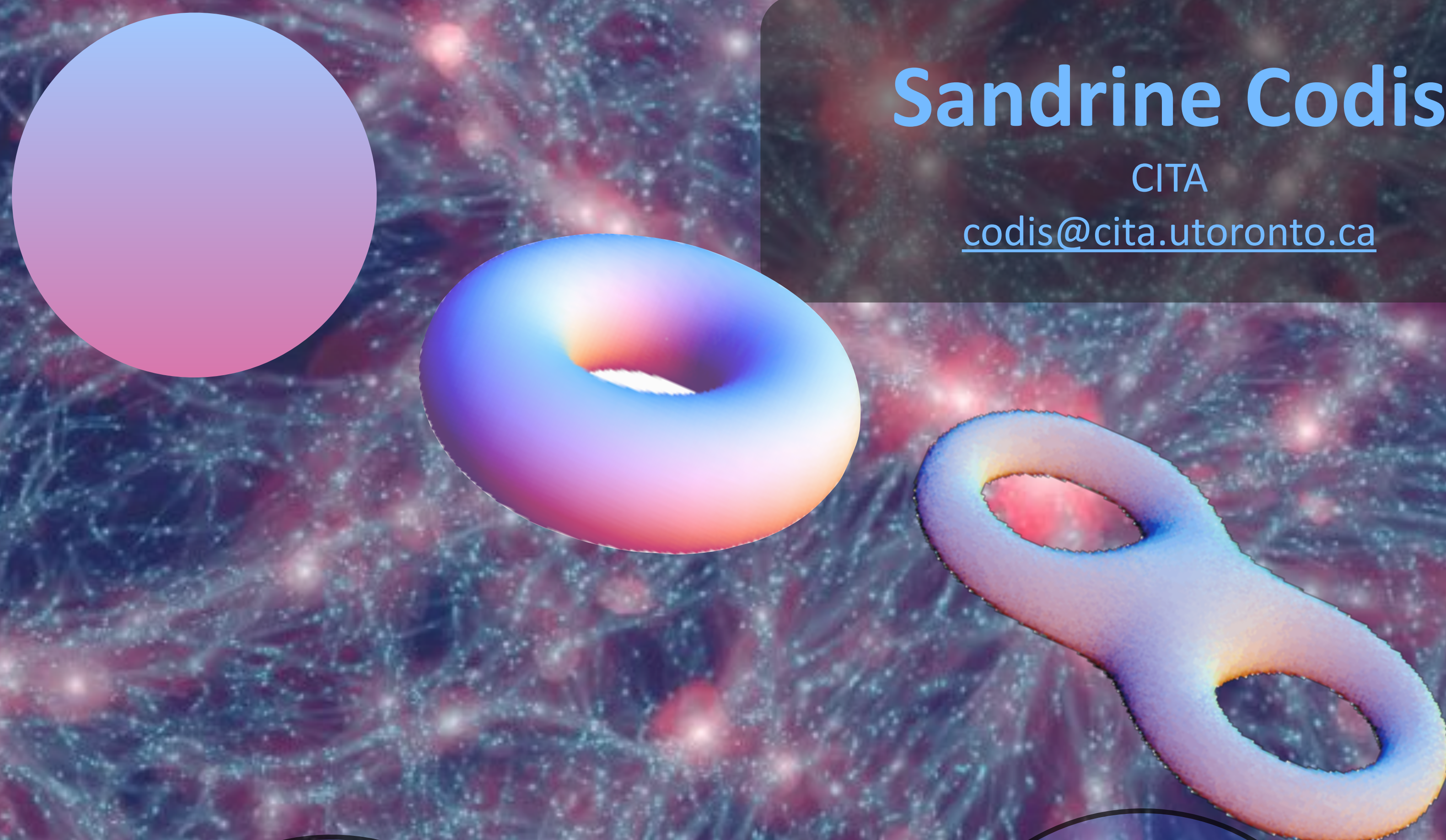
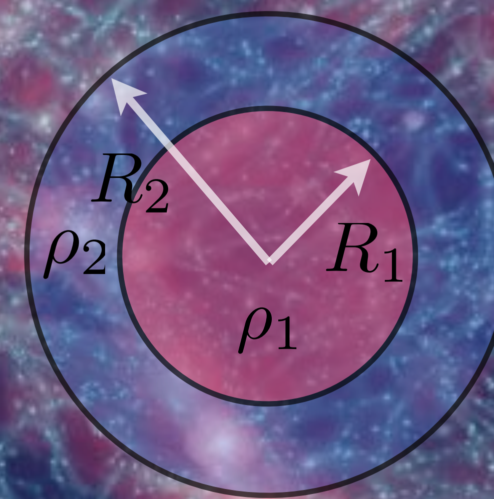
Statistiques d'ordre supérieur

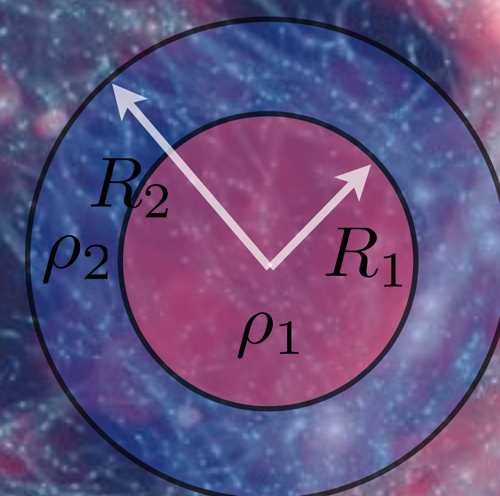
Sandrine Codis

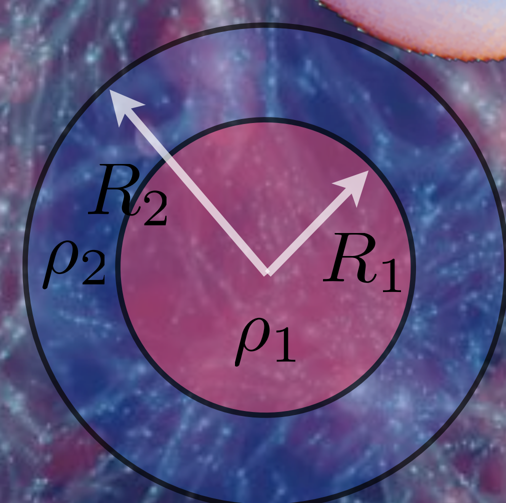
CITA

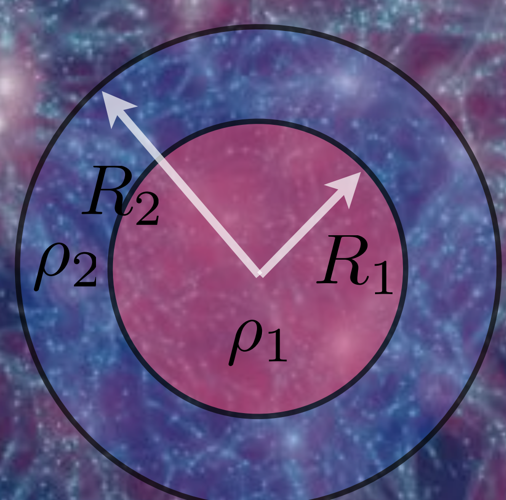
codis@cita.utoronto.ca

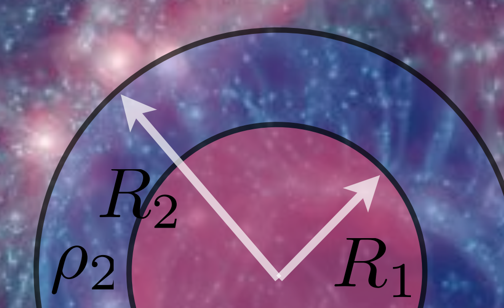












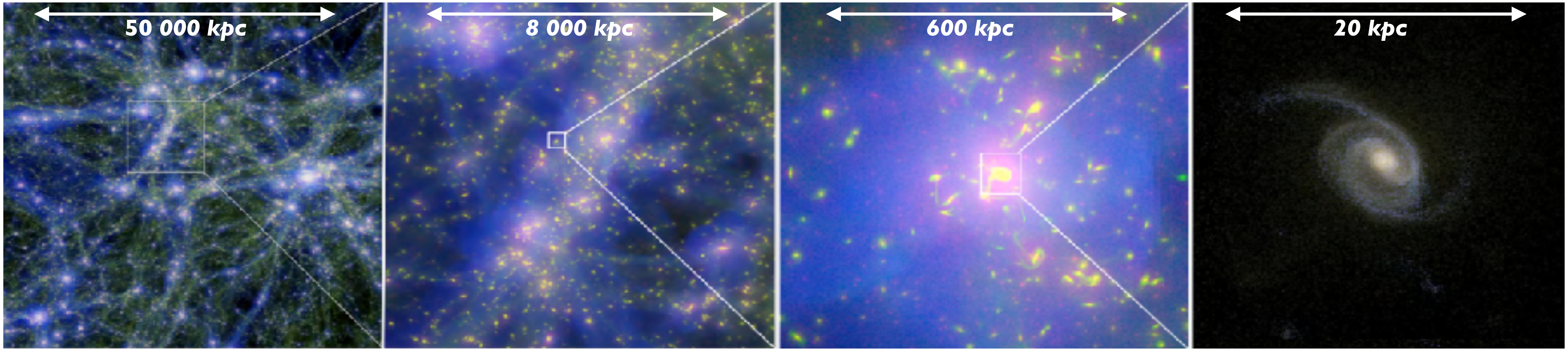
horizon-AGN

Ecole Euclid 2017, Frejus

Au menu…

Partie I : Introduction

Les grandes structures de l'Univers

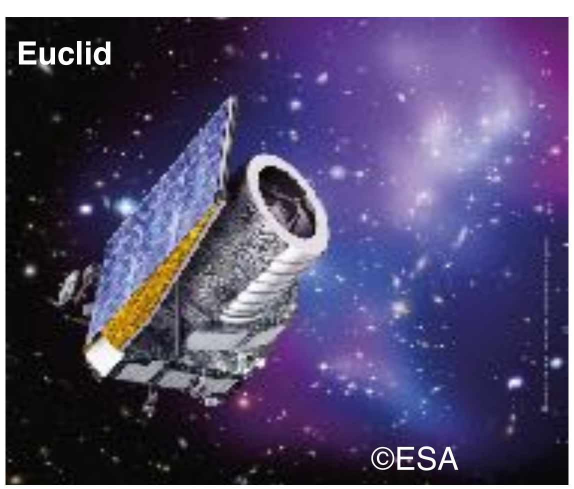


Grande échelle : clustering de la matière noire

Petite échelle : 'gastrophysique'

Comment utiliser les grandes structures pour contraindre la **cosmologie** et la **physique fondamentale** (énergie noire, neutrinos, tests de la RG, physique de l'inflation)?

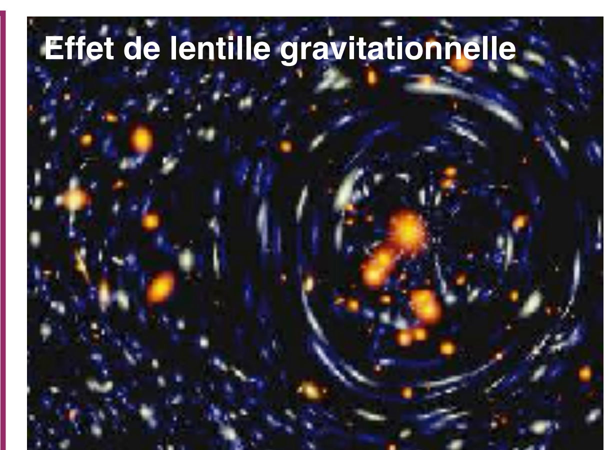
Comment modéliser la complexité des **petites échelles** et leur couplage aux grandes échelles ?



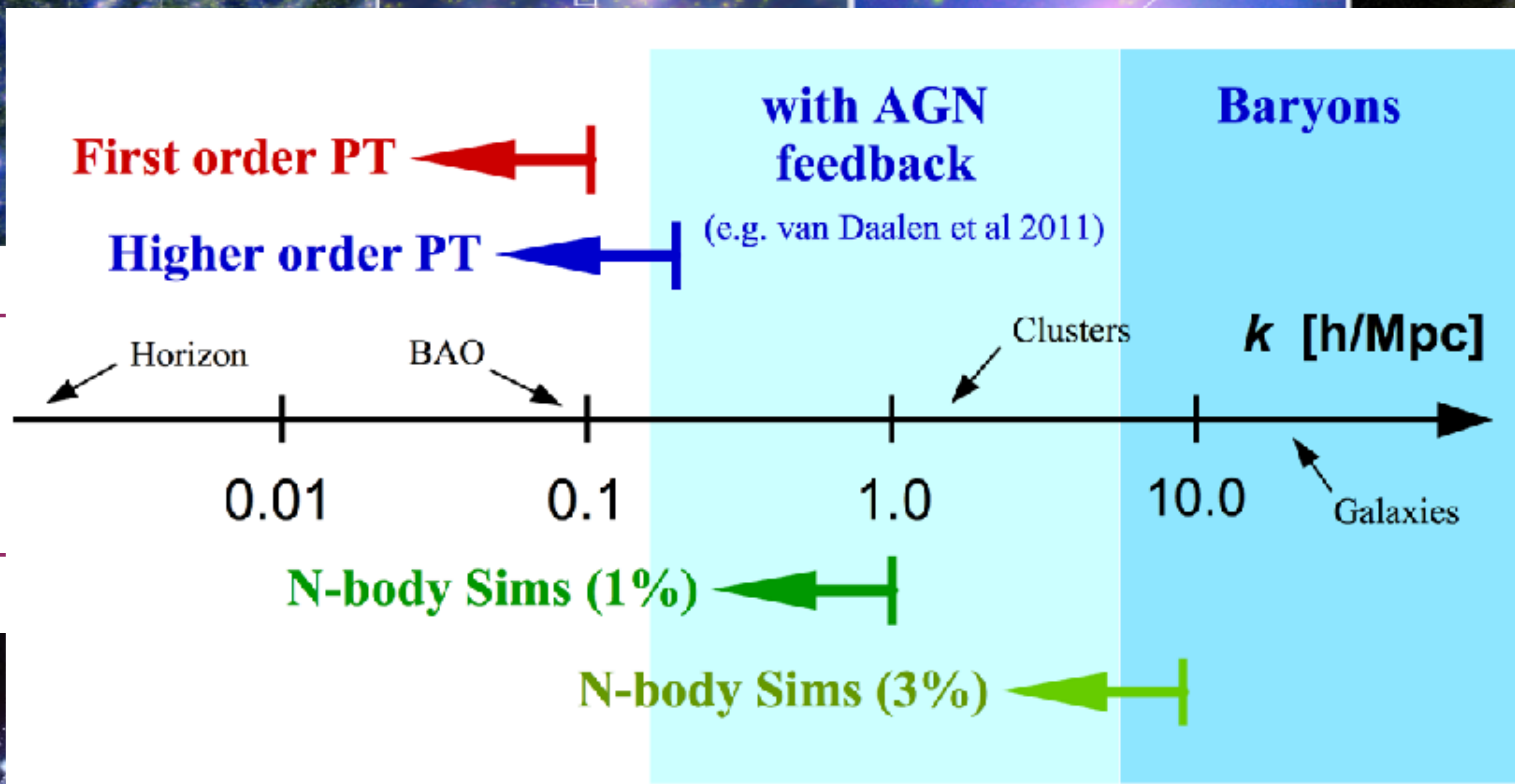
➡ Précisions des *mesures* avec l'arrivée de **grands relevés** de galaxies comme **Euclid** qui utiliseront :

- le **clustering** (traceur lumière)
- l'effet de **lentille gravitationnelle** (traceur masse)

➡ La précision des contraintes cosmologiques dépendra de notre capacité à *modéliser* ces observables à petite échelle.



Les grandes structures de l'Univers



es **petites**
es échelles ?

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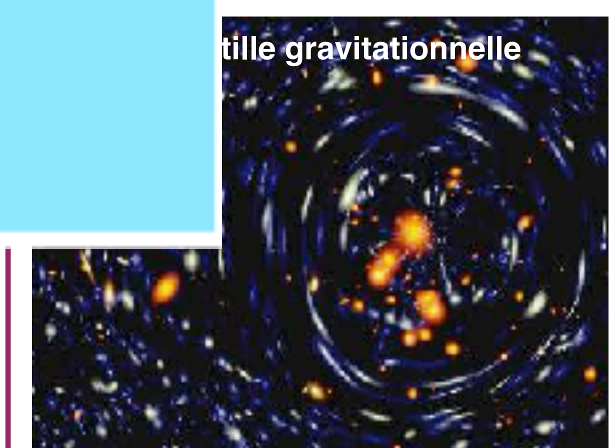

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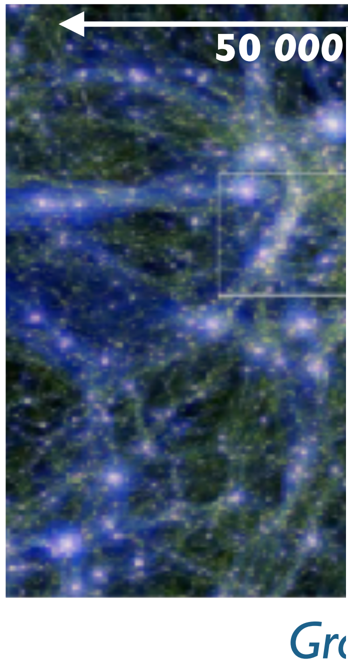

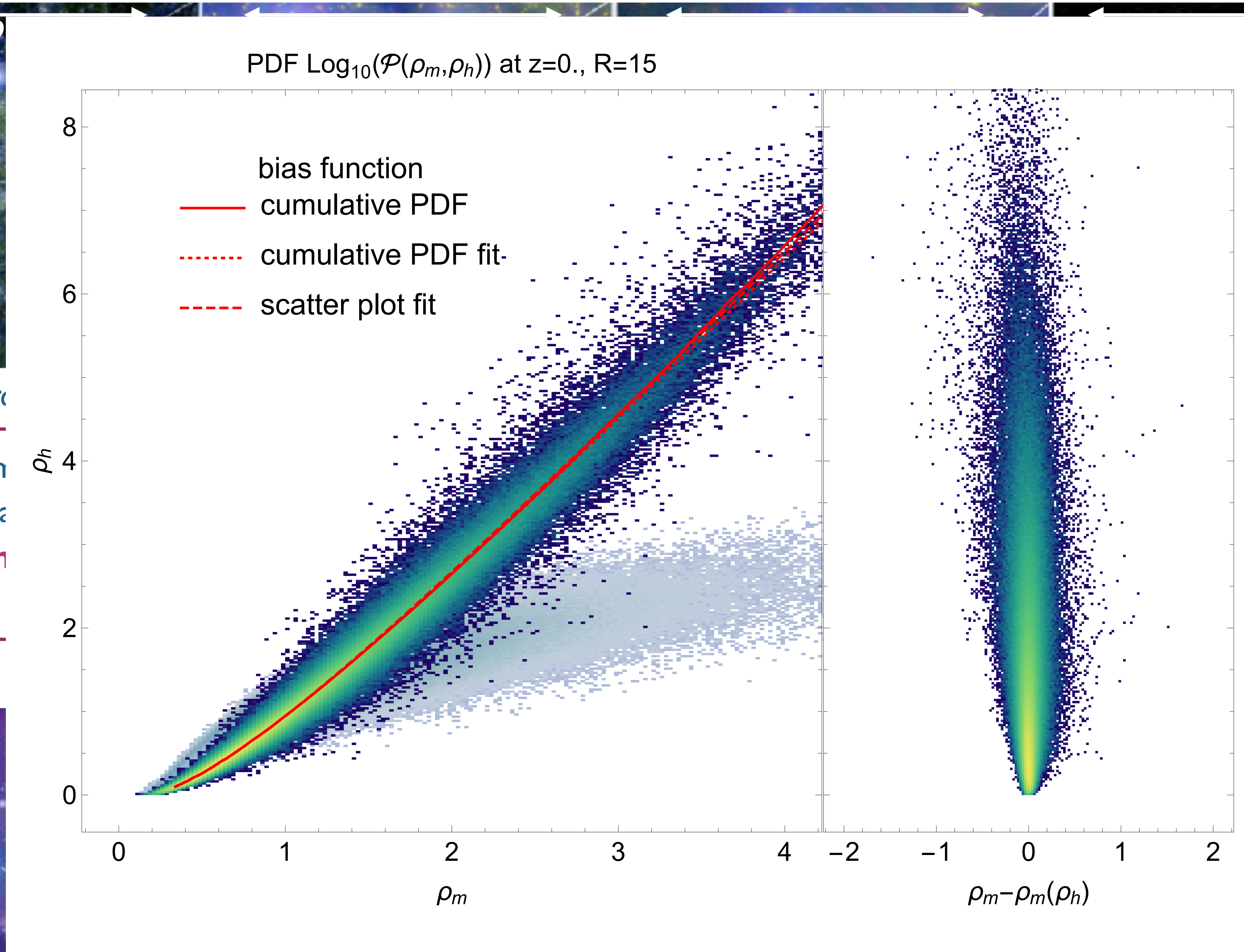

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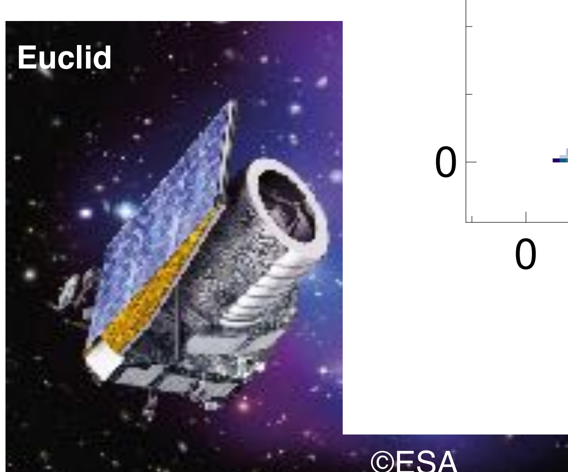
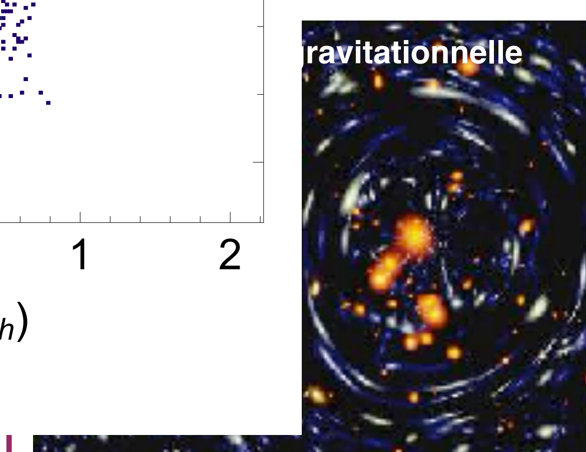
etites
chelles ?





notre capacité à *modéliser* ces observables à petite échelle.

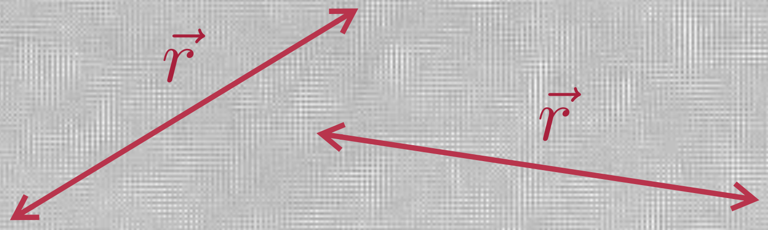
The LSS is sensitive to our cosmological model

Evolution of LSS is sensitive to cosmic expansion rate H(z) and growth rate of structures D(z).

Studying LSS then constrains cosmological parameters, dark energy e.o.s, modification of gravity...

It usually relies on correlation functions including Baryon Acoustic Oscillation peaks (e.g Cole+05, Anderson+14), redshift space distortions (e.g Guzzo+08, Samushia+14), lensing.

$$\langle \delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle = \xi_2(||\vec{r}||)$$



DM simulation by C. Pichon

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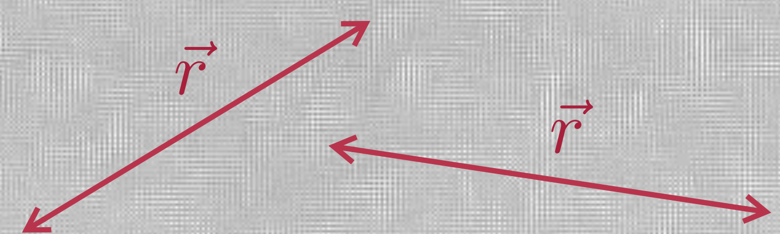

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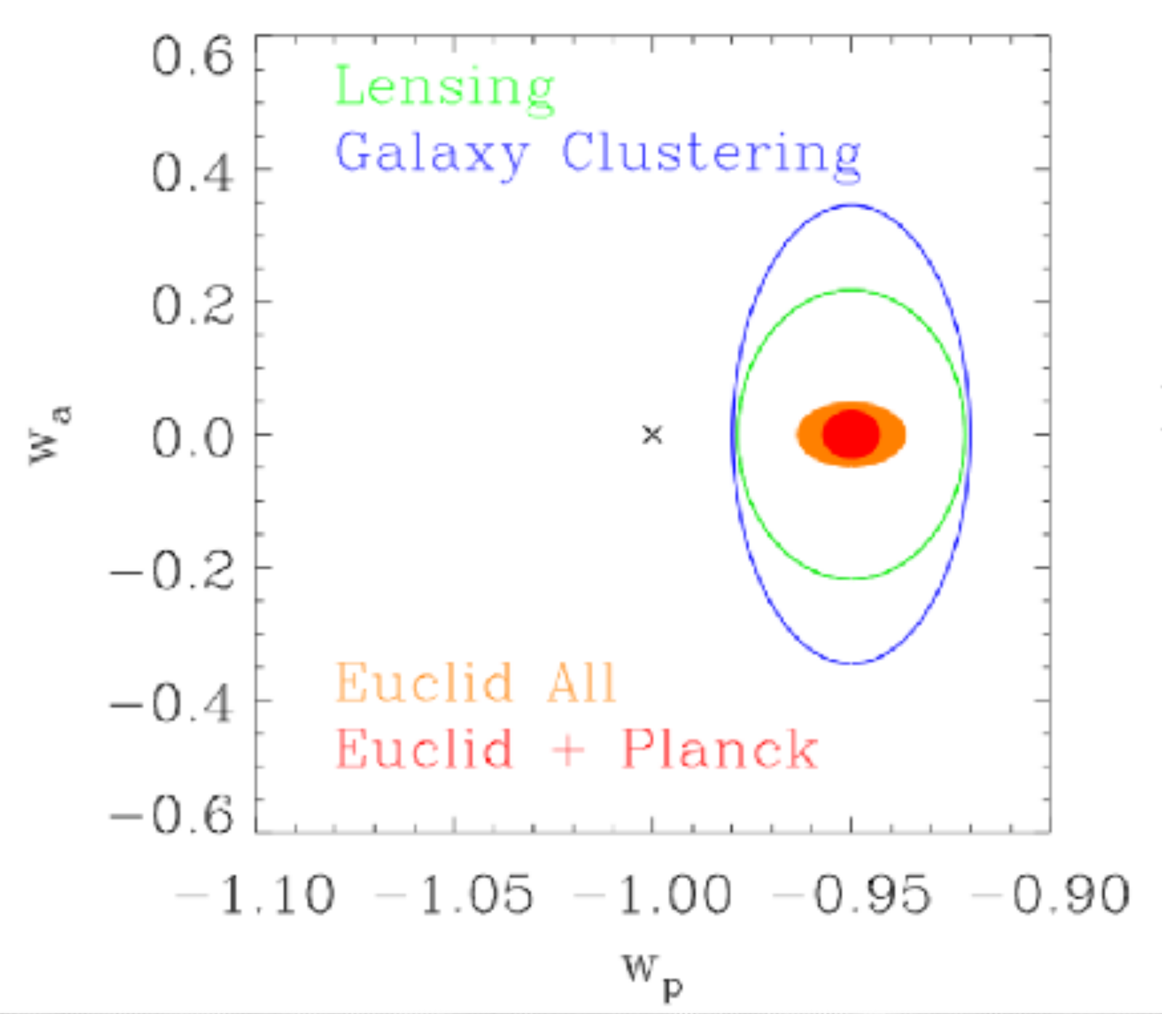


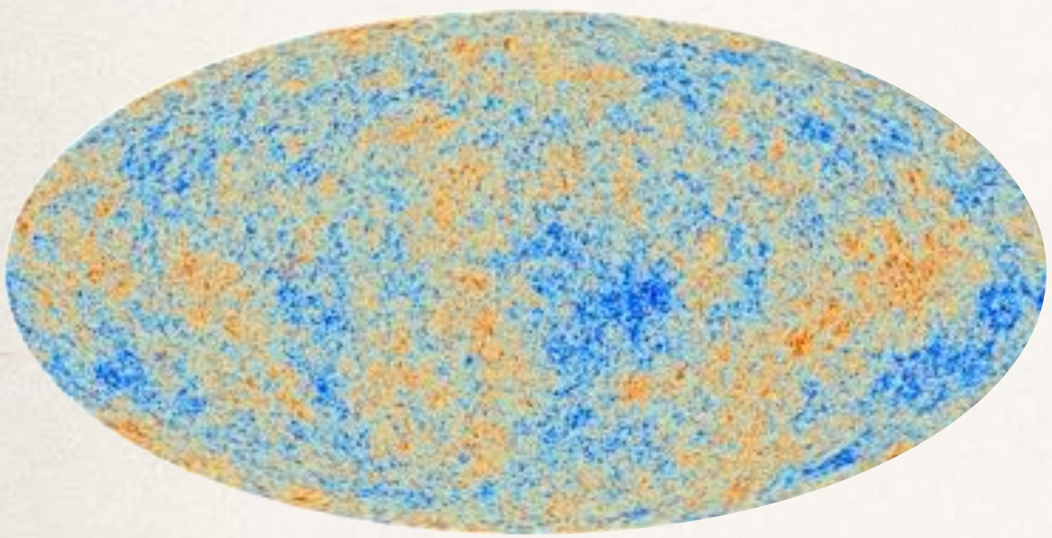
Expected constraints from Euclid on dark energy equation of state (credit: Euclid Red Book, Laureijs+11)

DM simulation by C. Pichon

How is the cosmic web woven?
How do structures grow in the Universe?

