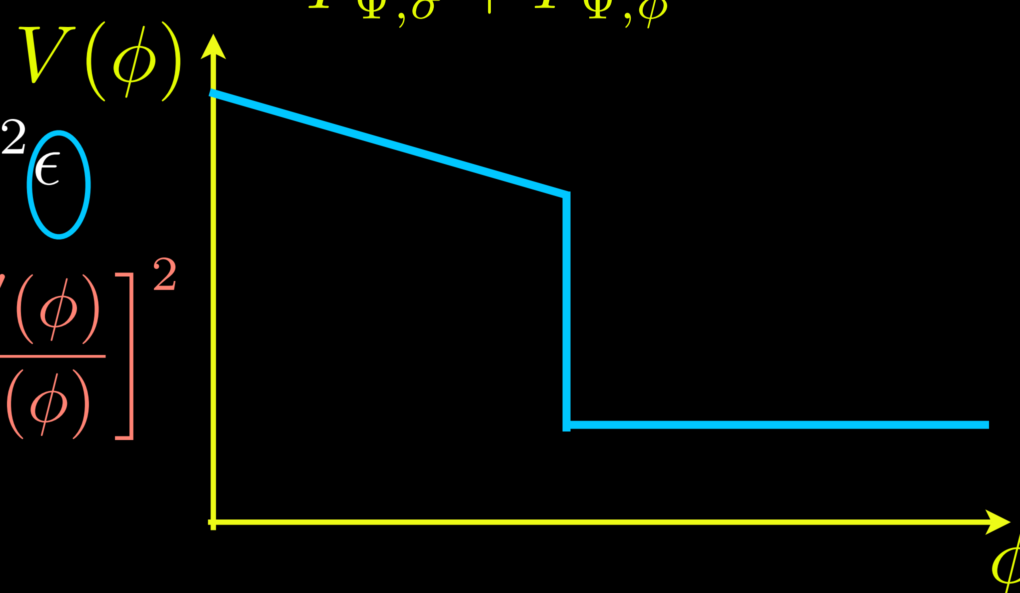
- A feature in $V(\phi)$ *Gordon 2007*

$$\xi = \frac{\tilde{\epsilon}}{\tilde{\epsilon}+1} \qquad \tilde{\epsilon} \equiv \frac{1}{9\pi}\left(\frac{m_{\mathrm{Pl}}}{\bar{\sigma}}\right)^2 R^2 \epsilon$$

$$\epsilon \downarrow \rightarrow P_{\Psi,\phi} \uparrow \rightarrow \xi \downarrow \qquad \epsilon \equiv \frac{m_{\mathrm{Pl}}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]^2$$

$$\downarrow$$

$$V(\phi) \downarrow \rightarrow P_{\Psi} \leftrightarrow$$



# A Scale-Dependent Asymmetry?

There are indications that only large scales are asymmetric.

- Asymmetry detected for $\ell = 5 - 40$.
- Some analyses see reduced asymmetry for $\ell \gtrsim 100$.

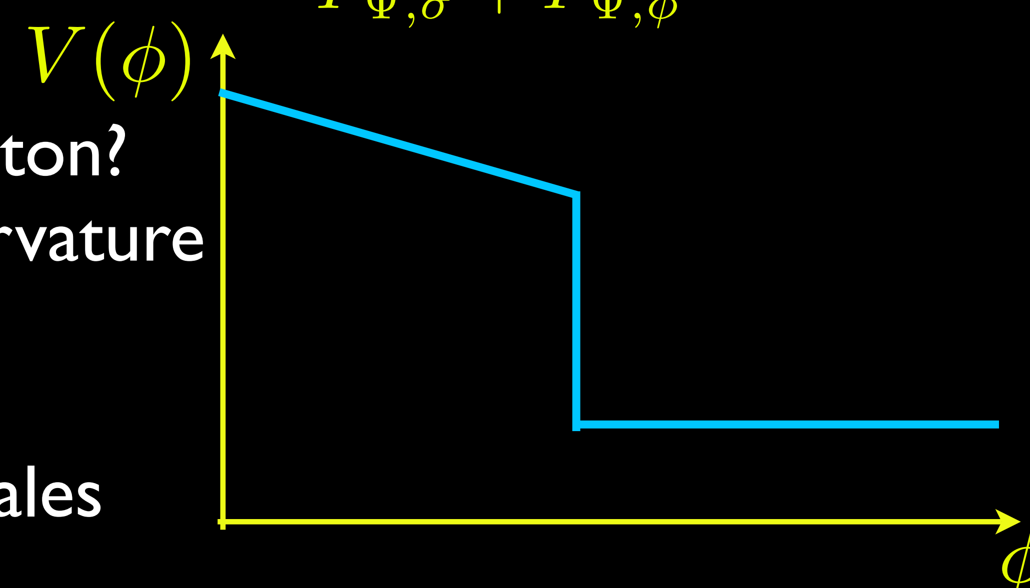
*Donoghue and Donoghue 2005; Lew 2008.*

*How could the asymmetry disappear at small scales?*

Only the perturbations from the curvaton are asymmetric; the inflaton perturbations are still statistically isotropic.

*Introduce scale dependence through* $\xi \equiv \dfrac{P_{\Psi,\sigma}}{P_{\Psi,\sigma} + P_{\Psi,\phi}}$.