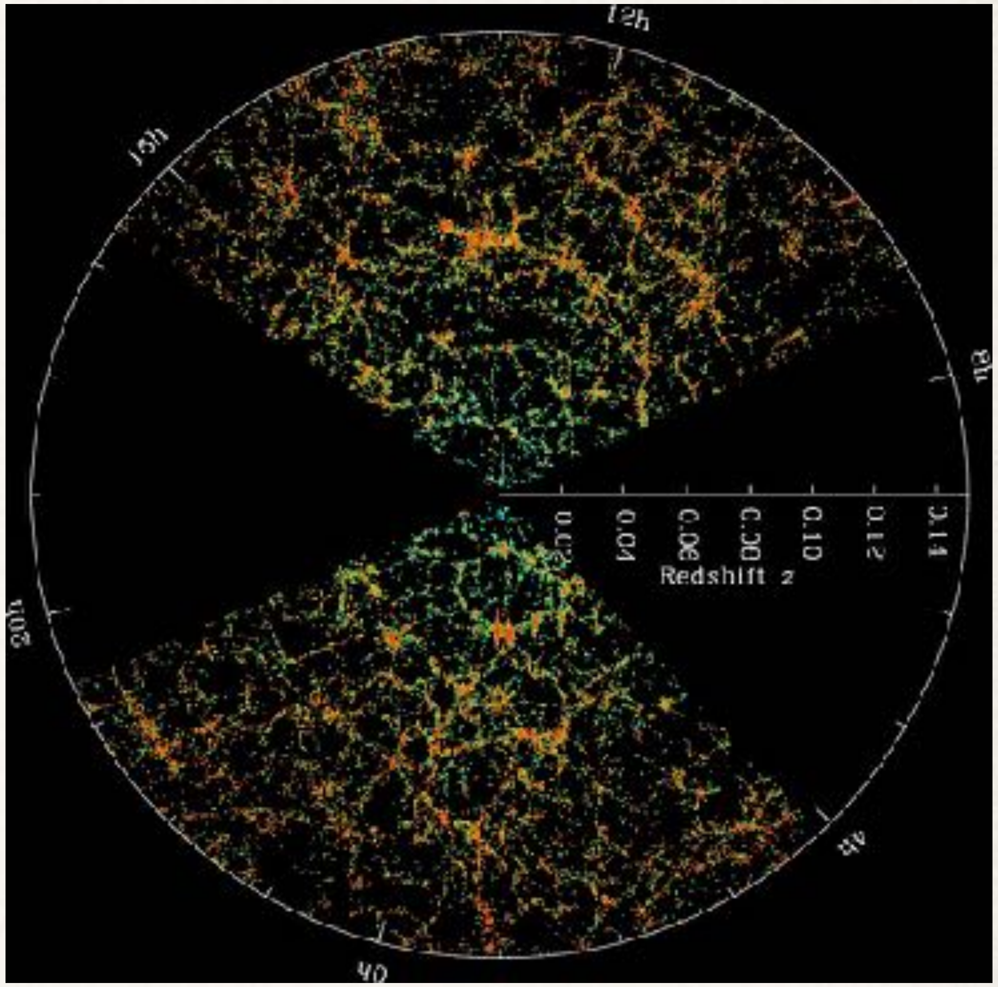
cosmic web: voids, walls, filaments, nodes

Gaussian primordial fluctuations



gravity

expansion



models of the early universe
predict
the statistics of the ICs

to be compared with
observations

theory of gravity + cosmological
model (dark energy, dark matter...)

theory can not predict our Universe (a single realization) on an object-by-object basis but
its statistics : expectation + cosmic variance

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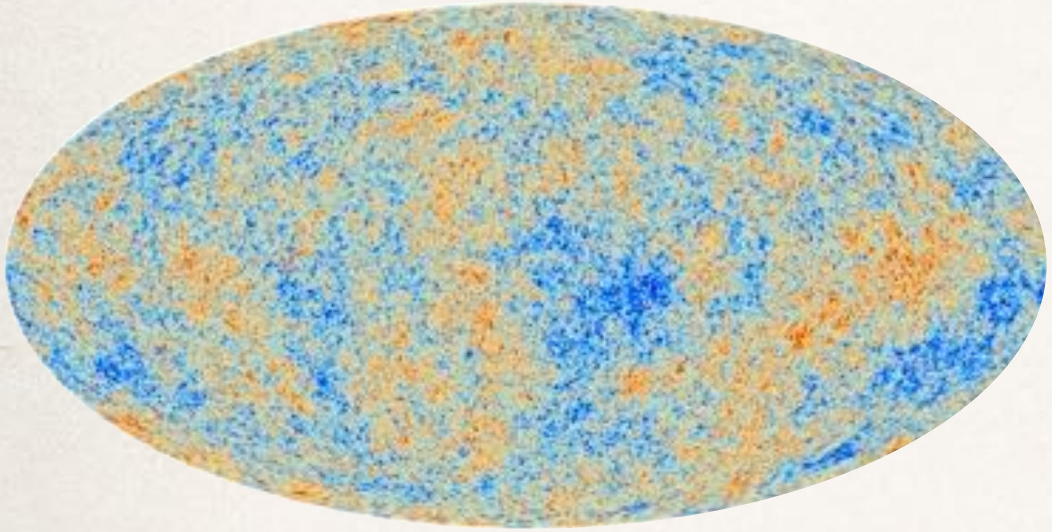
Gaussian primordial fluctuations



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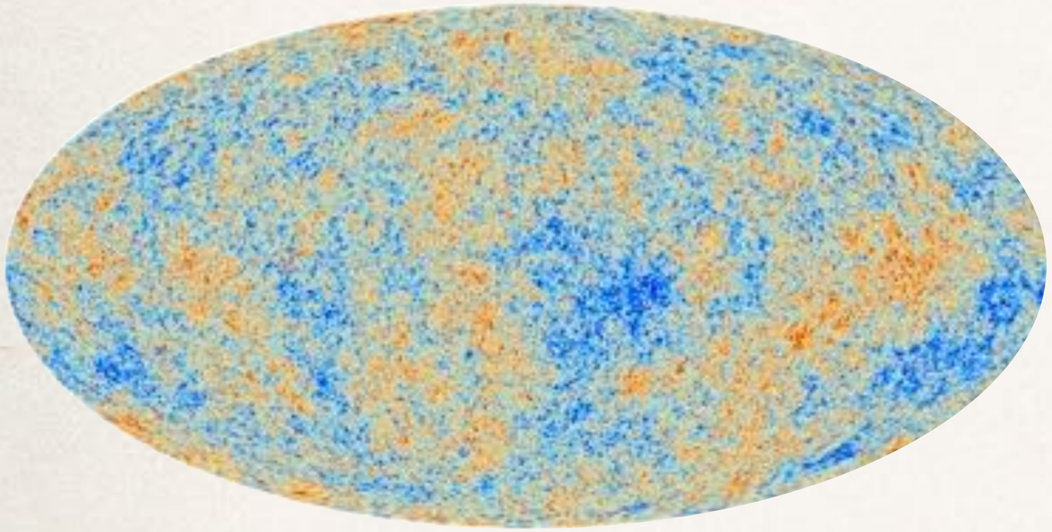

Comment caractériser entièrement les propriétés statistiques d'un GRF?

1) Avec sa moyenne et sa variance
2) Avec tous ses moments $<\delta^n>$
3) Avec sa moyenne et son spectre de puissance
4) Avec sa moyenne et sa fonction de corrélation à deux points
5) Avec l'ensemble (infini) de ses polyspectres successifs (spectre de puissance, bispectre, trispectre, etc).

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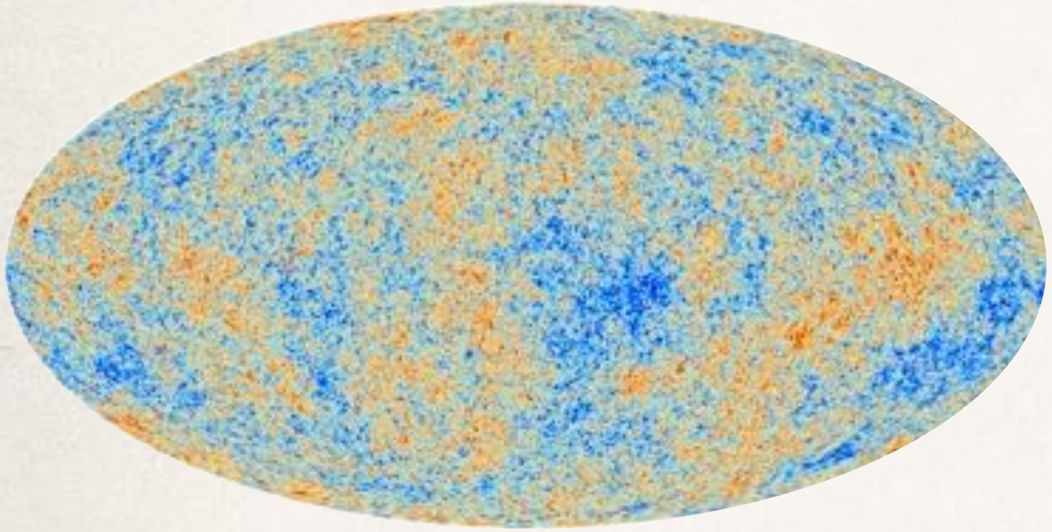

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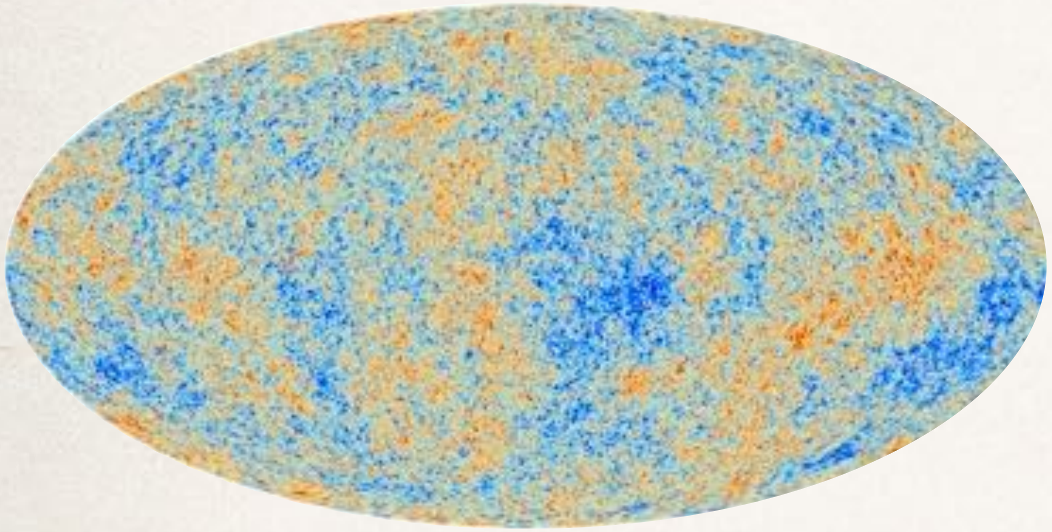

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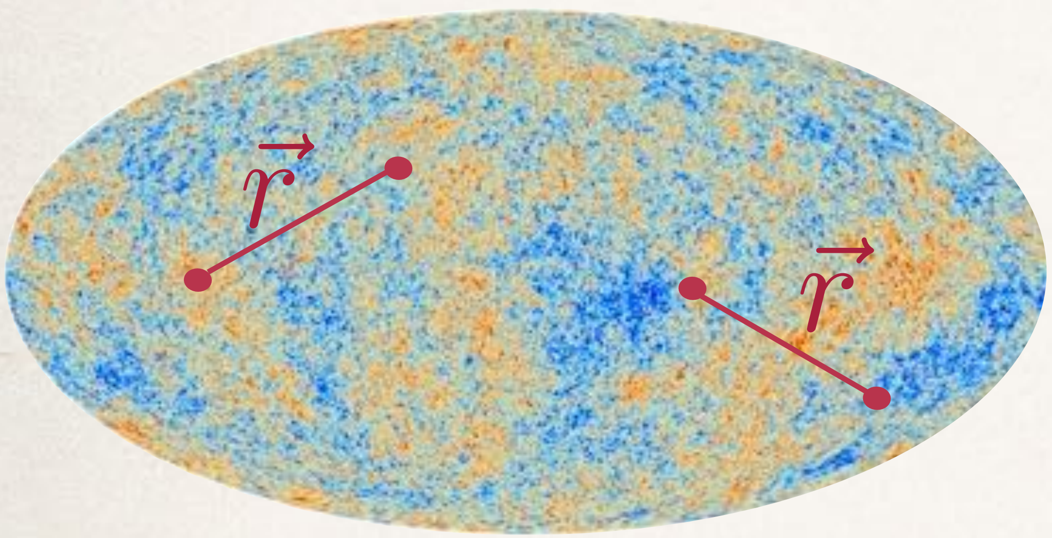

$$\langle \delta(\vec{x})\delta(\vec{x}+\vec{r}) \rangle = \xi_2(||\vec{r}||)$$

Initial state fully described by the 2-pt correlation function (= power spectrum)

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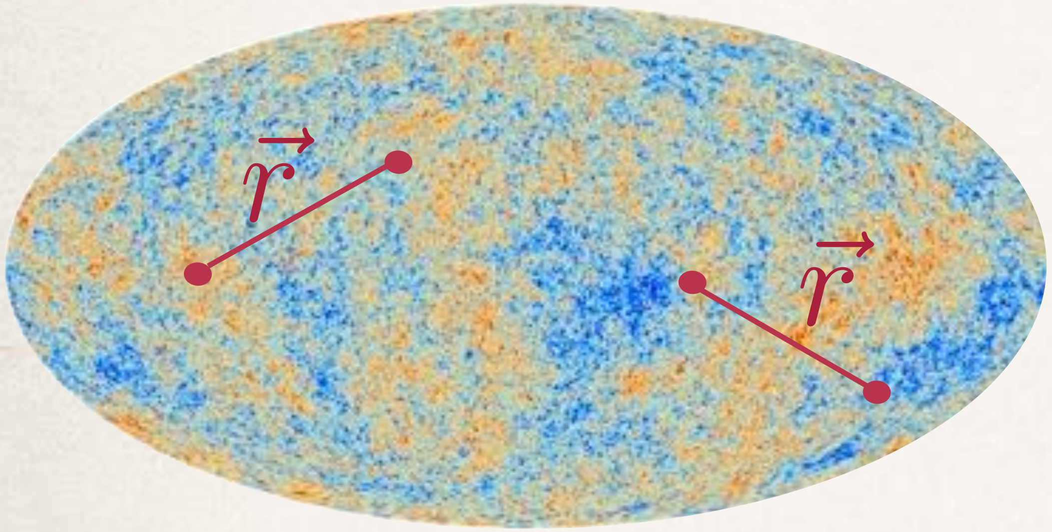

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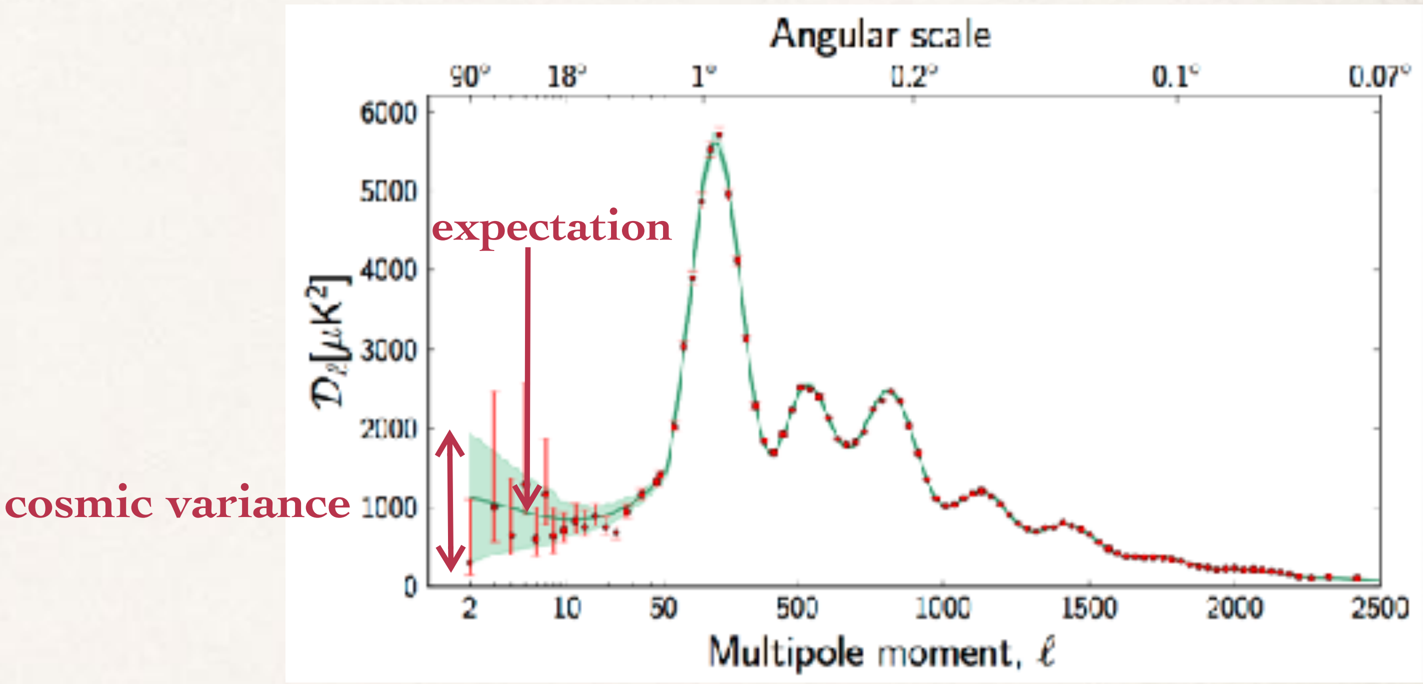
CMB as seen by Planck



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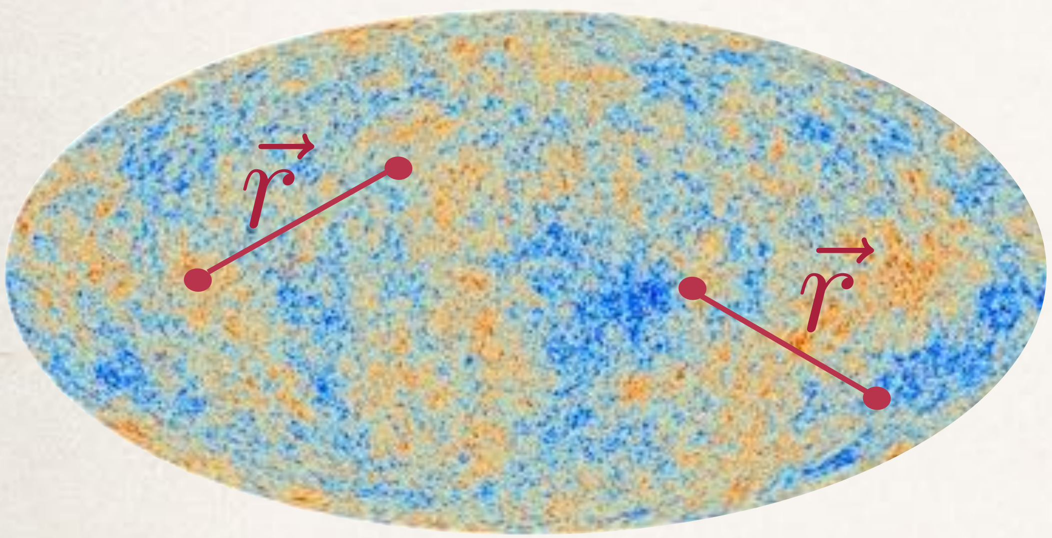

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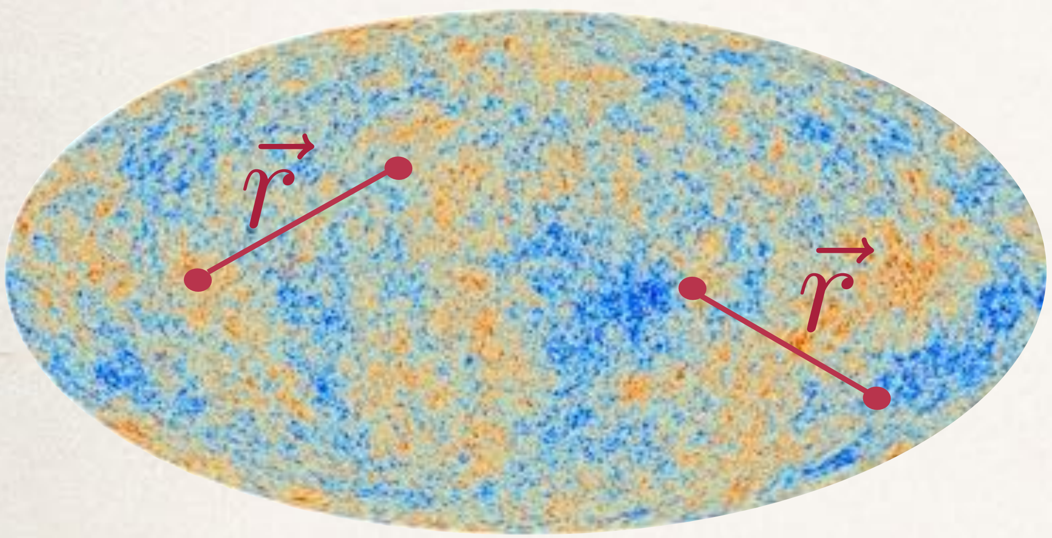

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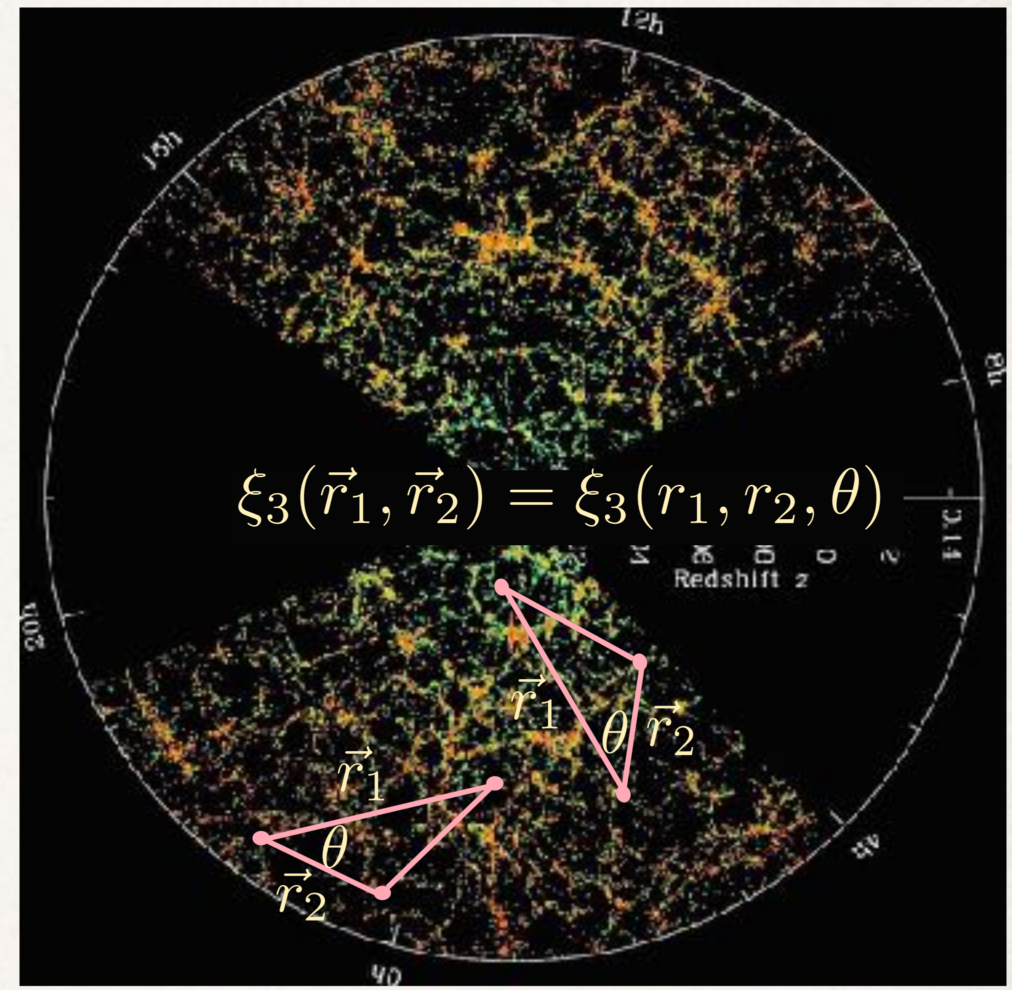
gravity

expansion



Initial state fully described by the 2-pt correlation function (= power spectrum)

Subsequent gravitational evolution is **non-Gaussian**: need to go beyond 2-pt and study **higher order statistics** e.g 3-pt correlation function (=bispectrum)

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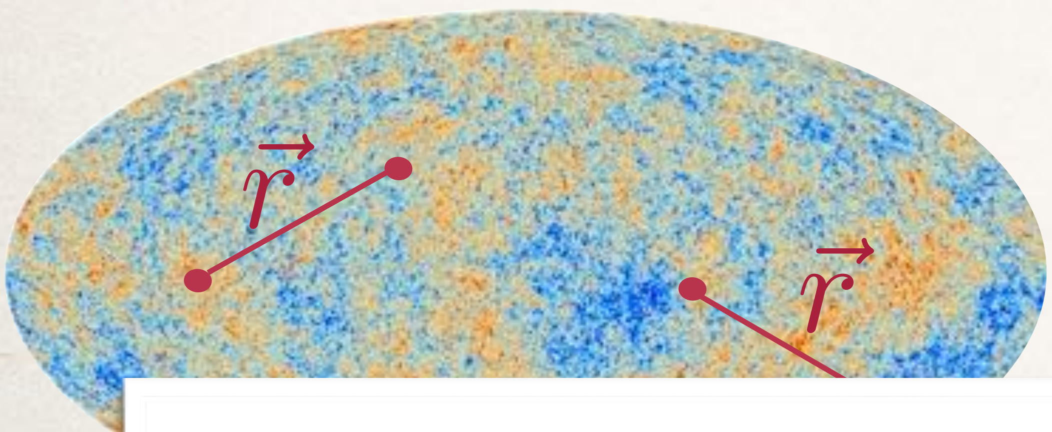
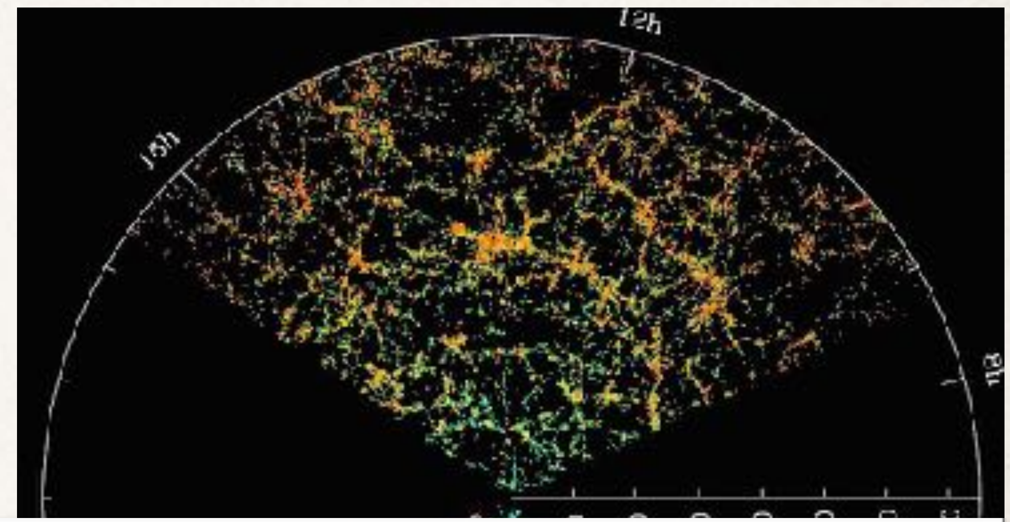
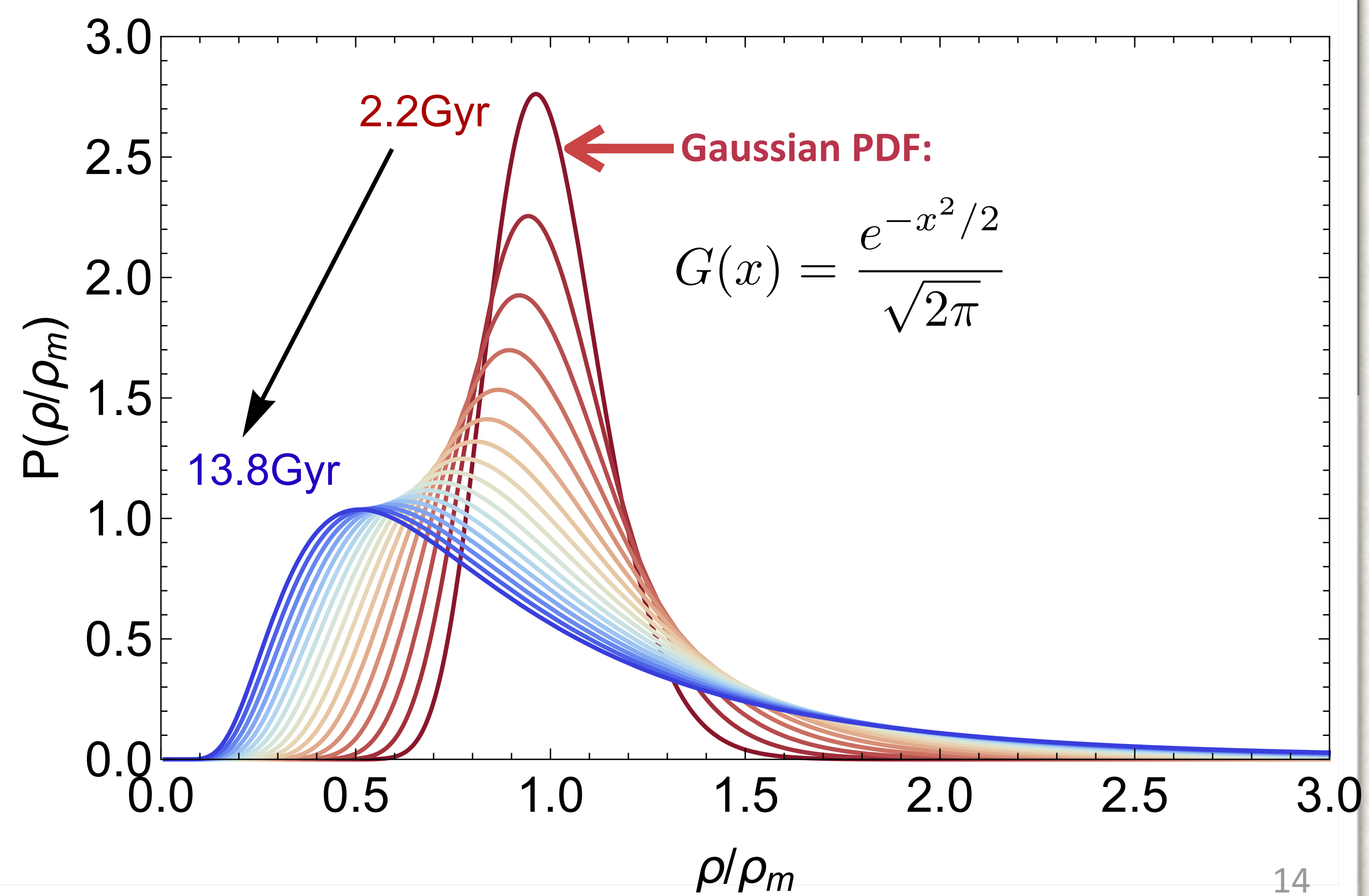


gravity





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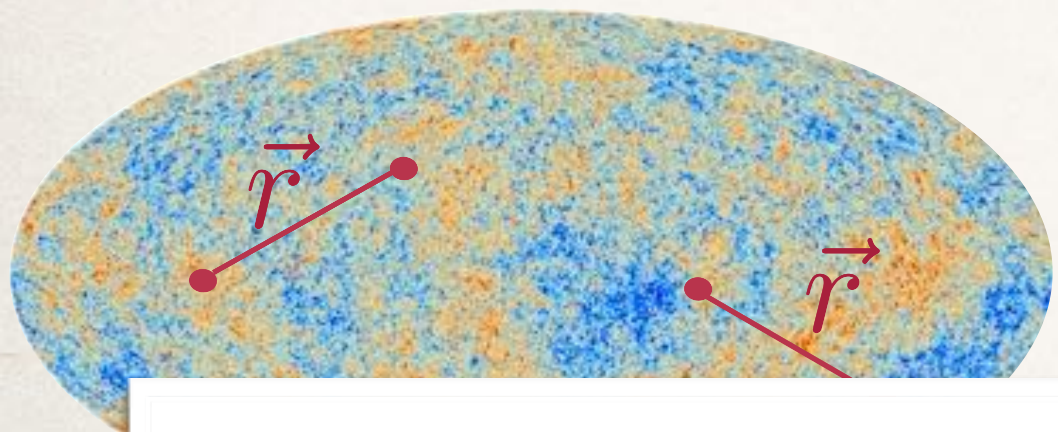

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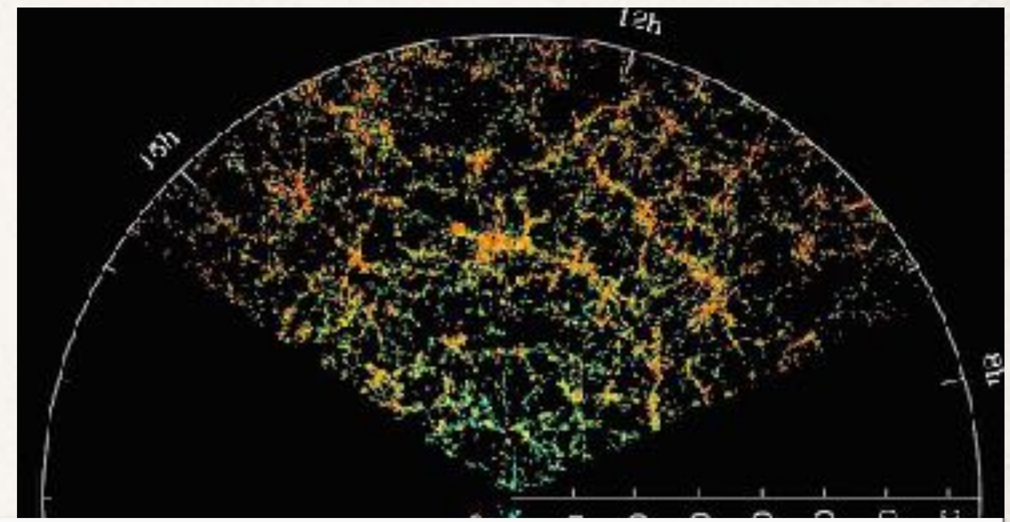

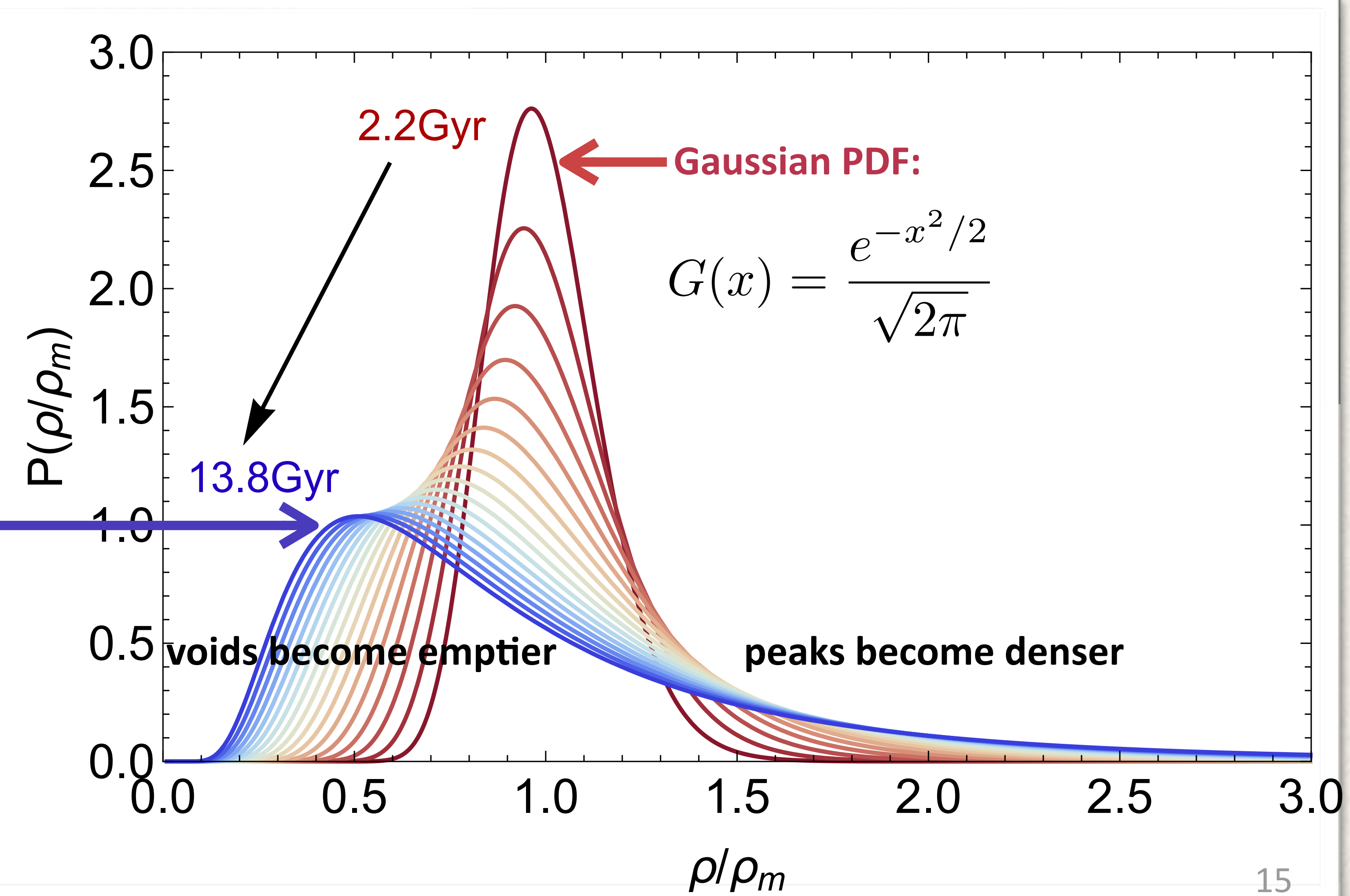

Positively skewed PDF:

$$P(x) = G(x)\left[1 + \frac{1}{3!}\langle x^3\rangle H_3(x) + \dots\right]$$

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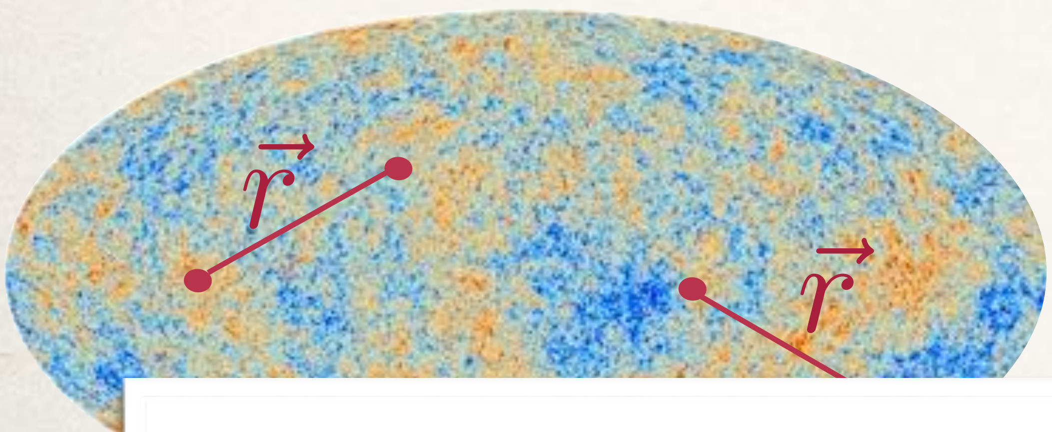

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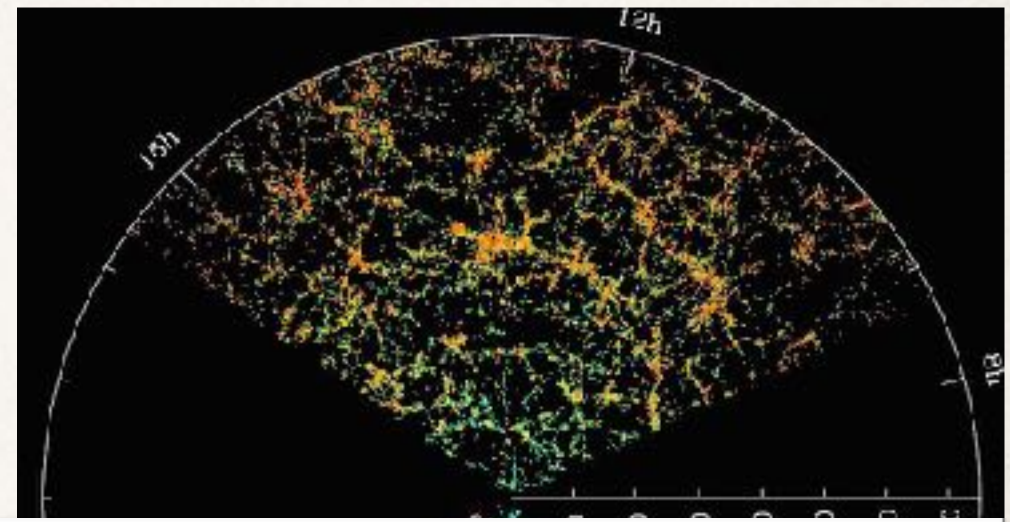

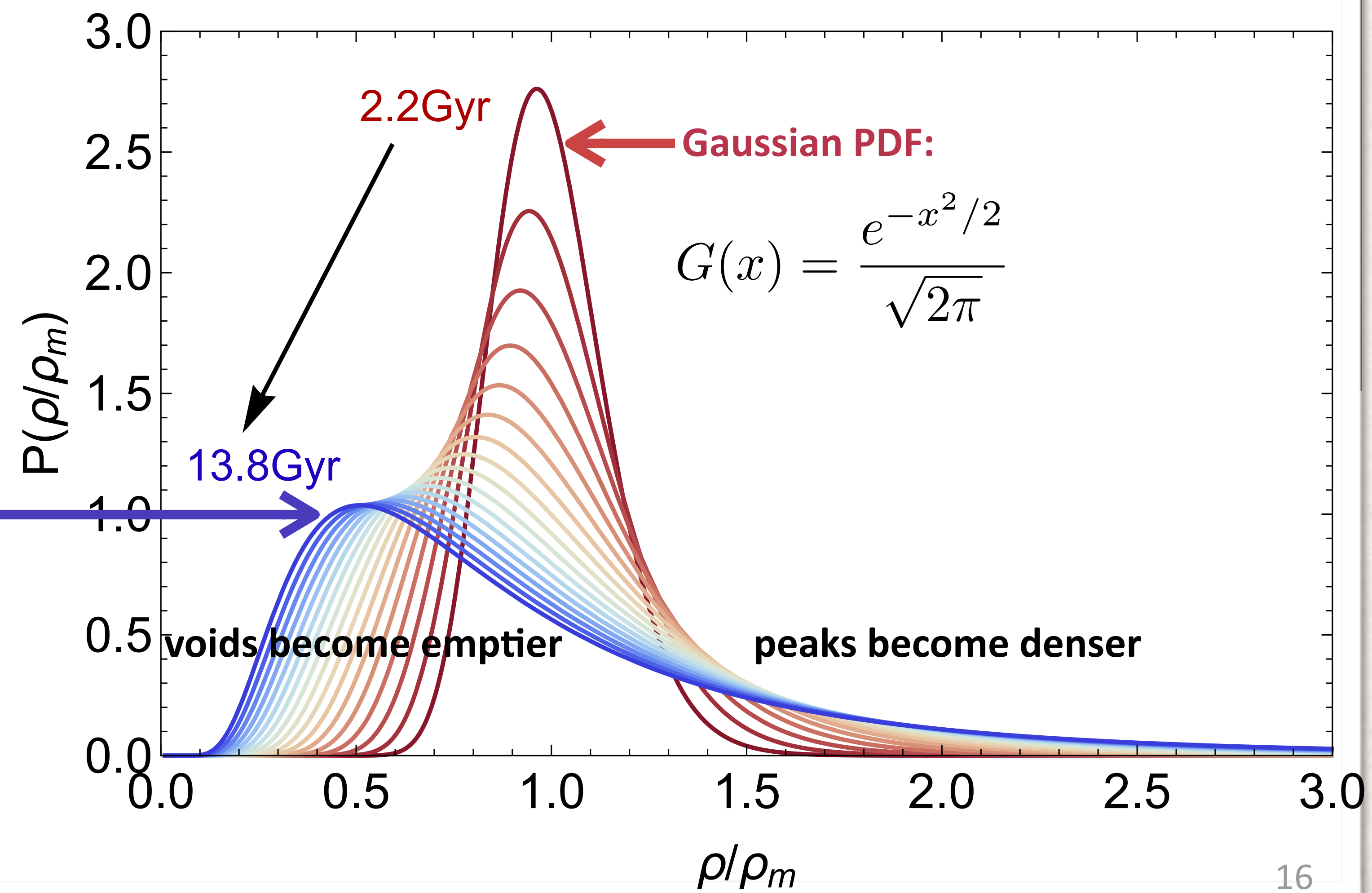

Positively skewed PDF:

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$=S_3\sigma^4$

NL evolution driven by σ

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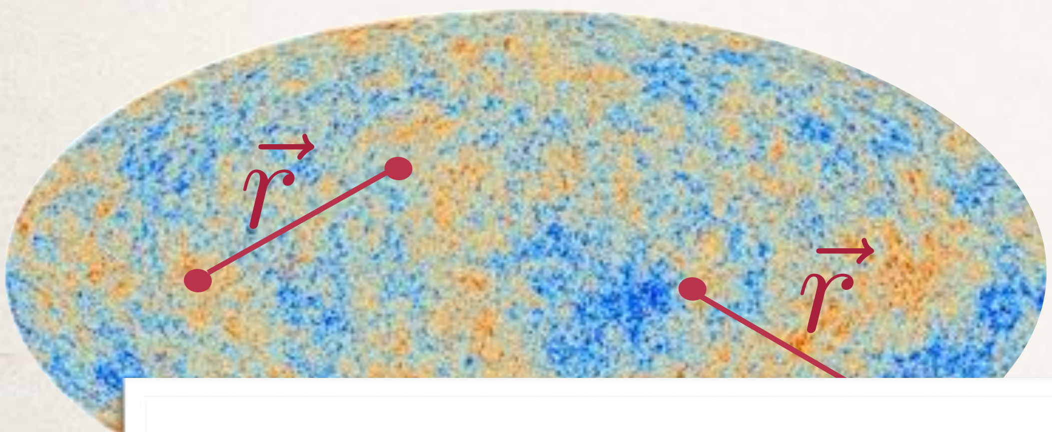
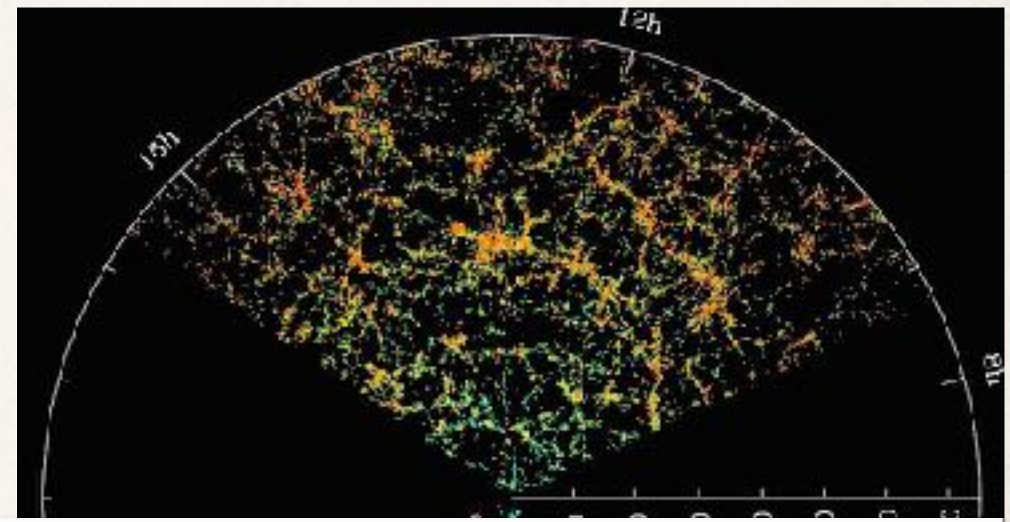
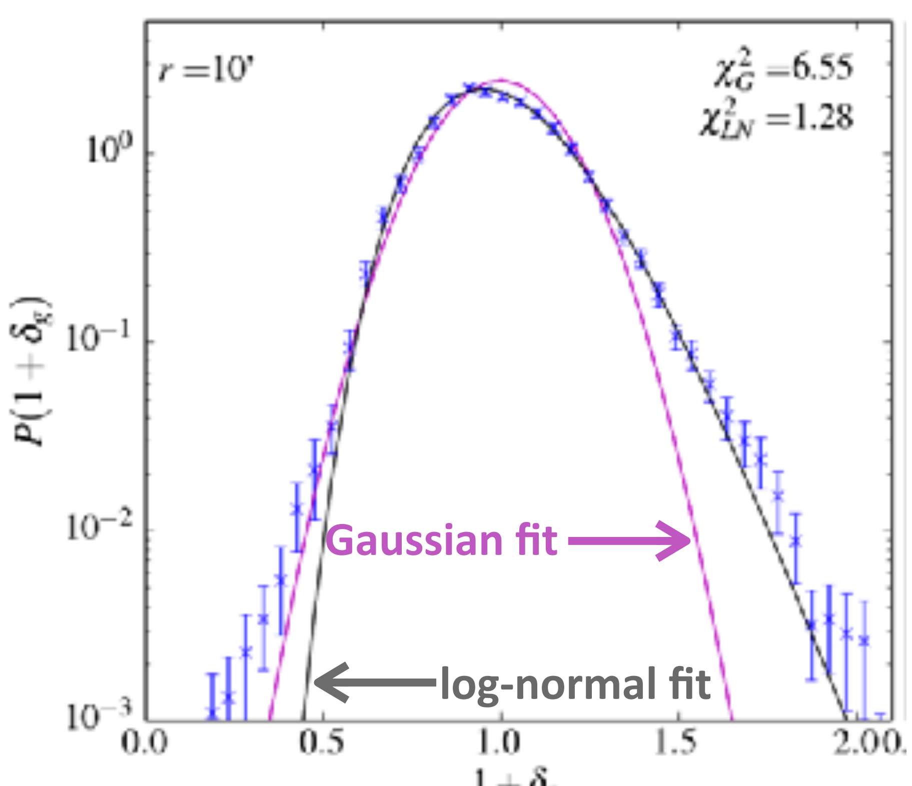
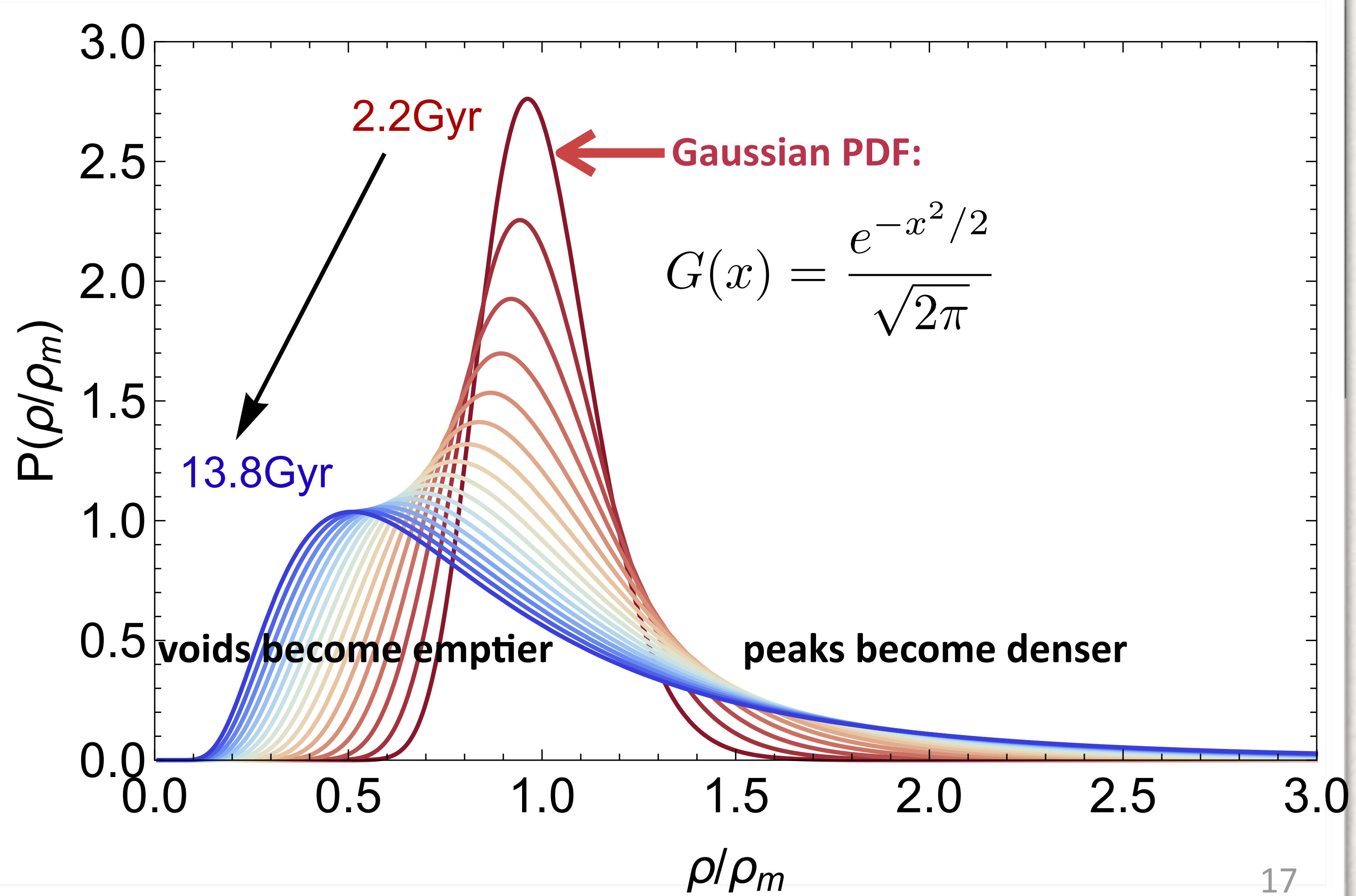


gravity

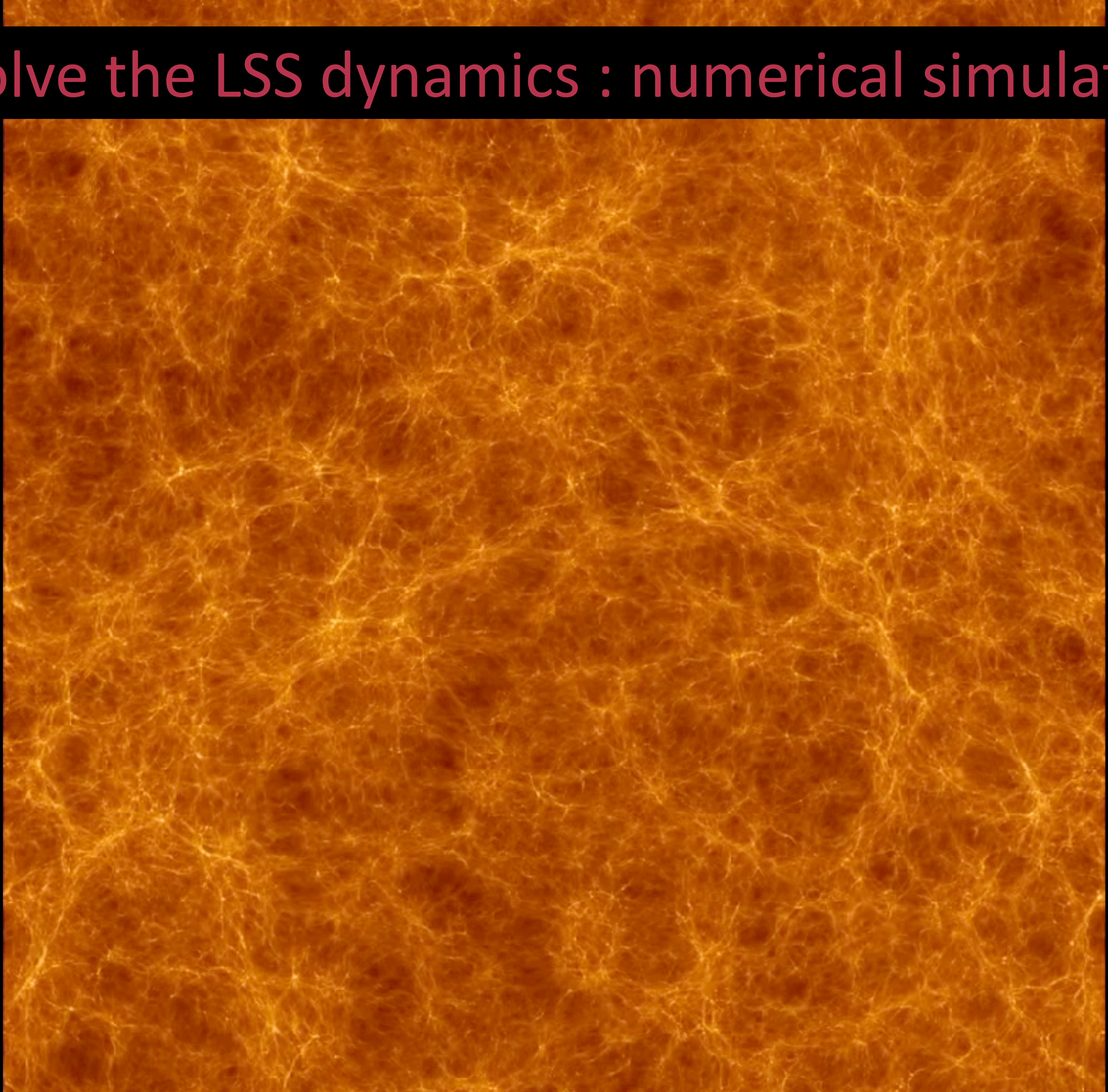


Clerkin'16 (Dark Energy Survey)