- A feature in $V(\phi)$ *Gordon 2007*

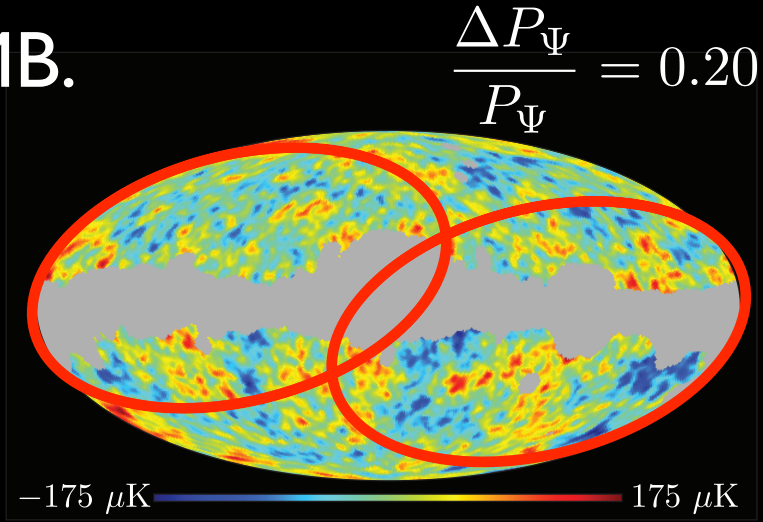



- Isocurvature modes from curvaton?
  - curvaton can produce isocurvature perturbations
  - isocurvature perturbations contribute more on large scales

***Work in progress....***

# Summary: How to Generate the Power Asymmetry

There is a power asymmetry in the CMB.




- present at the 99% confidence level
- detected on large scales

*Hansen, Banday, Gorski, 2004*
*Eriksen, Hansen, Banday, Gorski, Lilje 2004*
*Eriksen, Banday, Gorski, Hansen, Lilje 2007*

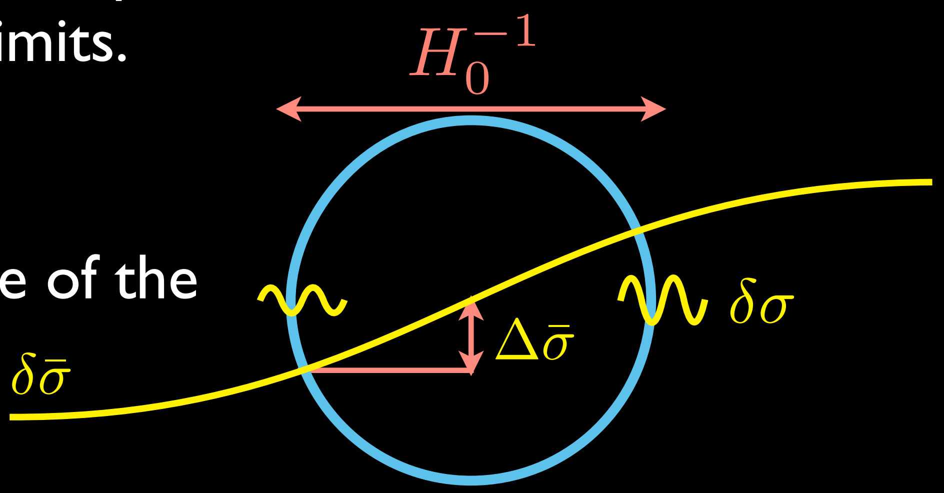
A superhorizon perturbation during inflation generates a power asymmetry.

- also generates large-scale CMB temperature perturbations
- no dipole; quadrupole and octupole set limits.

  *Erickcek, Carroll, Kamionkowski arXiv:0808.1570*

- an inflaton perturbation is ruled out
- a curvaton perturbation is a viable source of the observed asymmetry

  *Erickcek, Kamionkowski, Carroll arXiv:0806.0377*

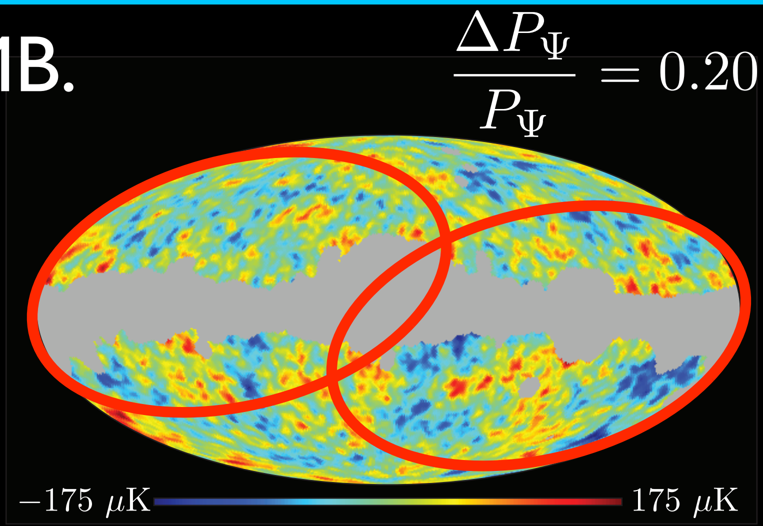


# Summary: How to Generate the Power Asymmetry

There is a power asymmetry in the CMB.

- present at the 99% confidence level
- detected on large scales

*Hansen, Banday, Gorski, 2004*
*Eriksen, Hansen, Banday, Gorski, Lilje 2004*
*Eriksen, Banday, Gorski, Hansen, Lilje 2007*




## Features of the Curvaton-Generated Power Asymmetry

- the superhorizon curvaton perturbation is not a quantum fluctuation
- the produced asymmetry is scale-invariant, but it is possible to modify that
- suppressed tensor-scalar ratio: $r \propto (1-\xi)$
- high non-Gaussianity: $f_{\mathrm{NL}} \gtrsim 50$