To solve the LSS dynamics : numerical simulations



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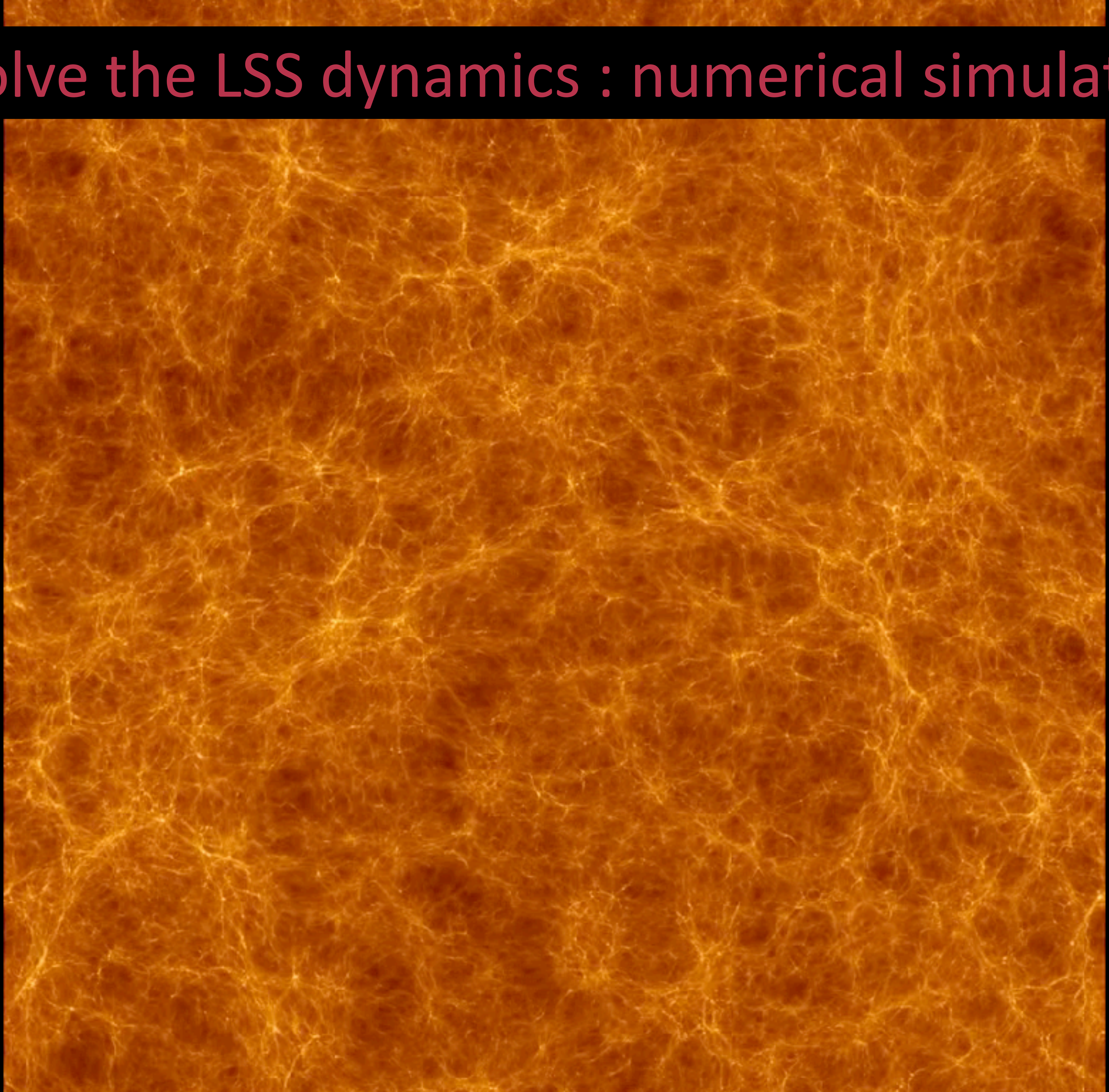


To solve the LSS dynamics : numerical simulations or theoretical predictions in some regimes

The Vlasov-Poisson equations (collisionless Boltzmann equation) - f(x,p) is the phase-space density distribution - are **fully nonlinear**:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial}{\partial t} f(\mathbf{x},\mathbf{p},t) + \frac{\mathbf{p}}{ma^2}\frac{\partial}{\partial \mathbf{x}} f(\mathbf{x},\mathbf{p},t) - m\frac{\partial}{\partial \mathbf{x}}\Phi(\mathbf{x})\frac{\partial}{\partial \mathbf{p}} f(\mathbf{x},\mathbf{p},t) = 0$$

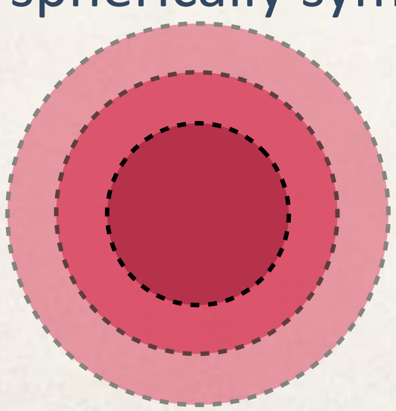
$$\Delta\Phi(\mathbf{x}) = \frac{4\pi G m}{a}\left(\int f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p} - \bar{n}\right)$$

➢ **single flow** equations until shell crossing for a self-gravitating cold fluid:

Peebles 1980; Fry 1984; Bernardeau 2002

$$\begin{aligned}
\frac{\partial}{\partial t}\delta(\mathbf{x},t) + \frac{1}{a}[(1+\delta(\mathbf{x},t))\mathbf{u}_i(\mathbf{x},t)]_{,i} &= 0 \\
\frac{\partial}{\partial t}\mathbf{u}_i(\mathbf{x},t) + \frac{\dot{a}}{a}\mathbf{u}_i(\mathbf{x},t) + \frac{1}{a}\mathbf{u}_j(\mathbf{x},t)\mathbf{u}_{i,j}(\mathbf{x},t) &= -\frac{1}{a}\Phi_{,i}(\mathbf{x},t) + \cancel{\ldots} \\
\Phi_{,ii}(\mathbf{x},t) - 4\pi G\bar{\rho}\, a^2\, \delta(\mathbf{x},t) &= 0
\end{aligned}$$

➢ **Exact solutions**: spherical collapse (gravitational evolution of a spherically symmetric field)



evolution of a shell of radius r and mass M: $\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{GM}{r^2} + \frac{\Lambda}{3} r$

To solve the LSS dynamics : numerical simulations or theoretical predictions in some regimes

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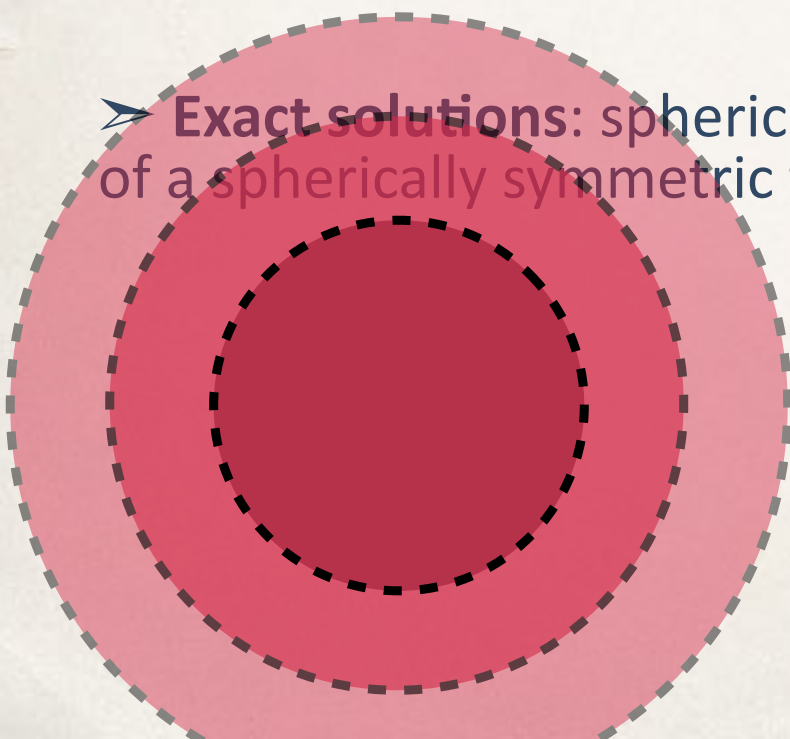

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$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial}{\partial t} f(\mathbf{x},\mathbf{p},t) + \frac{\mathbf{p}}{ma^2}\frac{\partial}{\partial \mathbf{x}} f(\mathbf{x},\mathbf{p},t) - m\frac{\partial}{\partial \mathbf{x}}\Phi(\mathbf{x})\frac{\partial}{\partial \mathbf{p}} f(\mathbf{x},\mathbf{p},t) = 0$$

$$\Delta\Phi(\mathbf{x}) = \frac{4\pi G m}{a}\left(\int f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p} - \bar{n}\right)$$

➢ **single flow** equations until shell crossing for a self-gravitating cold fluid:

Peebles 1980; Fry 1984; Bernardeau 2002

$$\begin{aligned}
\frac{\partial}{\partial t}\delta(\mathbf{x},t) + \frac{1}{a}[(1+\delta(\mathbf{x},t))\mathbf{u}_i(\mathbf{x},t)]_{,i} &= 0 \\
\frac{\partial}{\partial t}\mathbf{u}_i(\mathbf{x},t) + \frac{\dot{a}}{a}\mathbf{u}_i(\mathbf{x},t) + \frac{1}{a}\mathbf{u}_j(\mathbf{x},t)\mathbf{u}_{i,j}(\mathbf{x},t) &= -\frac{1}{a}\Phi_{,i}(\mathbf{x},t) + \cancel{\ldots} \\
\Phi_{,ii}(\mathbf{x},t) - 4\pi G\bar{\rho}\, a^2\, \delta(\mathbf{x},t) &= 0
\end{aligned}$$

➢ **Exact solutions**: spherical collapse (gravitational evolution of a spherically symmetric field)

evolution of a shell of radius r and mass M: $\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{GM}{r^2} + \frac{\Lambda}{3}r$

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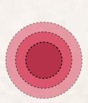

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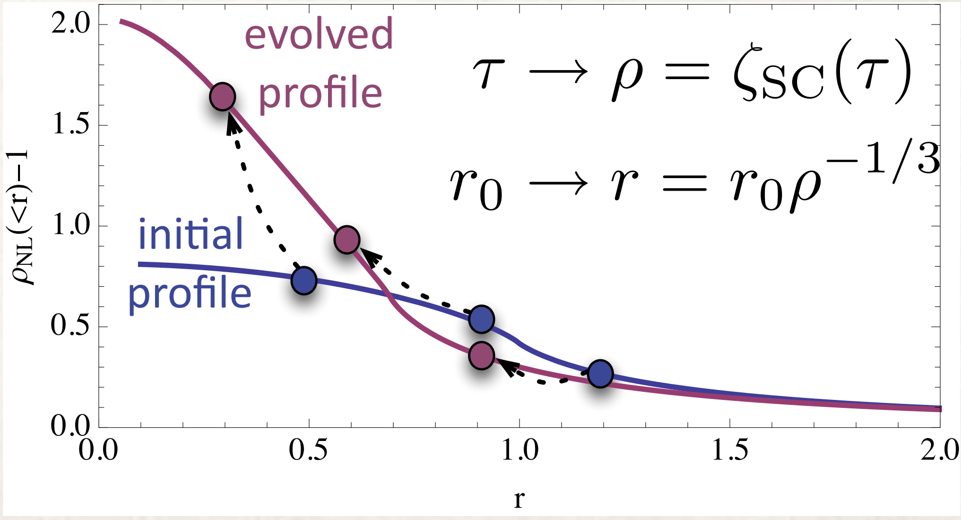

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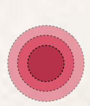

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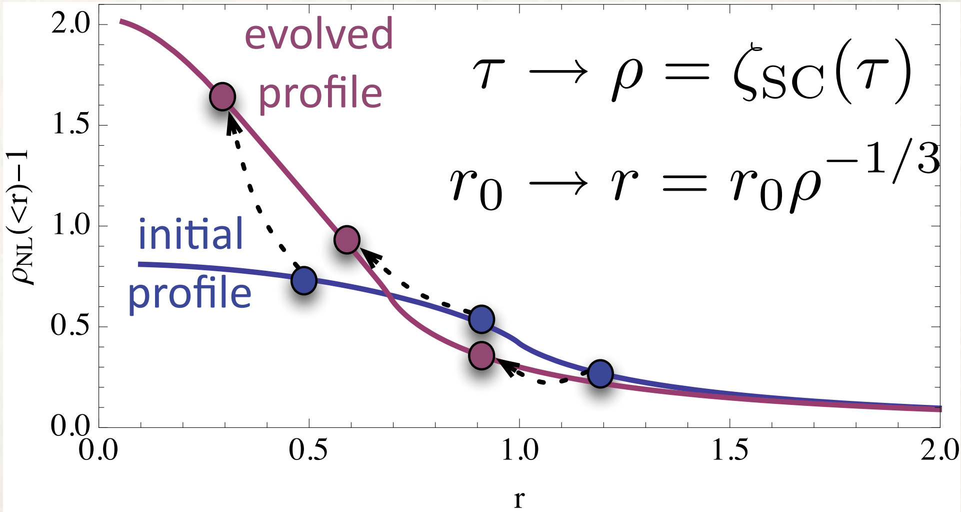

➢ **Perturbation Theory:** expand the cosmic fields with respect to initial density fields and solve perturbatively order by order $\delta(\mathbf{x},t) = \delta_1(\mathbf{x},t) + \delta_2(\mathbf{x},t) + \cdots$

Perturbation Theory

Single-flow equations + perturbative expansion yield the density at order n :

$$\delta_n(\mathbf{k}) = \int \mathrm{d}^3\mathbf{q}_1 \ldots \int \mathrm{d}^3\mathbf{q}_n \, \delta_D(\mathbf{k} - \mathbf{q}_{1\ldots n}) F_n(\mathbf{q}_1, \ldots, \mathbf{q}_n) \delta_1(\mathbf{q}_1) \ldots \delta_1(\mathbf{q}_n)$$

where F_n are the PT kernels and can be computed hierarchically in k space

$$F_2(\mathbf{q}_1, \mathbf{q}_2) = \frac{5}{7} + \frac{1}{2} \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_1 q_2} \left(\frac{q_1}{q_2} + \frac{q_2}{q_1}\right) + \frac{2}{7} \frac{(\mathbf{q}_1 \cdot \mathbf{q}_2)^2}{q_1^2 q_2^2}$$

$$F_3(q_1, q_2, q_3) = \frac{5\,(q_2+q_3).q_2\,(q_1+q_2+q_3).q_1}{36 q_1^2 q_2^2} + \frac{5\,(q_2+q_3).q_3\,(q_1+q_2+q_3).q_1}{36 q_1^2 q_3^2} +$$
$$\frac{q_3.q_2\,(q_2+q_3).(q_2+q_3)\,(q_1+q_2+q_3).q_1}{18 q_1^2 q_2^2 q_3^2} + \frac{(q_2+q_3).q_2\,(q_1+q_2+q_3).(q_2+q_3)}{12\,(q_2+q_3).(q_2+q_3)\,q_2^2} +$$
$$\frac{(q_2+q_3).q_1\,(q_2+q_3).q_2\,(q_1+q_2+q_3).(q_1+q_2+q_3)}{42\,(q_2+q_3).(q_2+q_3)\,q_1^2 q_2^2} + \frac{(q_2+q_3).q_3\,(q_1+q_2+q_3).(q_2+q_3)}{12\,(q_2+q_3).(q_2+q_3)\,q_3^2} +$$
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$$\frac{2 q_3.q_2\,(q_2+q_3).q_1\,(q_1+q_2+q_3).(q_1+q_2+q_3)}{63 q_1^2 q_2^2 q_3^2}$$

...

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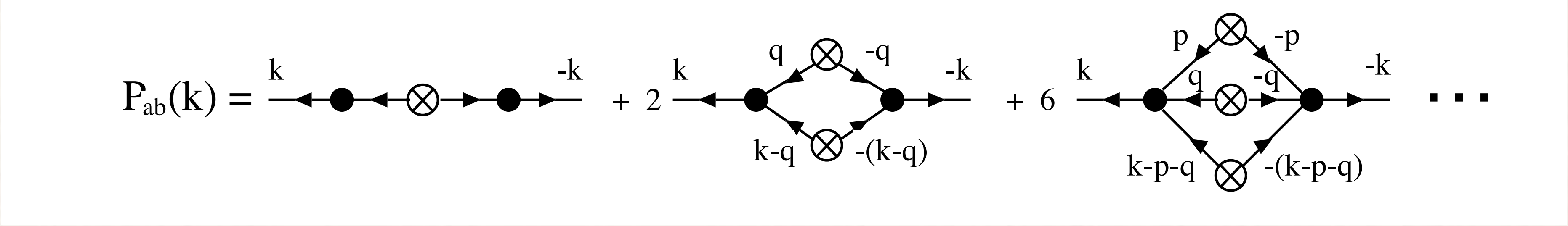
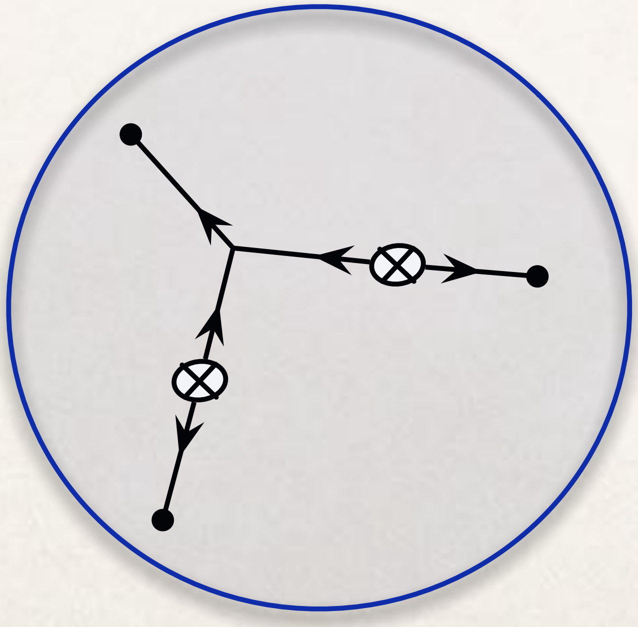
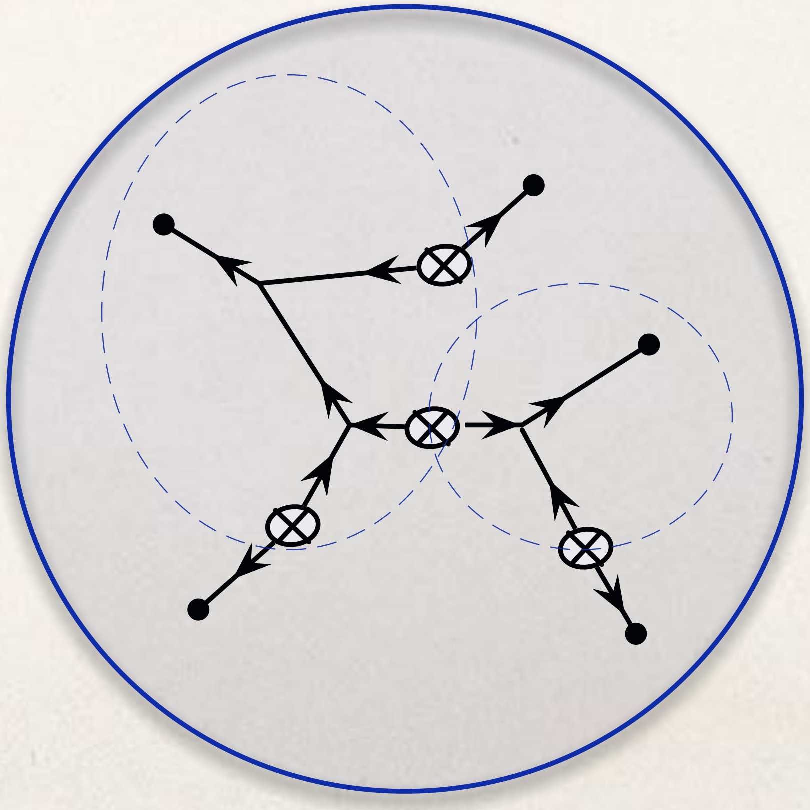
E.g the power spectrum $\langle\delta(\mathbf{k})\delta(\mathbf{k}')\rangle = P(\mathbf{k})\delta_{\mathrm{D}}(\mathbf{k}+\mathbf{k}')$ can then be predicted at any order



or any other N>2-point correlation function:



contribution to the 3-pt function (bispectrum)

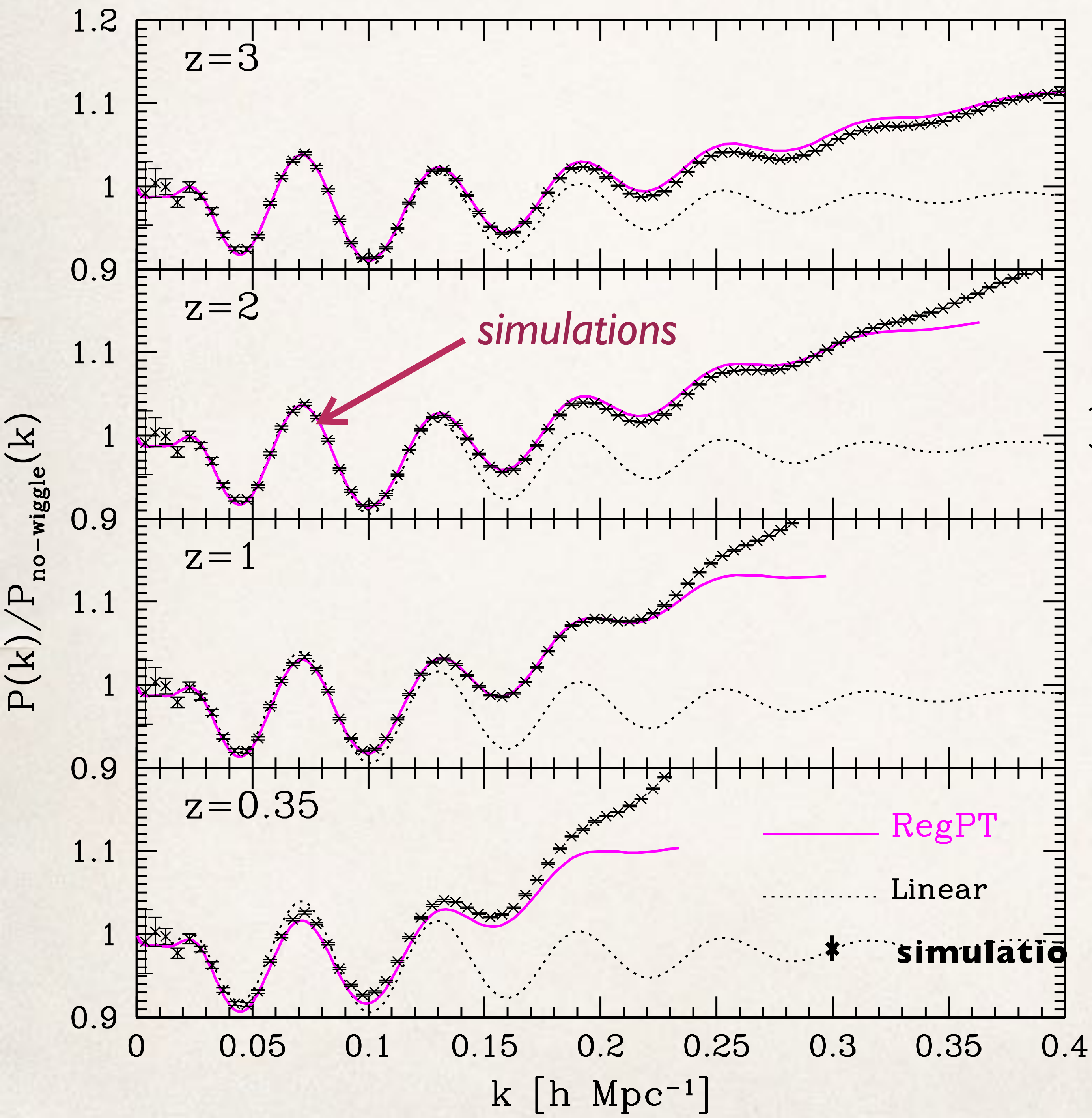


contribution to the 5-pt correlation function

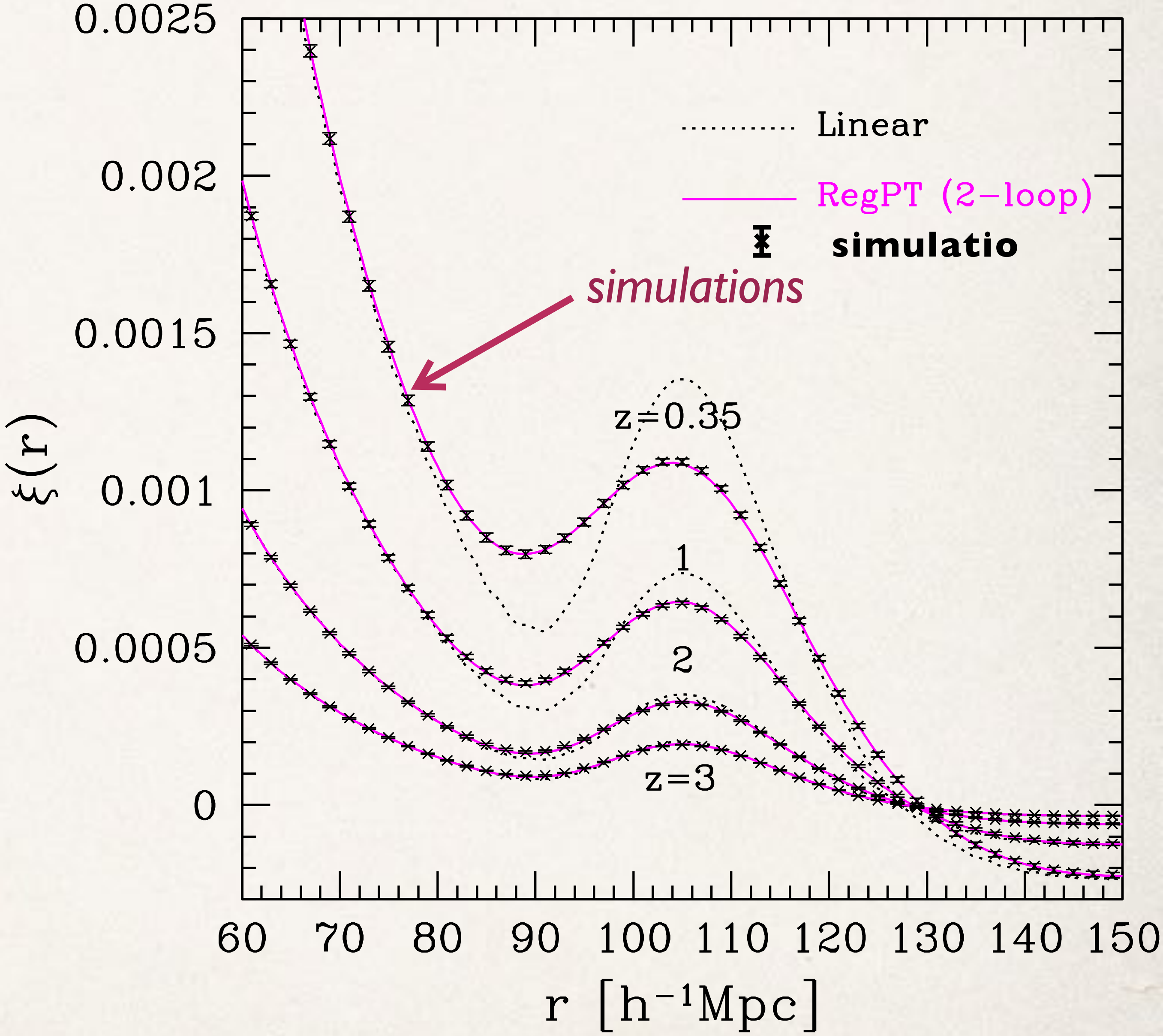
Taruya, Bernardeau, Nishimichi, Codis '12

Perturbation Theory

Power spectrum



2-pt correlation function



Charting PT

number of loops in standard PT for Gaussian Initial Conditions

Order of observable in field expansion

	leading order LO	order 1 NLO	order 2 NNLO	order 2.5	order 3	...order p
2-point statistics	OK	OK	OK	EFT	partial exact results	partial resum
3-point statistics	OK	OK (but not systematics				partial resummations
4-point statistics	OK	to be done... (cosmic variance)				
N-point statistics	OK, for topological estimators OK, in specific geometries (**counts in cells**)					

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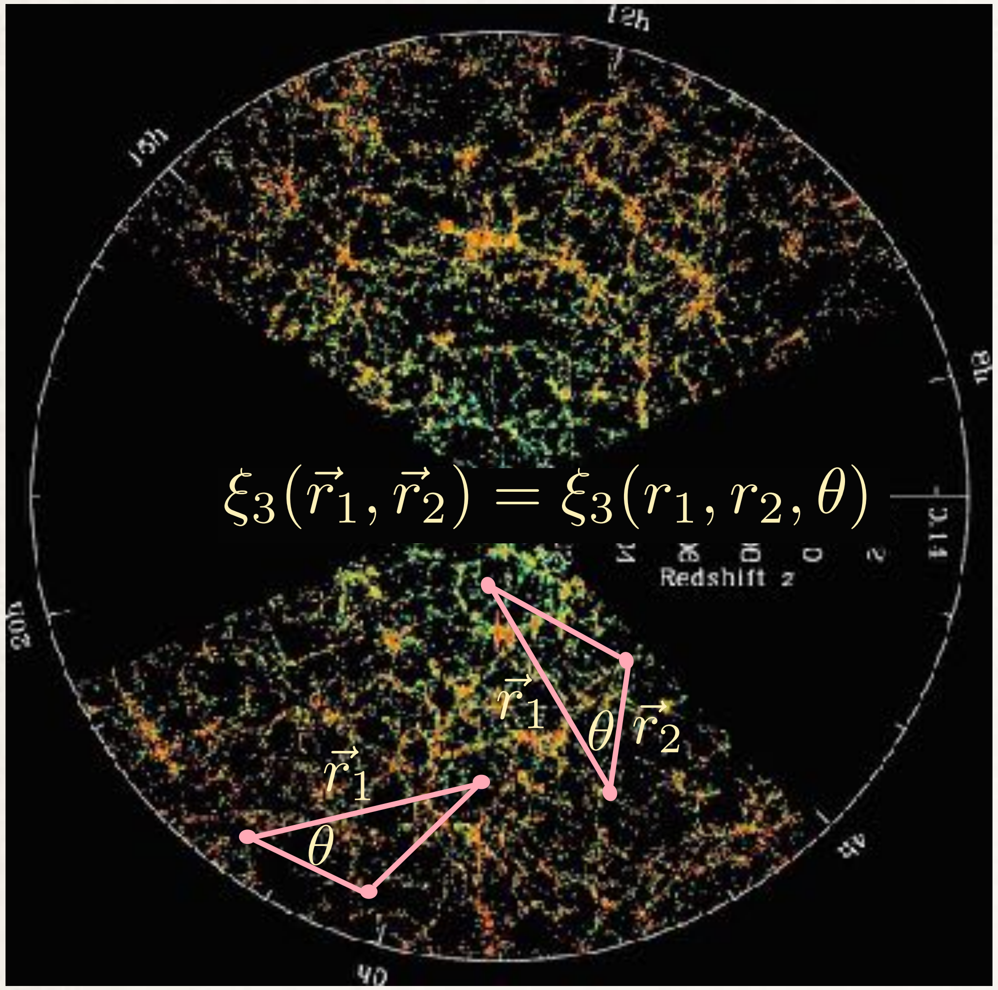
Bispectrum



$$\xi_3(\mathbf{r_1}, \mathbf{r_2}) = \langle \delta(0)\delta(\mathbf{r_1})\delta(\mathbf{r_2})\rangle$$

$$B(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3})\delta_D(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) = \langle \delta(\mathbf{k_1})\delta(\mathbf{k_2})\delta(\mathbf{k_3})\rangle$$

0 for GRF

tree-order PT= $2P(k_1)P(k_2)F_2(\mathbf{k_1}, \mathbf{k_2}) + cyc.$

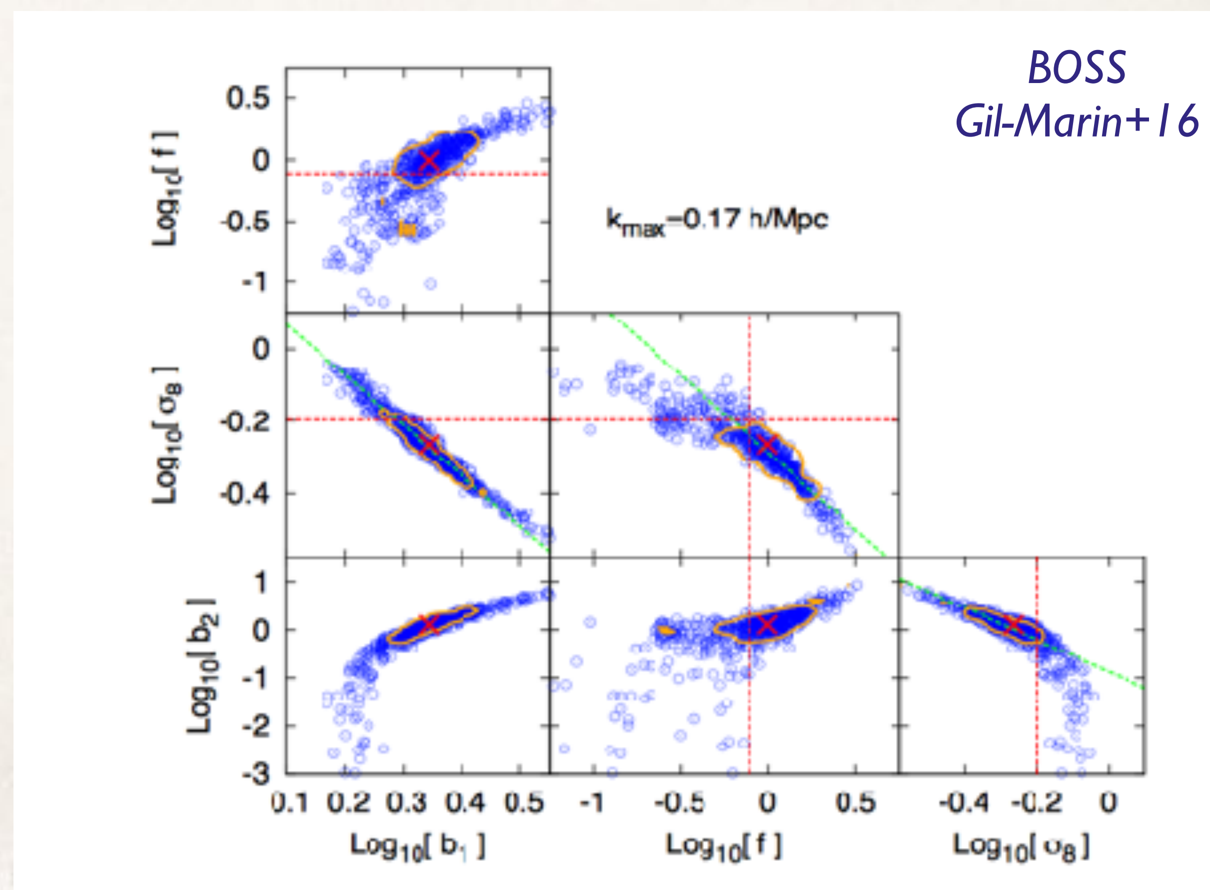
For late-time galaxy clustering, it allows to:

-measure the bias parameters

-measure primordial non-gaussianities

Tellarini+16

Sample	Power Spectrum $\sigma_{f_{NL}}$ bias float	Power Spectrum $\sigma_{f_{NL}}$ bias fixed	Bispectrum $\sigma_{f_{NL}}$ bias float	Bispectrum $\sigma_{f_{NL}}$ bias fixed
BOSS	21.30	13.28	$1.04^{(0.65)}_{(2.47)}$	$0.57^{(0.35)}_{(1.48)}$
eBOSS	14.21	11.12	$1.18^{(0.82)}_{(2.02)}$	$0.70^{(0.48)}_{(1.29)}$
Euclid	6.00	4.71	$0.45^{(0.18)}_{(0.71)}$	$0.32^{(0.12)}_{(0.35)}$
DESI	5.43	4.37	$0.31^{(0.17)}_{(0.48)}$	$0.21^{(0.12)}_{(0.37)}$
BOSS + Euclid	5.64	4.44	$0.39^{(0.17)}_{(0.59)}$	$0.28^{(0.11)}_{(0.34)}$



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only valid on very large scales

breaks down for $\sigma^2 \sim 0.1$

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The feature of spherical collapse leads to analytic predictions in the **mildly non-linear regime** @ few percent level until $\sigma^2 \sim 1$!!

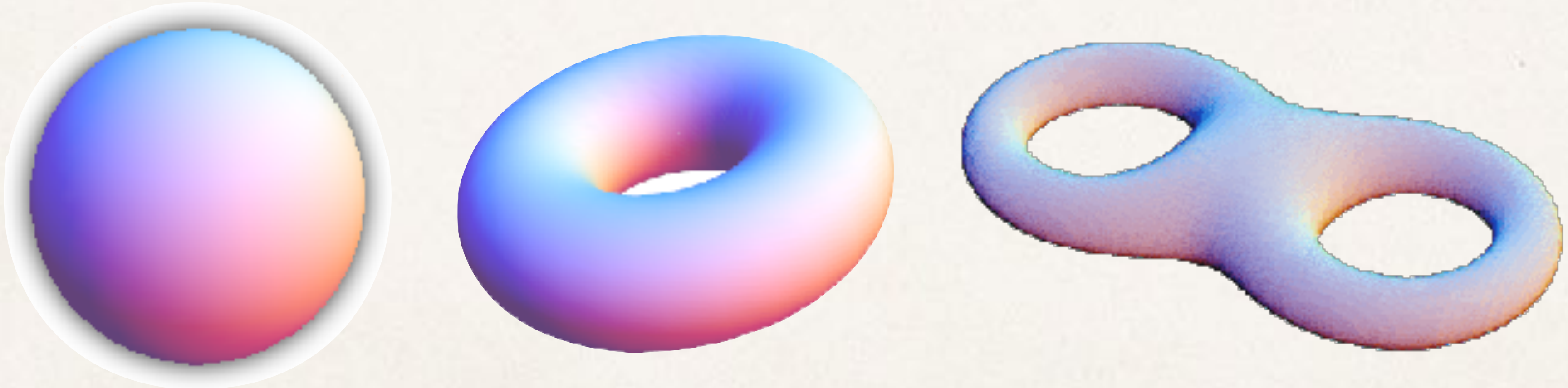
Partie II : Topologie

Topological estimators

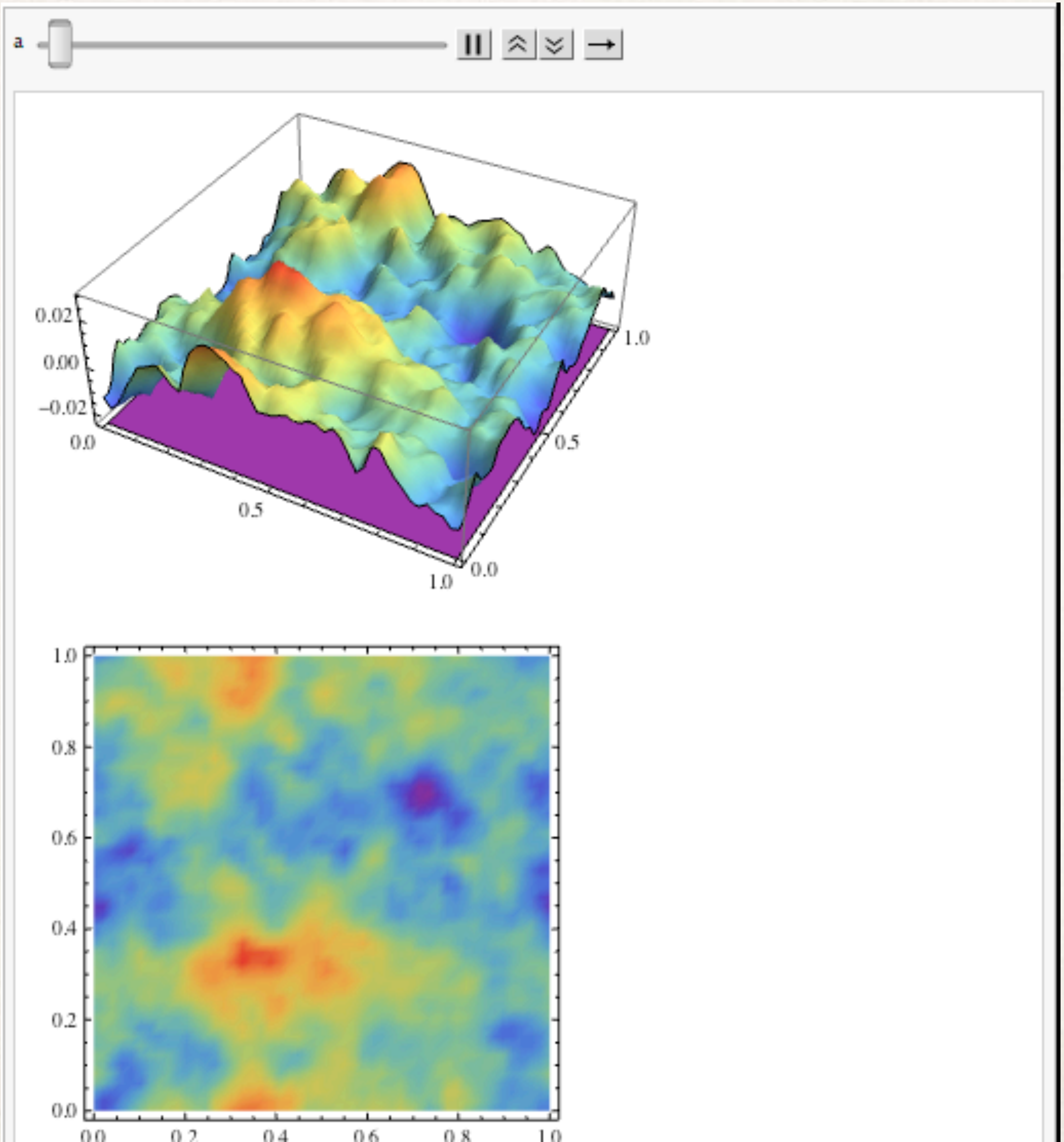
Alternative to the usual use of N-point correlation functions / poly-spectra,... which is :

- independent from bias (M/L ratio)
- easier to measure in the data (less sensitive to masks,...), more robust

Because topology is about shapes, connectivity, holes,... and is ***invariant*** under ***continuous*** deformation (stretching, twisting, bending...).



Topology of excursion sets



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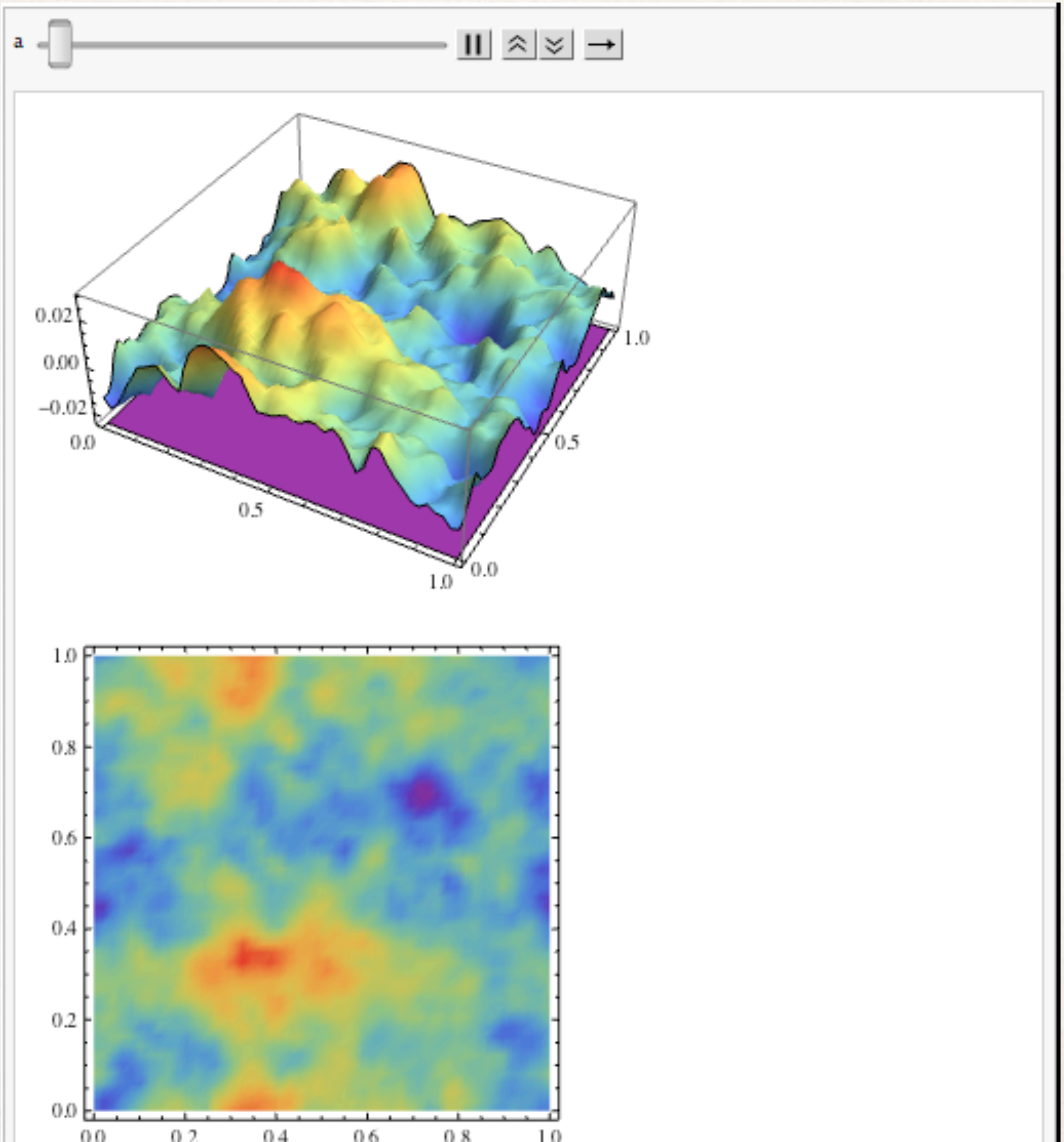
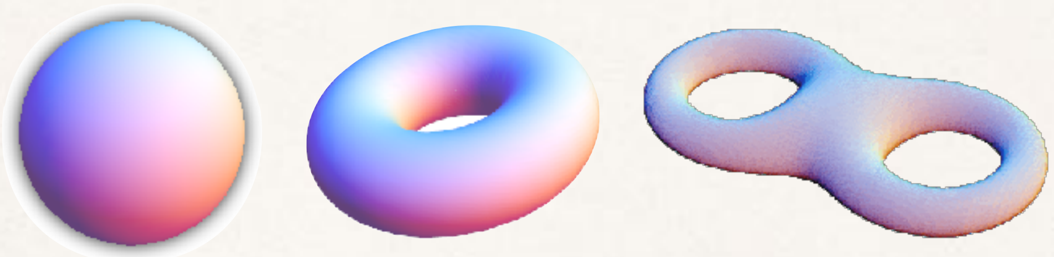


topological estimators?

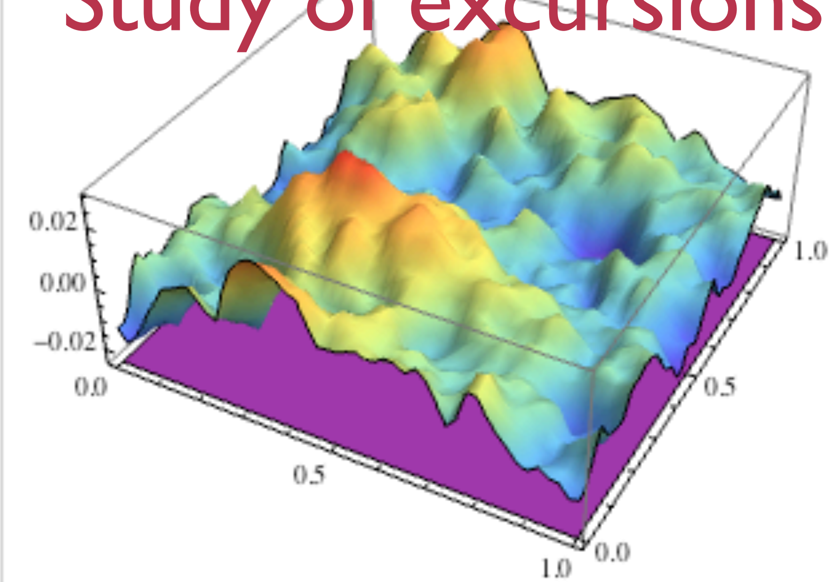
➢**Minkowski functionals (topological invariants):**

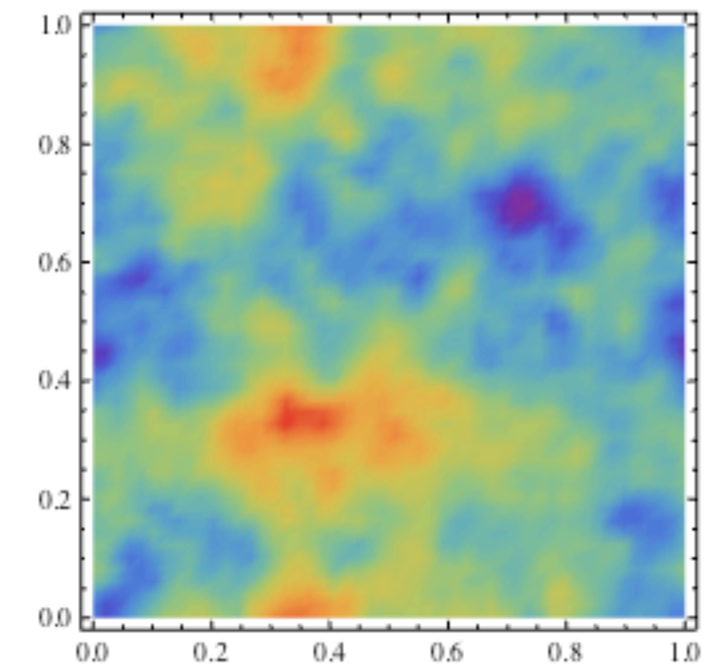
d+1 MFs in d dimensions.
Mathematical genus in 2D = number of handles/holes (max number of cuttings along closed curves without disconnecting the surface)



g=0 g=1 g=2

Study of excursions





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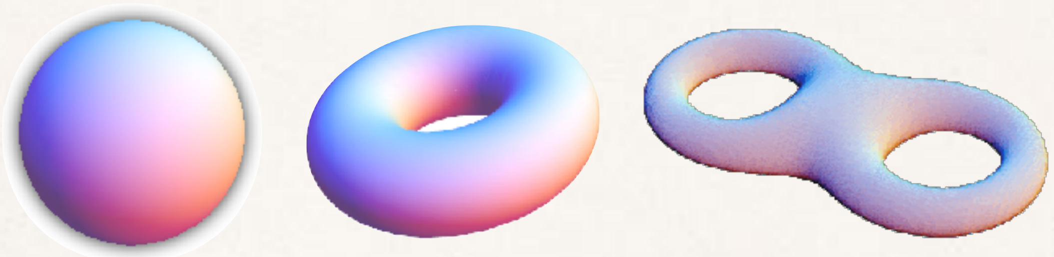

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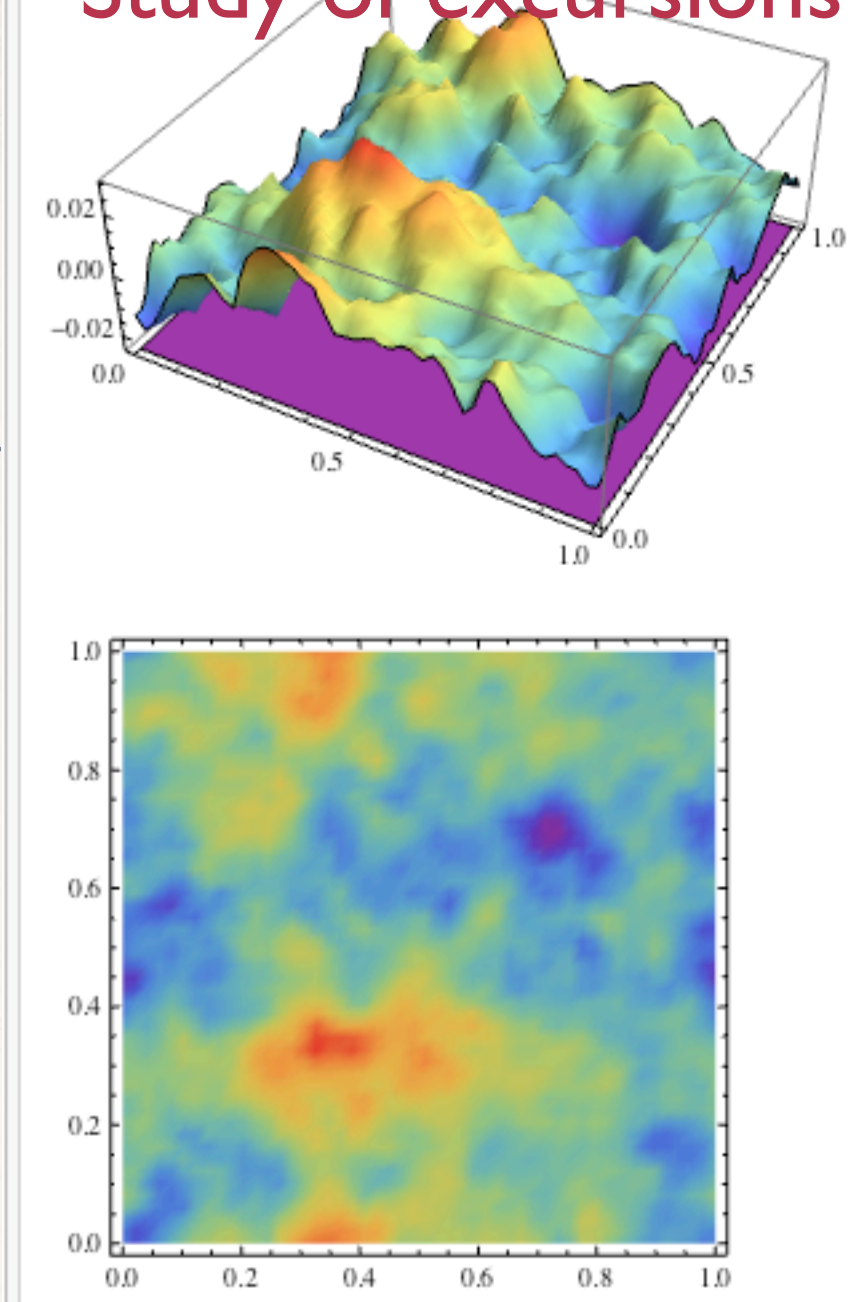

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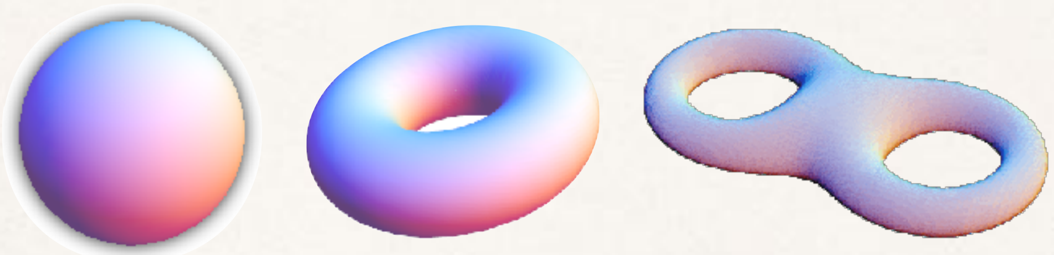

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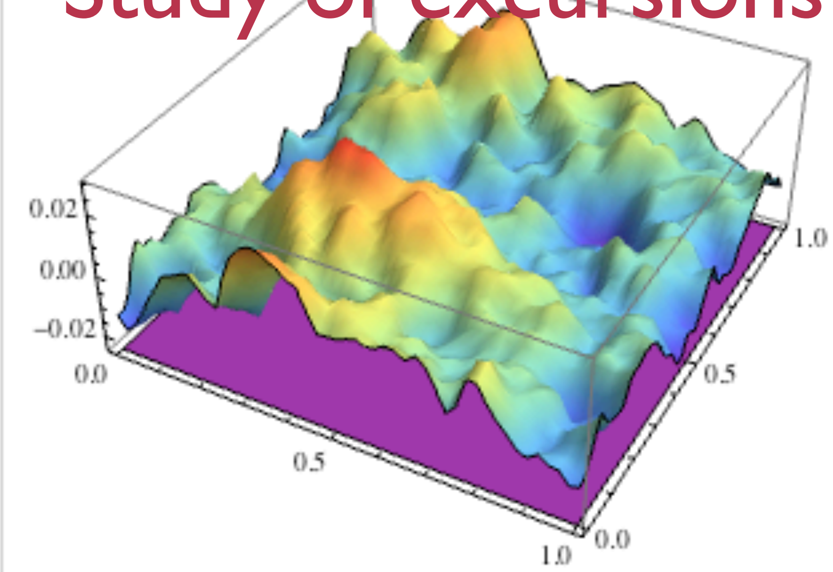

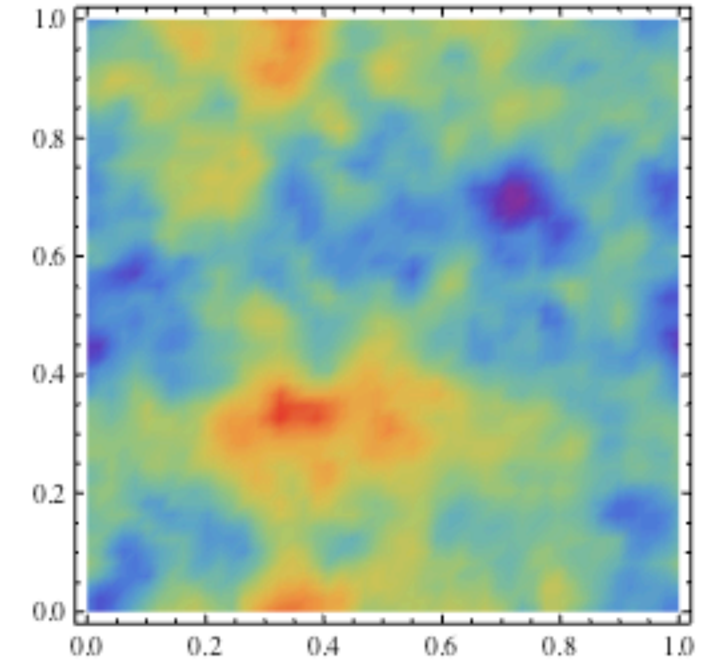

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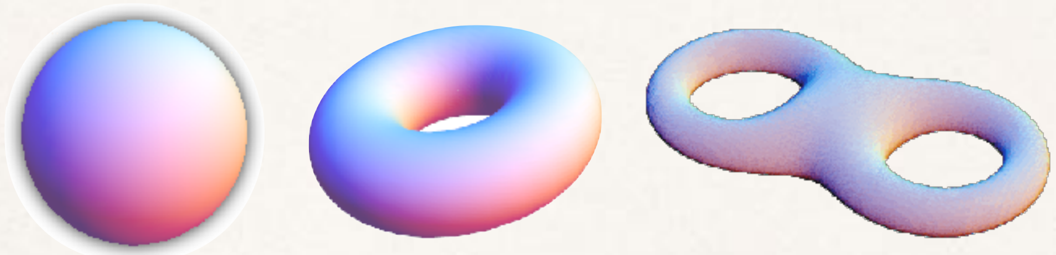

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This is a topological invariant: *deux surfaces sont homeomorphes si elles ont le meme genre.*

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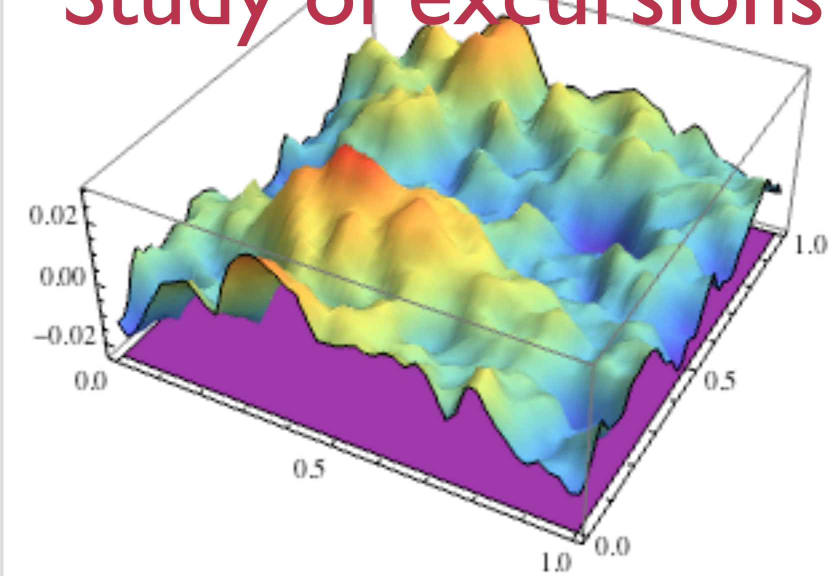

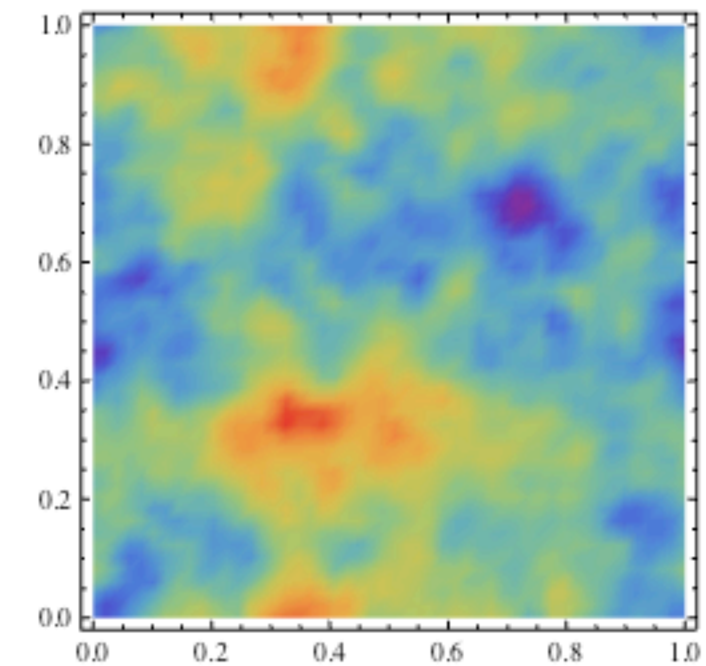

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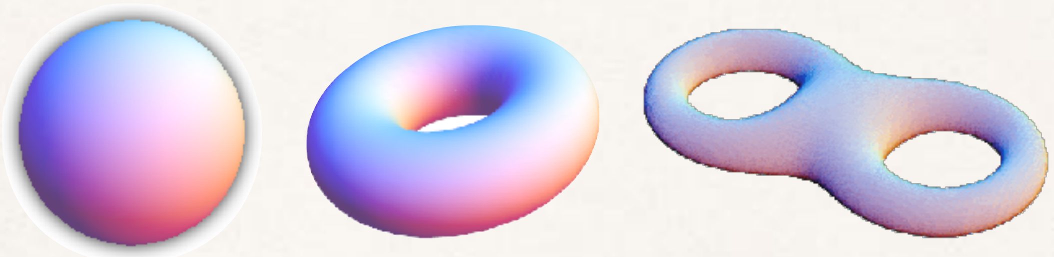

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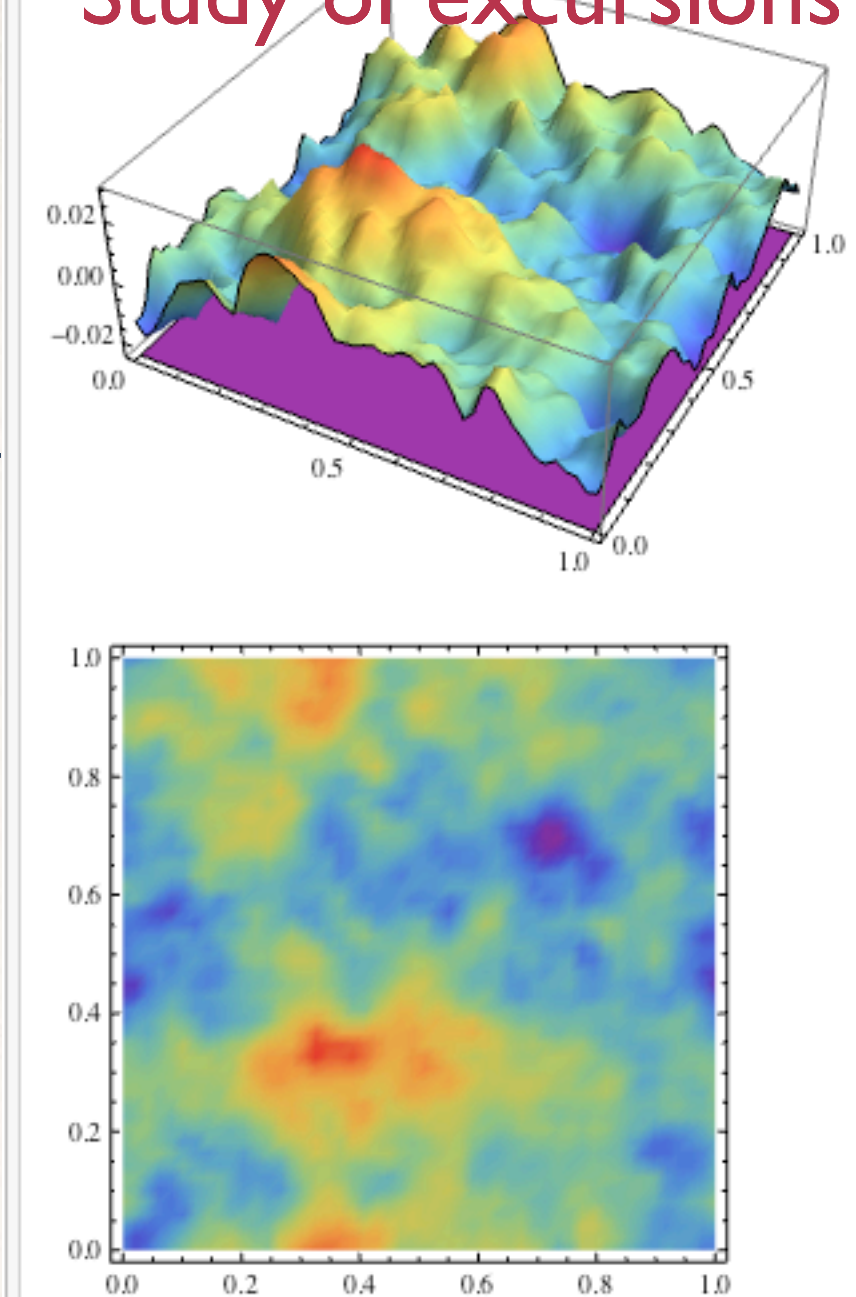
In ND, we define the **Euler-Poincaré characteristic** (in 2D, =2-2g) as the alternating sum of Betti numbers:

$$\chi = \sum_i (-1)^i b_i$$

where b_i is its rank of the i-th homology group (b_0=number of connected components, b_1=circular holes, b_2=cavities,…).
Gauss-Bonnet theorem: χ is the integral of the Gaussian curvature
Morse theory : it is the alternating sum of extrema.

The Euler characteristic obeys: **additivity, motion invariance and conditional continuity**, it is one of the MF.

Study of excursions



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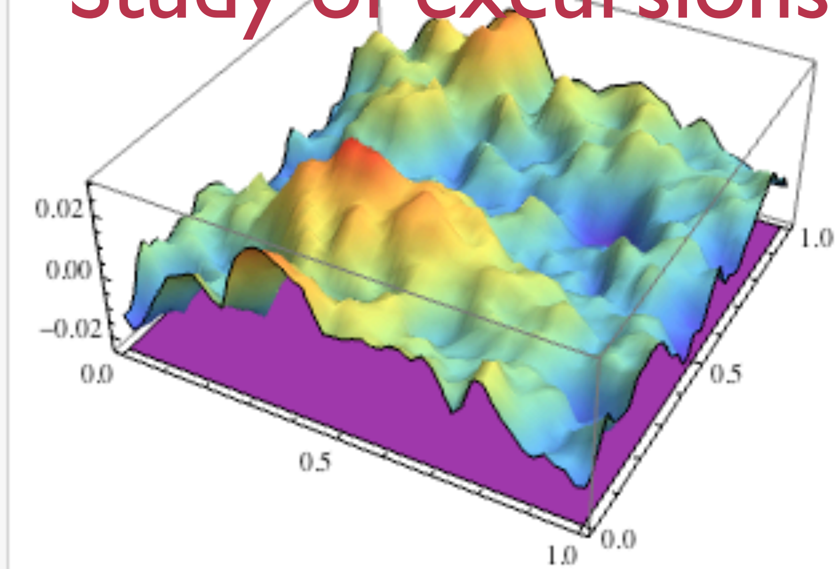

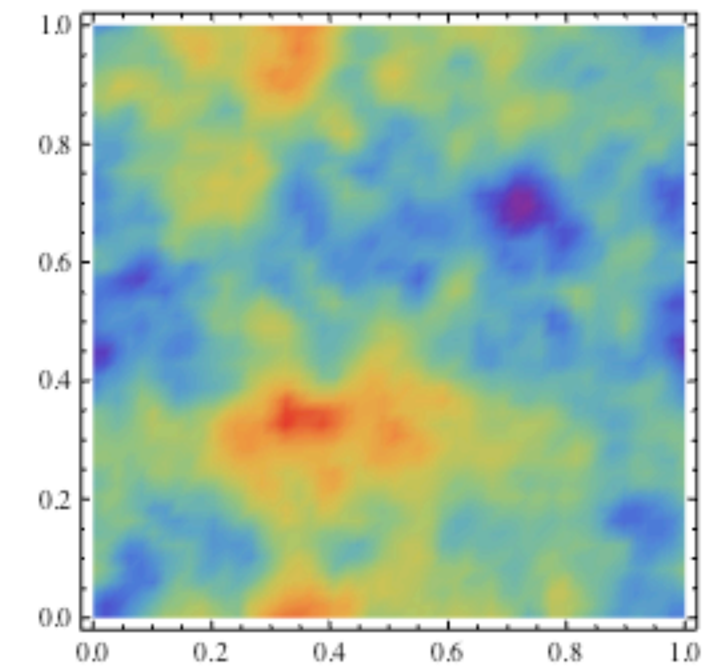

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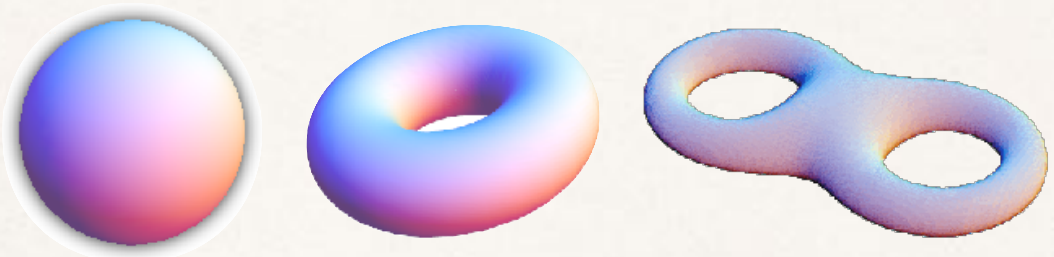


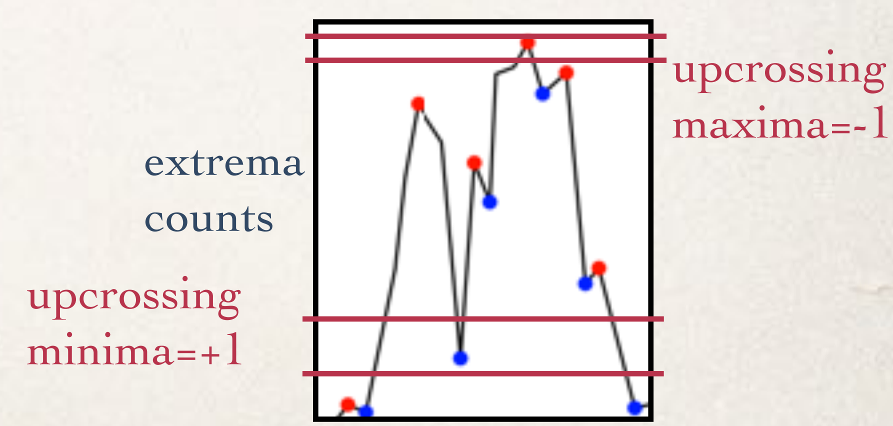
g=0 g=1 g=2

This is a topological invariant: *deux surfaces sont homeomorphes si elles ont le meme genre.*

In ND, we define the **Euler-Poincaré characteristic** (in 2D, =2-2g) as the alternating sum of Betti numbers:

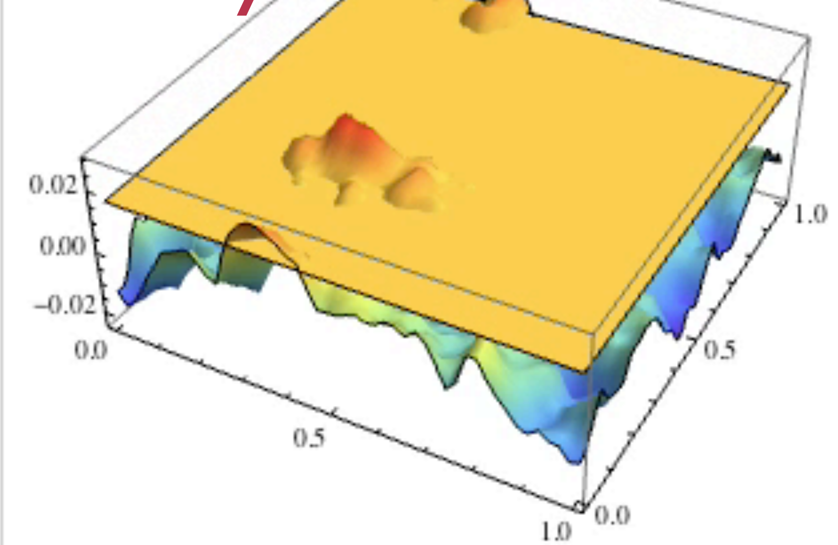
$$\chi = \sum_i (-1)^i b_i$$

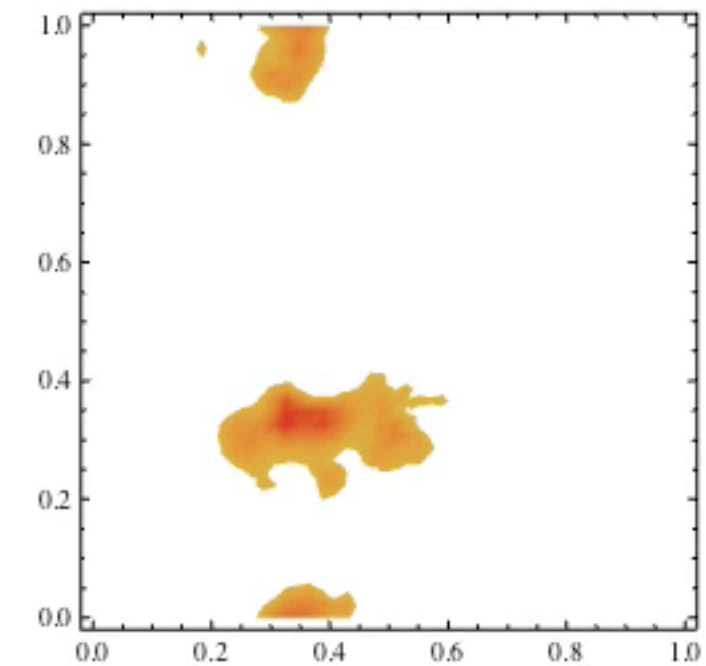
where b_i is its rank of the i-th homology group (b_0=number of connected components, b_1=circular holes, b_2=cavities,…).
Gauss-Bonnet theorem: χ is the integral of the Gaussian curvature
Morse theory : it is the alternating sum of extrema.



The Euler characteristic obeys: **additivity, motion invariance and conditional continuity**, it is one of the MF.

Study of excursions



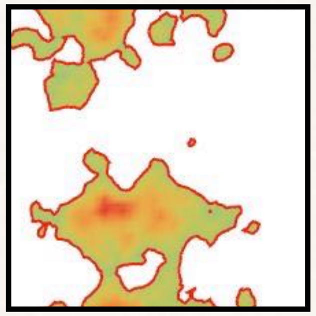


topological estimators?

➢**Minkowski functionals (topological invariants):**

d+1 MFs in d dimensions:
-Euler-Poincaré characteristic
and??
in 2D: length of isocontour + encompassed volume
in 3D: surface of isocontour+encompassed volume+integrated mean curvature

Euler characteristics
(related to genus)

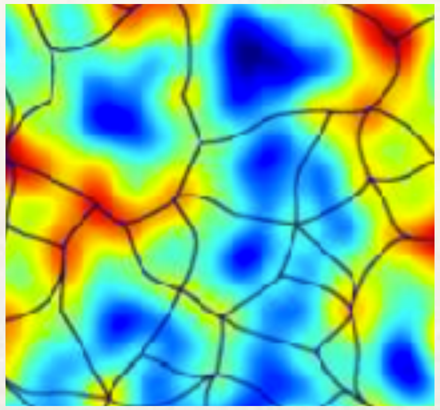


area/length of isocontours

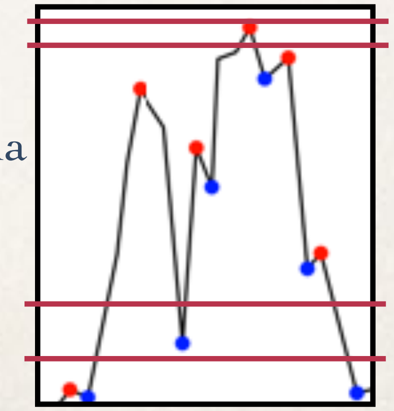
geometrical estimators?

➢**critical sets:**
peak/saddle/void counts
length of filaments
surface of walls
...

skeleton



extrema counts



topological estimators?

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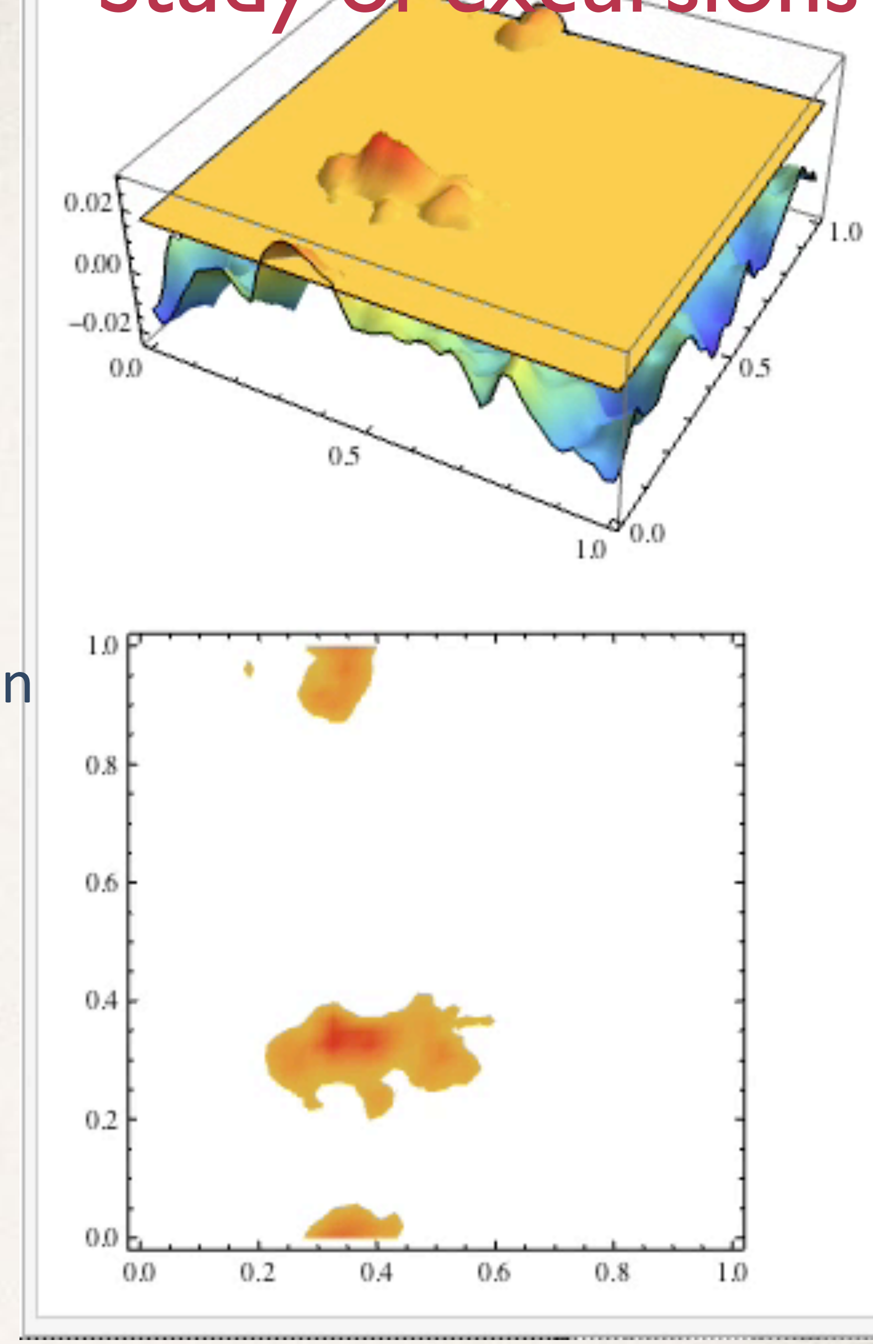

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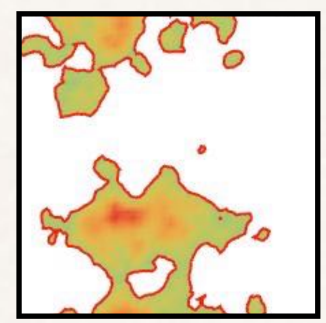

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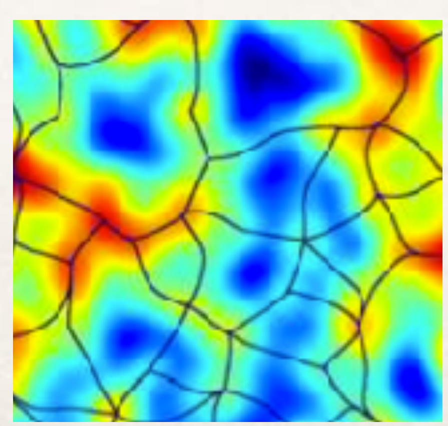
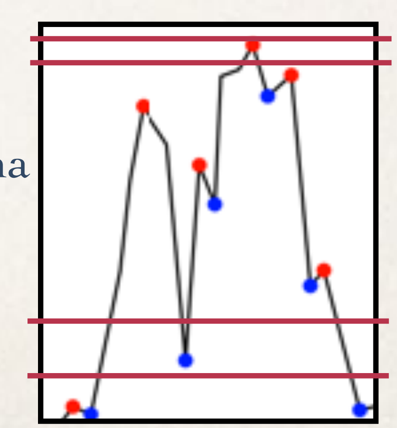


extrema
counts

upcrossing
minima=+1



upcrossing
maxima=-1

2.1 From topology to cumulants

2.2 From cumulants to D(z)

2.3 From D(z) to equation of state of Dark Energy

Joint PDF of the field

from ideas in BBKS' 86

Let us think about such properties of random fields as Euler characteristic (genus), density of extrema, ... Their computation requires the knowledge of the joint PDF :

$$P(x, x_i, x_{ij})$$

of the field x and its first x_i and second x_{ij} derivatives.

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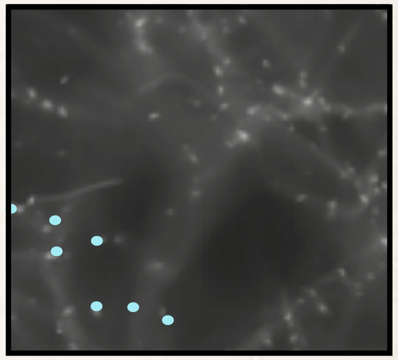
The number density of peaks is:

$$n_{\text{peak}}(\vec{r}) = \sum_k \delta_D(\vec{r} - \vec{r}_{peak\ k})$$

peaks = point process



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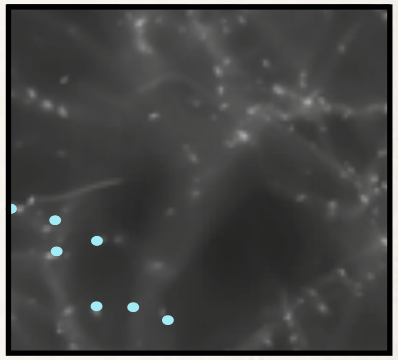

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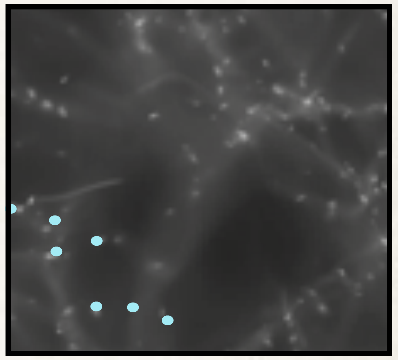

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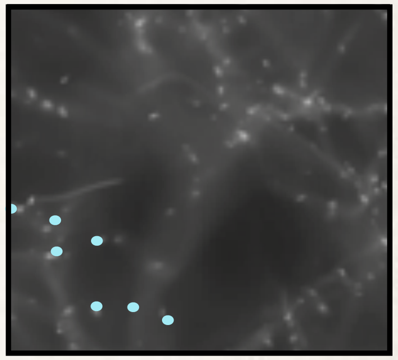

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So that in the end:

$$\langle n_{\text{peak}} \rangle = \int \frac{\mathrm{d}^3\vec{r}}{V} n_{\text{peak}}(\vec{r}) = \int \mathrm{d}x\, \mathrm{d}^3x_i\, \mathrm{d}^6x_{ij} P(x, x_i, x_{ij}) |\det x_{ij}| \delta_D(x_i)$$

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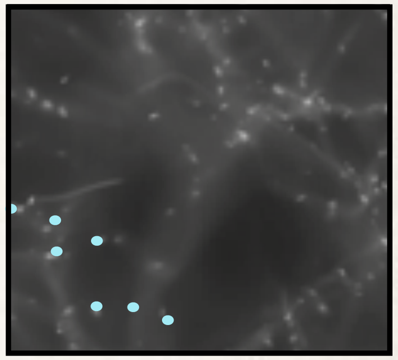

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ergodicity!
spatial average=ensemble average

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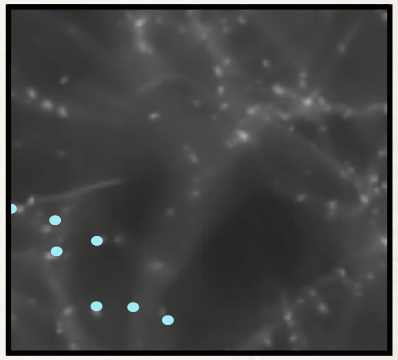

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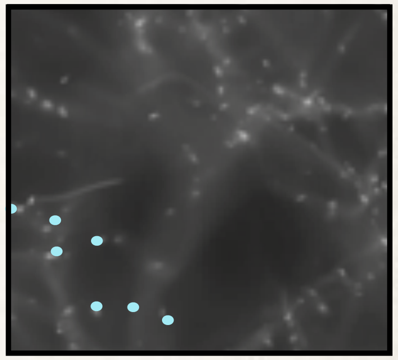

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So that in the end:

$$\langle n_{\rm peak} \rangle (\nu) = \int \frac{{\rm d}^3 \vec{r}}{V} n_{\rm peak}(\vec{r}) = \int {\rm d}x\, {\rm d}^3 x_i\, {\rm d}^6 x_{ij} P(x, x_i, x_{ij}) |\det x_{ij}| \delta_D(x_i) \times \Theta(-\lambda_1) \times \Theta(x - \sigma_0 \nu)$$

ergodicity!
spatial average=ensemble average

Gaussian JPDF

from ideas in BBKS' 86

Let us think about such properties of random fields as Euler characteristic (genus), density of extrema, ... Their computation requires the knowledge of the joint PDF :

$$P(x, x_i, x_{ij})$$

of the field x and its first x_i and second x_{ij} derivatives.

Minkowski functionals (Euler characteristic[genus] in 3 and 2D, area/length of isocontours, contour crossings), extrema counts, skeleton length, etc are then obtained by integration of the JPDF. For instance, the critical pt density and the 3D Euler characteristic read :

$$\langle n_{\rm crit} \rangle_{(\nu)} = \int {\rm d}x\, {\rm d}^3 x_i\, {\rm d}^6 x_{ij} P(x, x_i, x_{ij}) |\det x_{ij}| \delta_D(x_i) \times \Theta(x - \sigma_0 \nu)$$

$$\chi_{\rm 3D}(\nu) = -\int P(x, x_i, x_{ij}) \delta_{\rm D}(x_i) \det x_{ij} \Theta(x - \sigma_0 \nu)$$

Those integrations can in principle be computed for any PDF.

the trick: use the **invariants** of the field $(x, x_i^2, {\rm tr}(x_{ij}), \det(x_{ij}), \ldots)$!

5=10-5 in r-space
8=10-2 in z-space

The result for the **Gaussian** 3D Euler characteristic is:

$$\chi_{\rm 3D}(\nu) \propto e^{-\nu^2/2} H_2(\nu)$$

Non-Gaussian expansion

see also Pogosyan' 00

Let us think about such properties of random fields as Euler characteristic (genus), density of extrema, ... Their computation requires the knowledge of the joint PDF :

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How to go beyond Gaussianity?

Non-Gaussian expansion

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How to go beyond Gaussianity?

Gram-Charlier expansion (analogous to the Taylor expansion for PDF)***:***
The moment expansion of the general JPDF P(x) around a Gaussian PDF G(x) is an Hermite expansion:

$$P(x) = G(x)\left[1 + \sum_{n=3}^{\infty} \frac{1}{n!} \langle x^n \rangle_{GC} H_n(x)\right]$$ ***to all order in non gaussianity***

where Hermite polynomials are polynomials of order n in x, orthogonal wrt the Gaussian kernel G.

The same kind of expansion holds for $P(x, x_i, x_{ij})$

Moment expansion for NG statistics

Minkowski functionals (Euler characteristic[genus] in 3 and 2D, area/length of isocontours, contour crossings), extrema counts, skeleton length, etc are then obtained by integration of the JPDF. For instance, the 3D Euler characteristic reads :

$$\chi_{3\mathrm{D}}(\nu) = -\int P(x, x_i, x_{ij})\delta_{\mathrm{D}}(x_i)\,\det x_{ij}\Theta(x-\sigma_0\nu)$$

Those integrations can in principle be computed to all orders in non-Gaussianity.

The trick: use the **invariants** of the field ($x, x_i^2, \mathrm{tr}(x_{ij}), \det(x_{ij}),\ldots$)
+Gram-Charlier expansion of the JPDF!

5=10-5 in r-space
8=10-2 in z-space

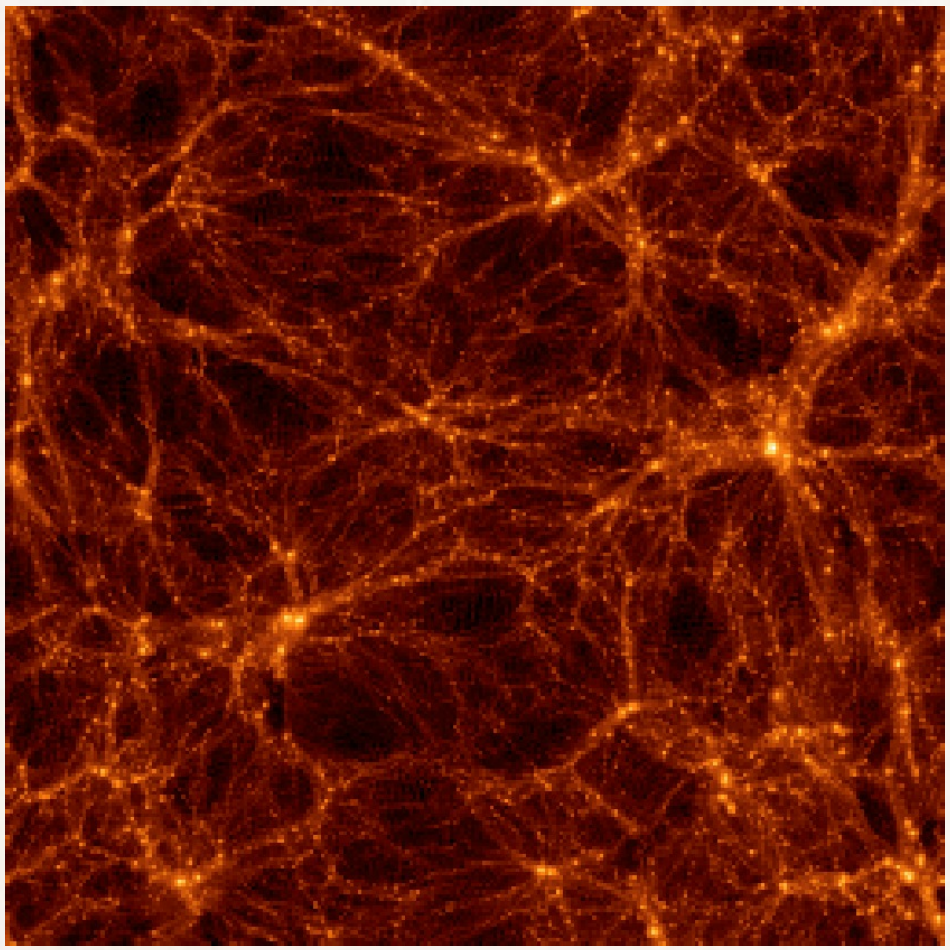
We finally get moment expansion for each NG statistics in real space (*Gay et al '12*) and in redshift space (*Codis et al '13*) e.g

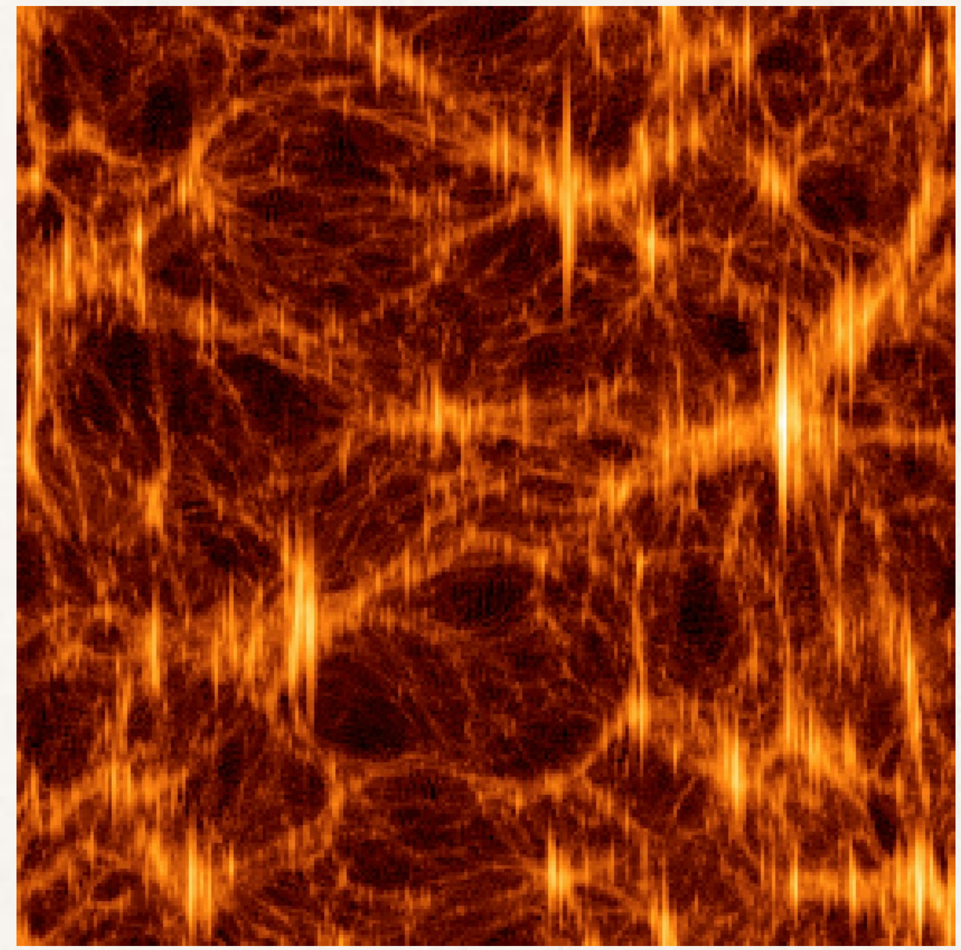
$$\chi^{\mathrm{s}}_{3\mathrm{D}}(\nu) = \frac{e^{-\nu^2/2}}{8\pi^2}\frac{\sigma_{1||}\sigma_{1\perp}^2}{\sigma^3}\left[H_2(\nu) + \frac{1}{\gamma_\perp^2}\sum_{n=3}^{\infty}\sum_{\sigma_{n-2}}\frac{(-1)^{j+m}}{i!\,j!m!(2m-1)2^m}H_i(\nu)\left(\left\langle x^i q_\perp^{2j} J_{2\perp}\, x_3^{2m}\right\rangle_{\mathrm{GC}} - (1-\gamma_\perp^2)\left\langle x^i q_\perp^{2j}\zeta^2 x_3^{2m}\right\rangle_{\mathrm{GC}}\right)\right.$$
$$\left. + 2\frac{\sqrt{1-\gamma_\perp^2}}{\gamma_\perp}\sum_{n=3}^{\infty}\sum_{\sigma_{n-1}}\frac{(-1)^{j+m}}{i!\,j!m!(2m-1)2^m}\left\langle x^i q_\perp^{2j}\zeta\, x_3^{2m}\right\rangle_{\mathrm{GC}} H_i(\nu)H_1(\nu) - \sum_{n=3}^{\infty}\sum_{\sigma_n}\frac{(-1)^{j+m}}{i!\,j!m!(2m-1)2^m}\left\langle x^i q_\perp^{2j} x_3^{2m}\right\rangle_{\mathrm{GC}} H_i(\nu)H_2(\nu)\right],$$

generalizing the result of *Matsubara '96* (Gaussian term) to **all orders in non-Gaussianity**.

key ingredient: genus (DM)=genus(light) if bias is monotonic!

Effect of redshift space distortion

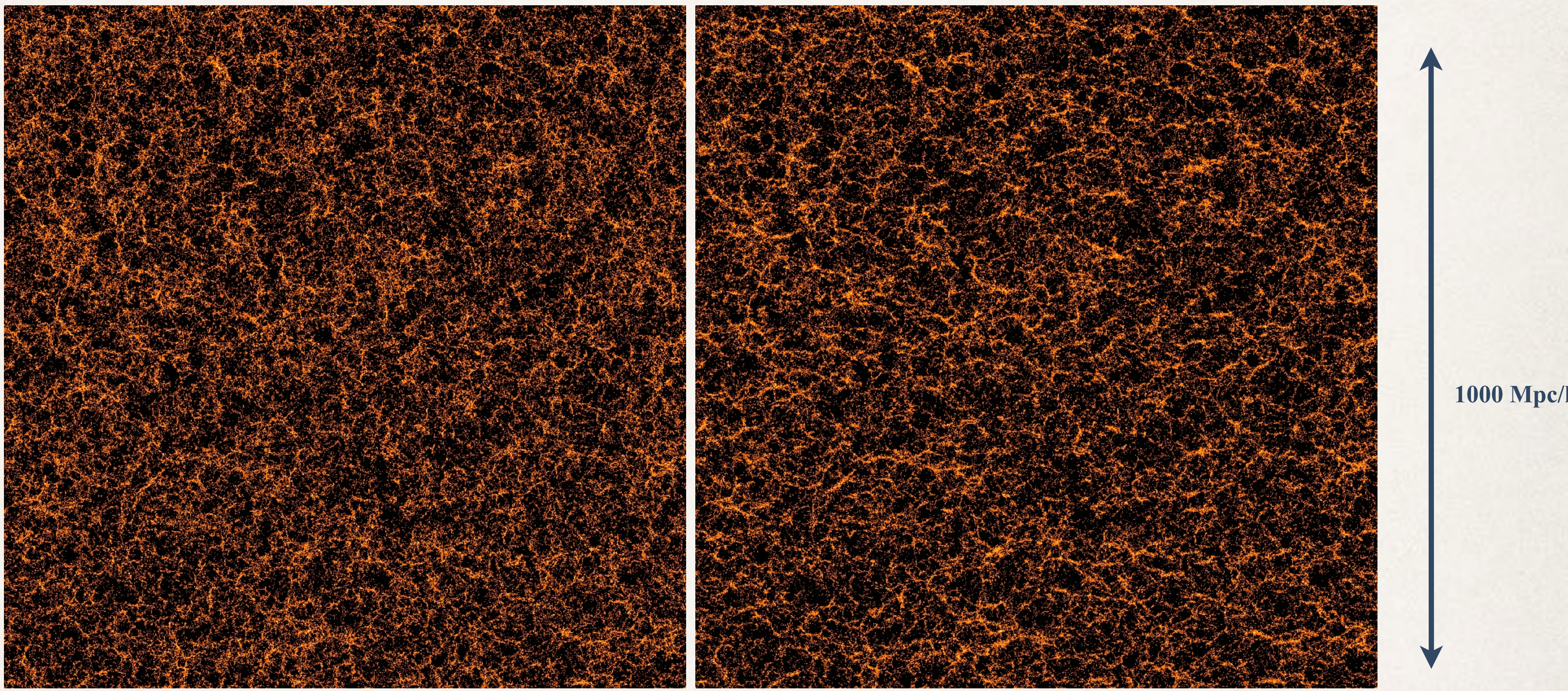




100 Mpc/h

Finger-of-God Effect

Effect of redshift space distortion

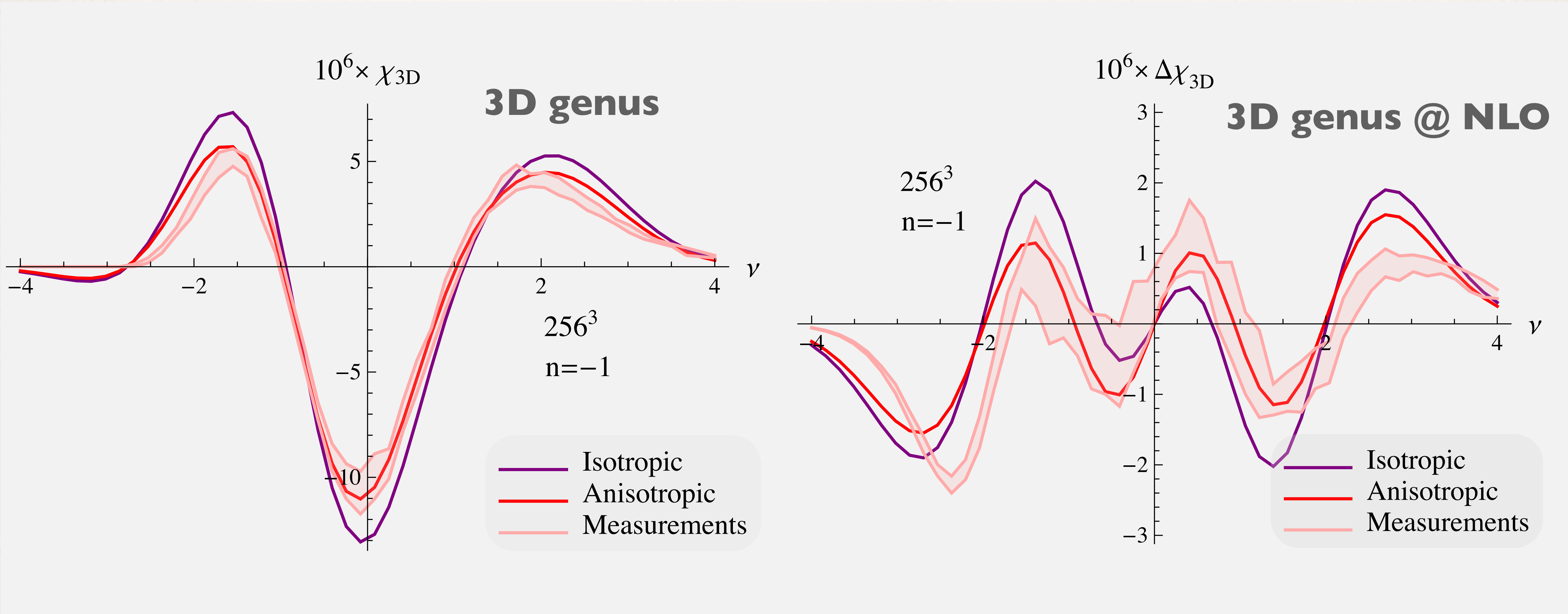


Kaiser Effect

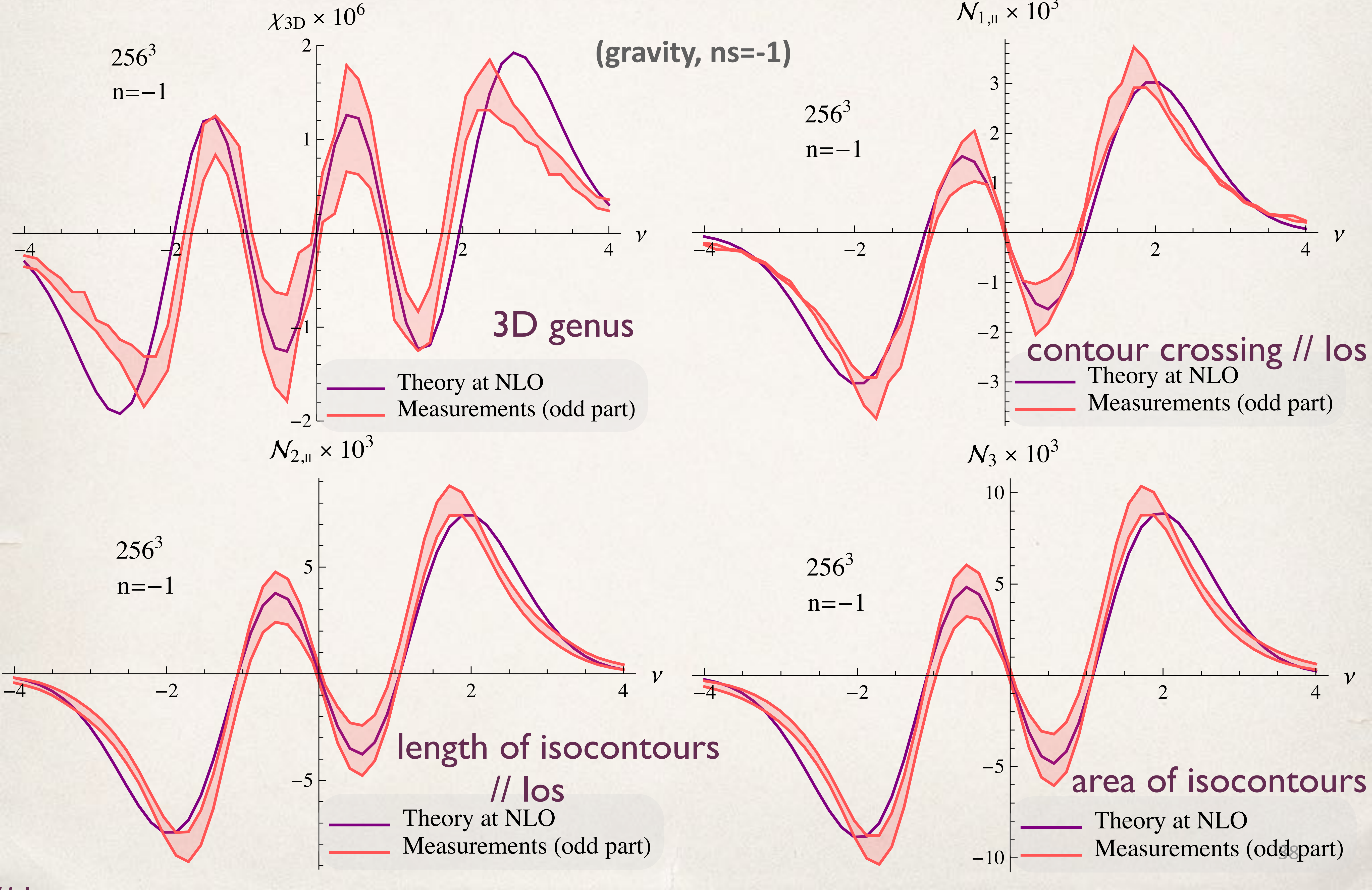
$$\delta_g^{(z)} = (1 + \beta\mu^2)\delta_g^{(r)}$$

$\Omega_m{}^{\gamma}/b$, $\gamma \simeq 0.55$(GR)
dynamical parameter

Effect of redshift space distortion



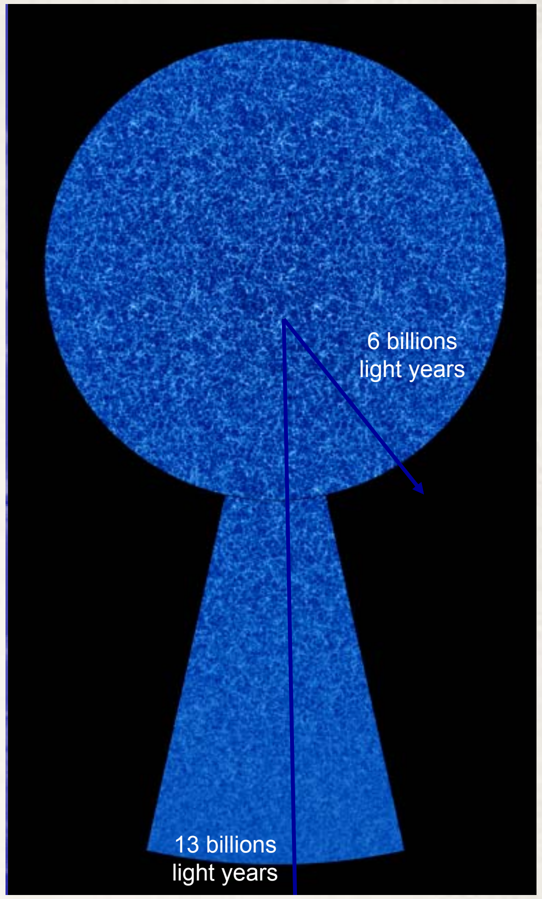
MFs for scale-invariant power spectra : NG corrections



Horizon 4π simulation: Critical point Counts

4096^3
2Gpc/h across

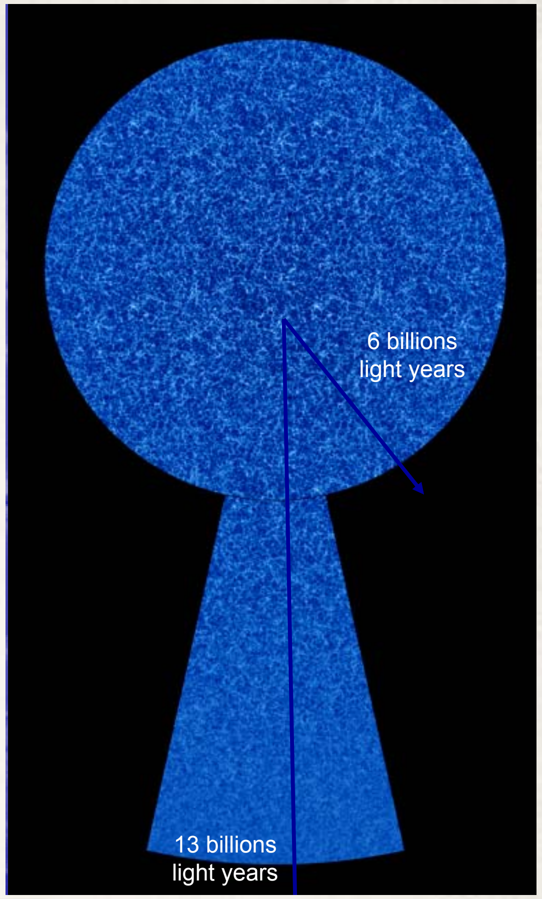
from a catalog of
haloes above 10^{11} M_{sun}



Horizon light cone

contrast

Horizon 4π simulation : 3D and 2D genus



Horizon light cone

4096^3
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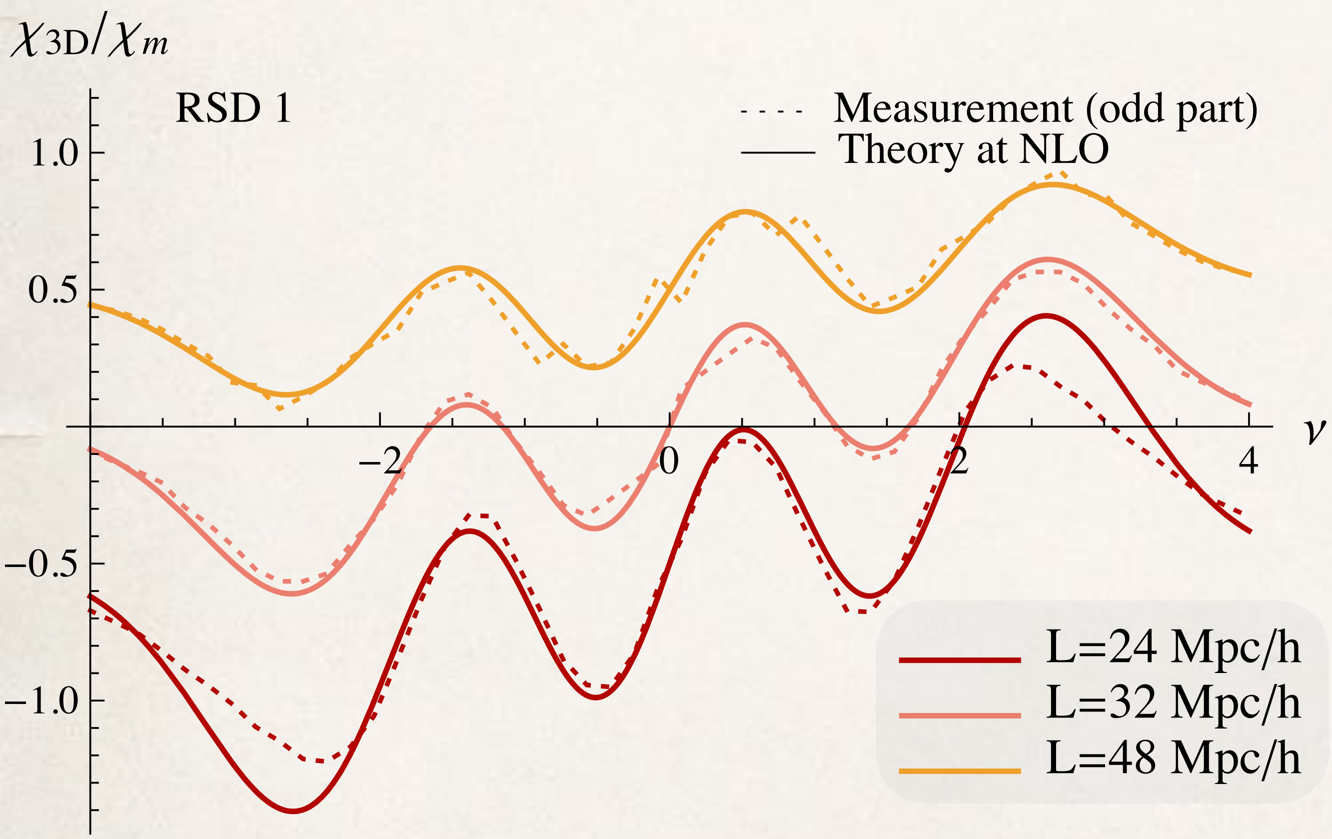
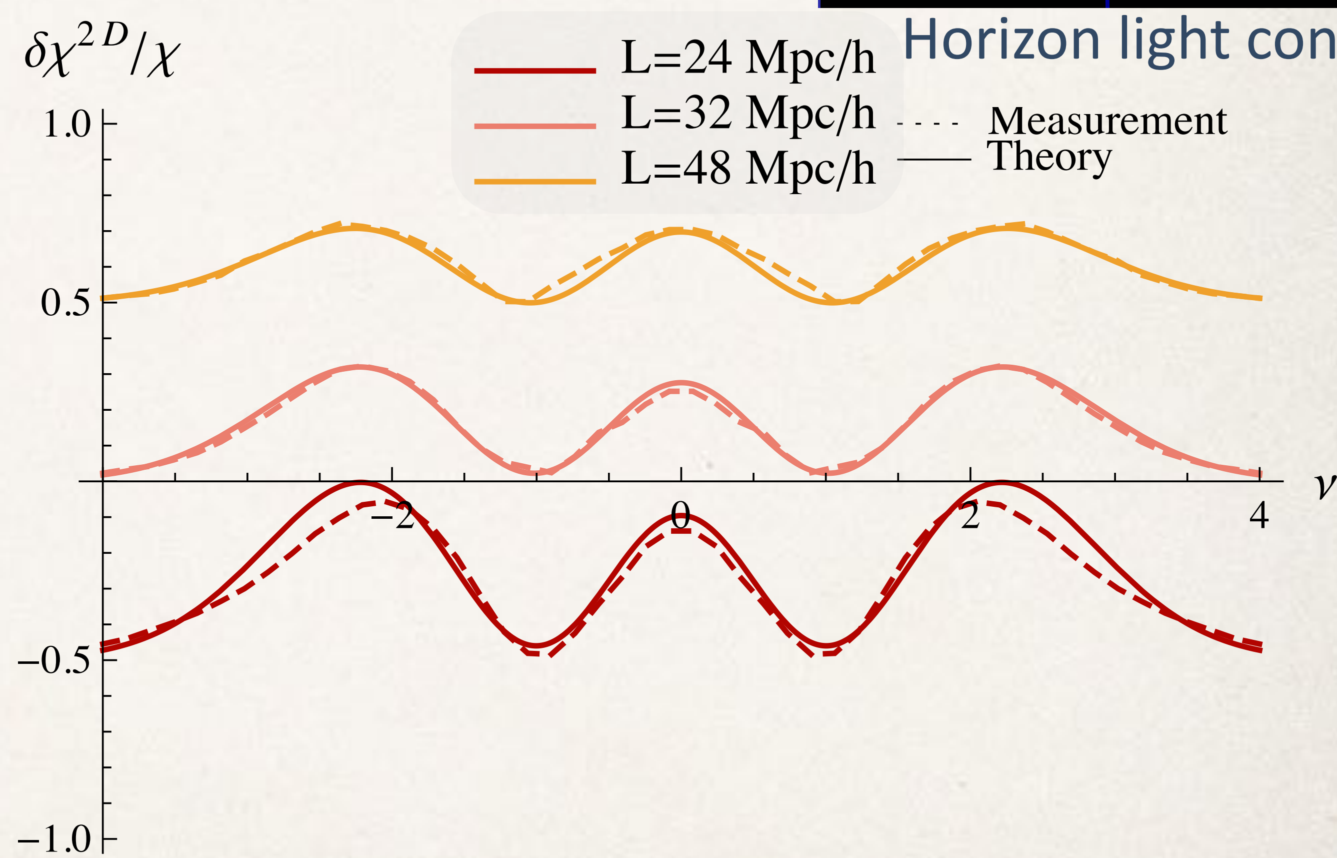
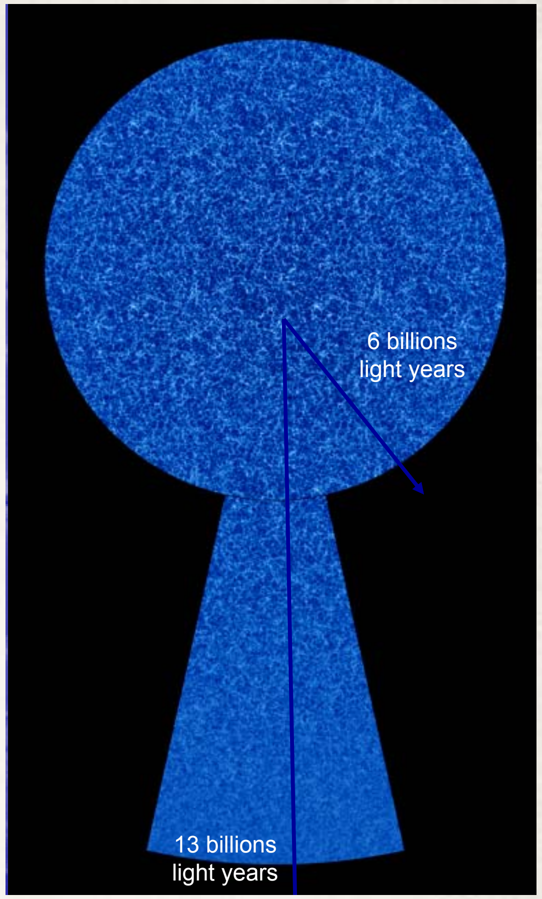




Horizon 4π simulation : measure β?

4096^3
2Gpc/h across



Horizon light cone

weakly affected by RSD

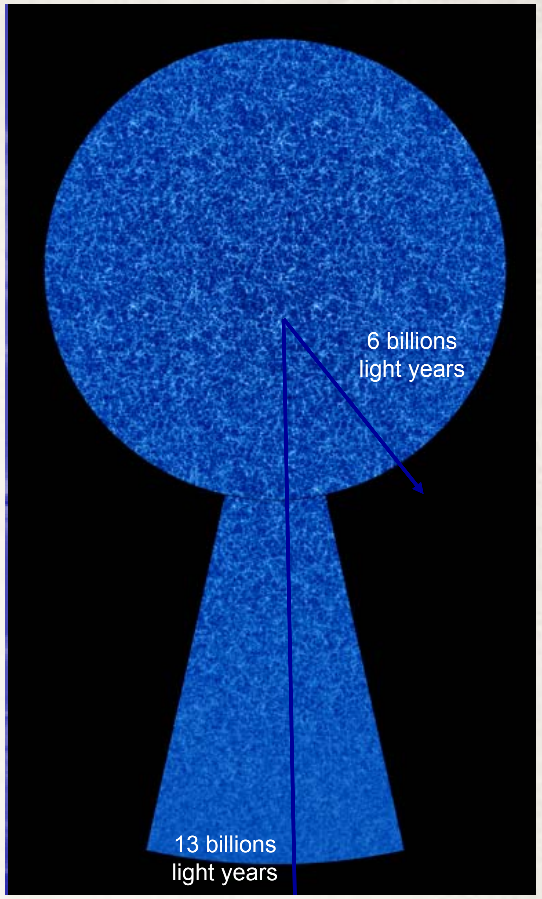
3D genus

Horizon 4π simulation : 2D slices // LOS

4096^3
2Gpc/h across

sensitivity to $\beta = f/b_1$

2D genus



Horizon light cone

2.1 From topology to cumulants

2.2 From cumulants to D(z)

2.3 From D(z) to equation of state of Dark Energy

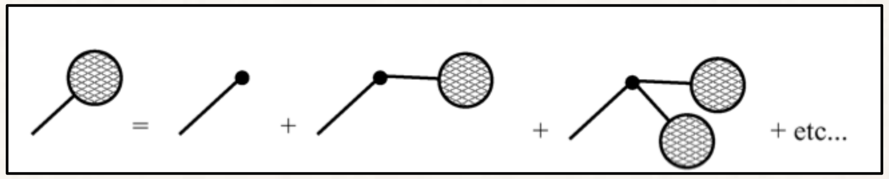
Generalized geometrical S_n

Purpose: Express the invariant **cumulants** in terms of σ (hence D(z)) through Perturbation theory

$$F_2(\mathbf{k_1},\mathbf{k_2}) = \frac{5}{7} + \frac{\mathbf{k_1}\cdot\mathbf{k_2}}{{k_1}^2} + \frac{2}{7}\frac{(\mathbf{k_1}\cdot\mathbf{k_2})^2}{{k_1}^2{k_2}^2} \implies \mathcal{F}_{\alpha,\beta,\gamma}(\mathbf{k_1},\mathbf{k_2}) = F_2(\mathbf{k_1},\mathbf{k_2})\mathcal{G}_{\alpha,\beta,\gamma}(\mathbf{k_1},\mathbf{k_2})$$

GRAVITY

Geometric shape factor= powers of k



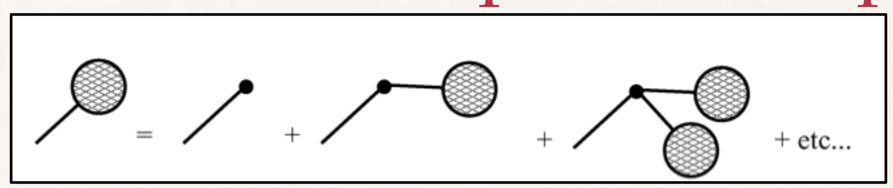
Generalized geometrical S_n

Purpose: Express the invariant **cumulants** in terms of σ (hence D(z)) through Perturbation theory

$$F_2(\mathbf{k_1},\mathbf{k_2}) = \frac{5}{7} + \frac{\mathbf{k_1}\cdot\mathbf{k_2}}{{k_1}^2} + \frac{2}{7}\frac{(\mathbf{k_1}\cdot\mathbf{k_2})^2}{{k_1}^2{k_2}^2} \implies \mathcal{F}_{\alpha,\beta,\gamma}(\mathbf{k_1},\mathbf{k_2}) = F_2(\mathbf{k_1},\mathbf{k_2})\mathcal{G}_{\alpha,\beta,\gamma}(\mathbf{k_1},\mathbf{k_2})$$

GRAVITY

Geometric shape factor= powers of k



power spectrum index

$$\frac{1}{\sigma}\langle x^3\rangle = 3\ {}_2F_1\left(\frac{3+n}{2},\frac{3+n}{2},\frac{3}{2},\frac{1}{4}\right) - \frac{1}{7}(8+7n)\ {}_2F_1\left(\frac{3+n}{2},\frac{3+n}{2},\frac{5}{2},\frac{1}{4}\right).$$

skewness at tree order

Lokas 94

$$\frac{1}{\sigma}\langle x{x_1}^2\rangle = \frac{4(48+62n+21n^2)}{21n^2}\ {}_2F_1\left(\frac{3+n}{2},\frac{3+n}{2},\frac{3}{2},\frac{1}{4}\right) - \frac{6(3+n)(8+7n)}{21n^2}\ {}_2F_1\left(\frac{3+n}{2},\frac{5+n}{2},\frac{3}{2},\frac{1}{4}\right)$$

3pt field- gradient cumulant

n= -3 :

$$\frac{1}{\sigma}\left\langle x^3\right\rangle = \frac{34}{7} \implies \frac{1}{\sigma}\left\langle x x_1^2\right\rangle = \frac{34}{7}\frac{2}{3^2}$$

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GRAVITY

G ... owers of k

	$n_s = 0$	
	prediction	measurement
$\langle x^3\rangle/\sigma$	3.144	3.08 ± 0.08
$\langle xq^2\rangle/\sigma$	2.096	2.05 ± 0.03
$\langle x^2 J_1\rangle/\sigma$	-3.248	-3.15 ± 0.06
$\langle x{J_1}^2\rangle/\sigma$	3.871	3.75 ± 0.06
$\langle xJ_2\rangle/\sigma$	1.545	1.54 ± 0.02
$\langle q^2 J_1\rangle/\sigma$	-1.335	-1.28 ± 0.02
$\langle {J_1}^3\rangle/\sigma$	-4.644	-4.50 ± 0.08
$\langle J_1 J_2\rangle/\sigma$	-0.679	-0.65 ± 0.01
$\langle J_3\rangle/\sigma$	1.304	1.28 ± 0.03

power spectrum index

$$\frac{1}{\sigma}\langle x^3\rangle = 3\,{}_2F_1\left(\frac{3+n}{2}, \frac{3\ldots}{2}, \frac{\ldots}{2}, \frac{\ldots}{4}\right) - \frac{\ldots}{7}(8+7n)\,{}_2F_1\left(\frac{3+n}{2}, \frac{3+n}{2}, \frac{5}{2}, \frac{1}{4}\right).$$

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n= -3 : $$\frac{1}{\sigma}\langle x^3\rangle = \frac{34}{7} \Longrightarrow \frac{1}{\sigma}\langle xx_1^2\rangle = \frac{34}{7}\frac{2}{3^2}$$

Generalized geometrical S_n

We can express the invariant **cumulants** in terms of σ (hence D(z)) through Perturbation theory

generalized S_3:

	$n_s = 0$	
	prediction	measurement
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Remember, we have analytical predictions e.g.

$$\chi^{\rm s}_{\rm 3D}(\nu) = \frac{e^{-\nu^2/2}}{8\pi^2}\frac{\sigma_{1||}\sigma_{1\perp}^2}{\sigma^3}\Bigg[H_2(\nu) + \frac{1}{\gamma_\perp^2}\sum_{n=3}^{\infty}\sum_{\sigma_{n-2}}\frac{(-1)^{j+m}}{i!\,j!m!(2m-1)2^m}H_i(\nu)\left(\left\langle x^i q_\perp^{2j} J_{2\perp}\, x_3^{2m}\right\rangle_{\rm GC} - (1-\gamma_\perp^2)\left\langle x^i q_\perp^{2j}\zeta^2 x_3^{2m}\right\rangle_{\rm GC}\right)$$

$$+\,2\frac{\sqrt{1-\gamma_\perp^2}}{\gamma_\perp}\sum_{n=3}^{\infty}\sum_{\sigma_{n-1}}\frac{(-1)^{j+m}}{i!\,j!m!(2m-1)2^m}\left\langle x^i q_\perp^{2j}\zeta\, x_3^{2m}\right\rangle_{\rm GC} H_i(\nu)H_1(\nu) - \sum_{n=3}^{\infty}\sum_{\sigma_n}\frac{(-1)^{j+m}}{i!\,j!m!(2m-1)2^m}\left\langle x^i q_\perp^{2j} x_3^{2m}\right\rangle_{\rm GC} H_i(\nu)H_2(\nu)\Bigg],$$

that depend on generalized S_3 times σ at first order i.e **some numbers** times σ where $\sigma = \sigma_{DM}(z) = D(z)\sigma_0$

predicted from PT

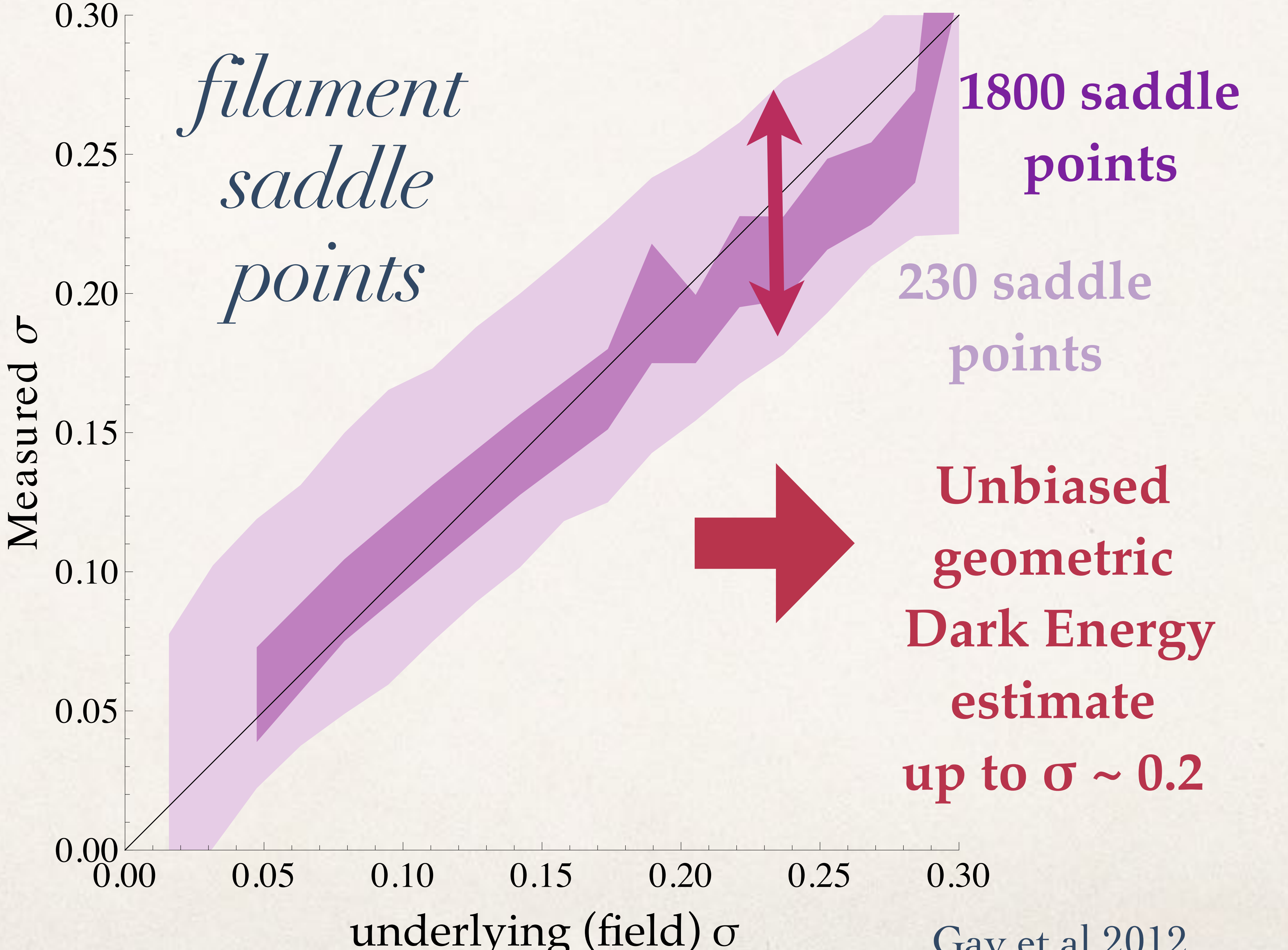
2.1 From topology to cumulants

2.2 From cumulants to D(z)

2.3 From D(z) to equation of state of Dark Energy

Fiducial DE experiment

- Generate scale invariant ICs
- Evolve them with gravity
- identify critical sets
- compute differential counts
- estimate amplitude of NG **distorsion** via PT
- deduce **geometric** critical set σ

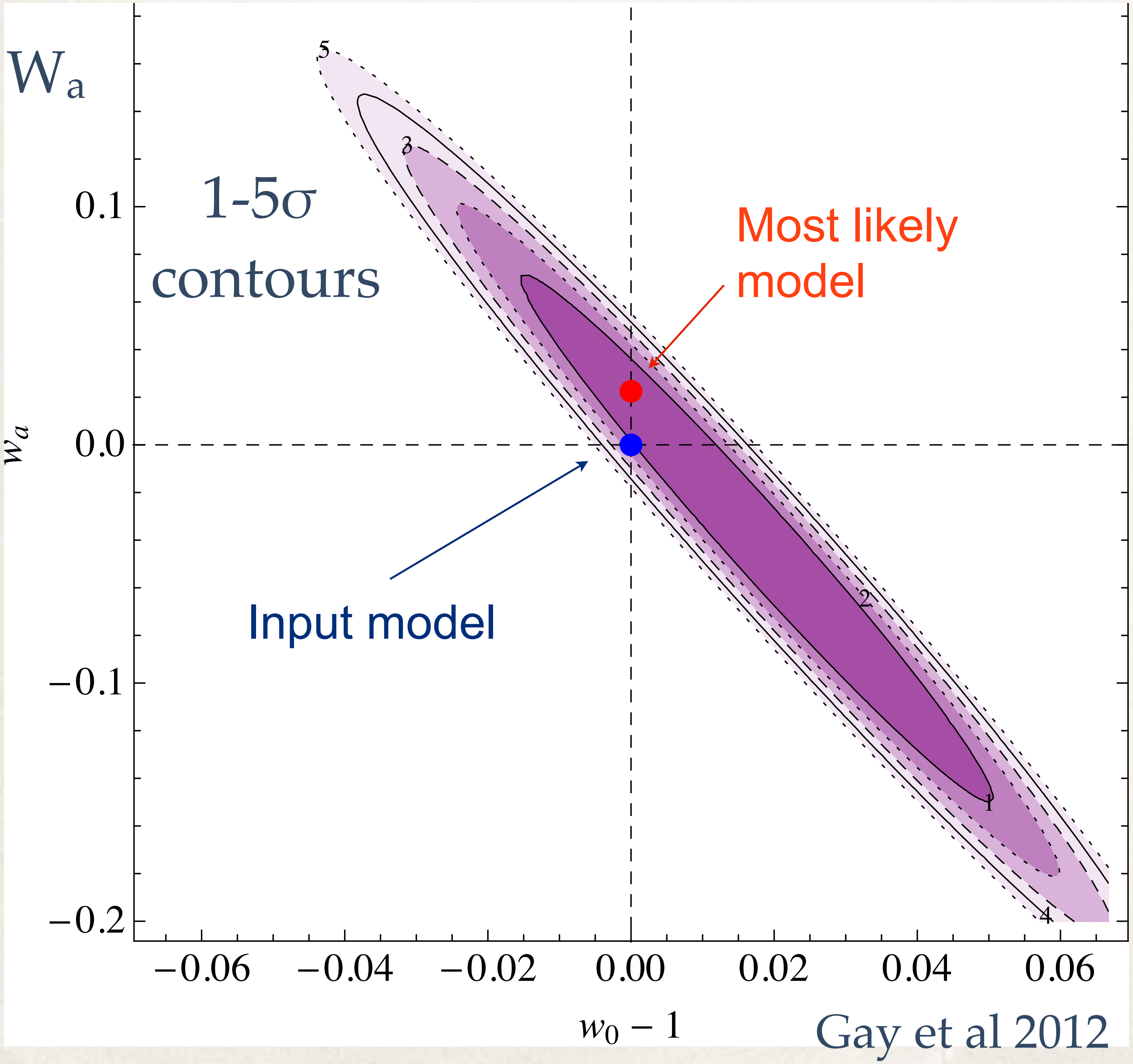


Unbiased geometric Dark Energy estimate up to σ ~ 0.2

Gay et al 2012

Figure of merit : 3D dark energy probe

- Assume error on D[z]
- Explore likelihood w.r.t w_a and w_0



$$H^2(a) = H_0^2\left[\frac{\Omega_0}{a^3} + \Omega_\Lambda \exp\left(3\int_0^z \frac{1+w(z')}{1+z'}\mathrm{d}z'\right)\right],$$

$$D(z|w_0, w_a) = \frac{5\Omega_0 H_0^2}{2} H(a)\int_0^a \frac{\mathrm{d}a'}{a'^3 H^3(a')},$$

5 % @ 3σ on w_0

10 % @ 3σ on w_a

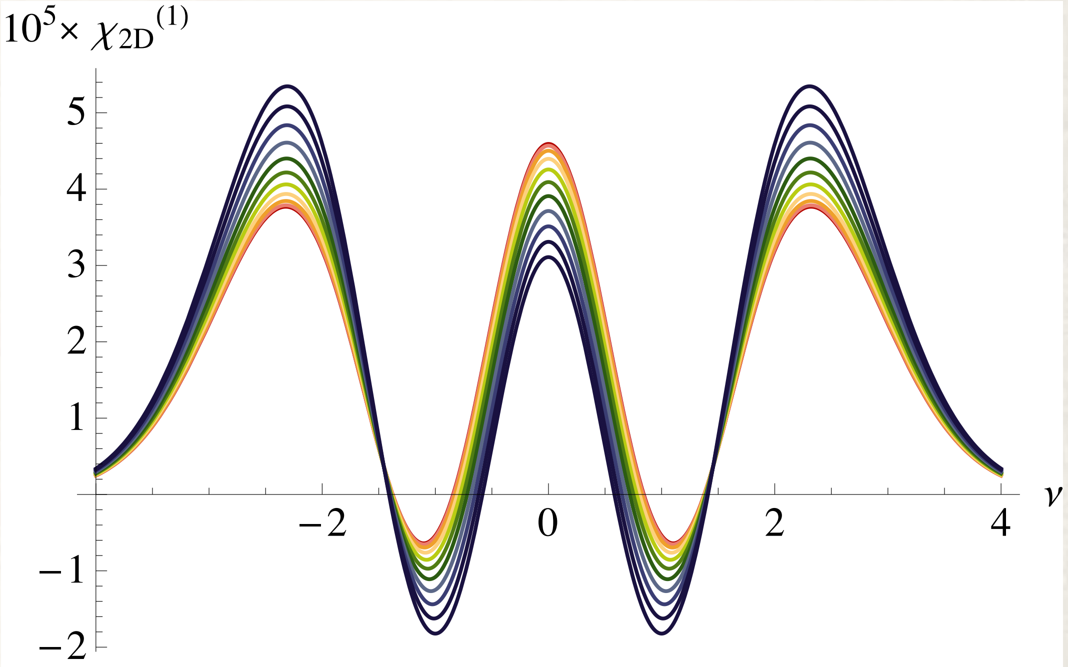
2D genus measures β

2D MFs (and in particular 2D genus) can give access to $\beta=\Omega_m{}^{\gamma}/b$ varying the orientation of slices and measuring e.g the amplitude of 2D genus (or other 2D MFs):

$$\chi^{(0)}_{2\mathrm{D}}(\nu,\theta_S)=\frac{H_1(\nu)e^{-\nu^2/2}}{2(2\pi)^{3/2}}\frac{\sigma_{1\perp}\sqrt{2\cos^2\theta_S\sigma^2_{1\|}+\sin^2\theta_S\sigma^2_{1\perp}}}{\sigma^2}$$

so that:

$$\frac{\chi^{(0)}_{2\mathrm{D}}(\nu,\theta_1)}{\chi^{(0)}_{2\mathrm{D}}(\nu,\theta_2)}=\sqrt{\frac{2\cos^2\theta_1\sigma^2_{1\|}+\sin^2\theta_1\sigma^2_{1\perp}}{2\cos^2\theta_2\sigma^2_{1\|}+\sin^2\theta_2\sigma^2_{1\perp}}}$$

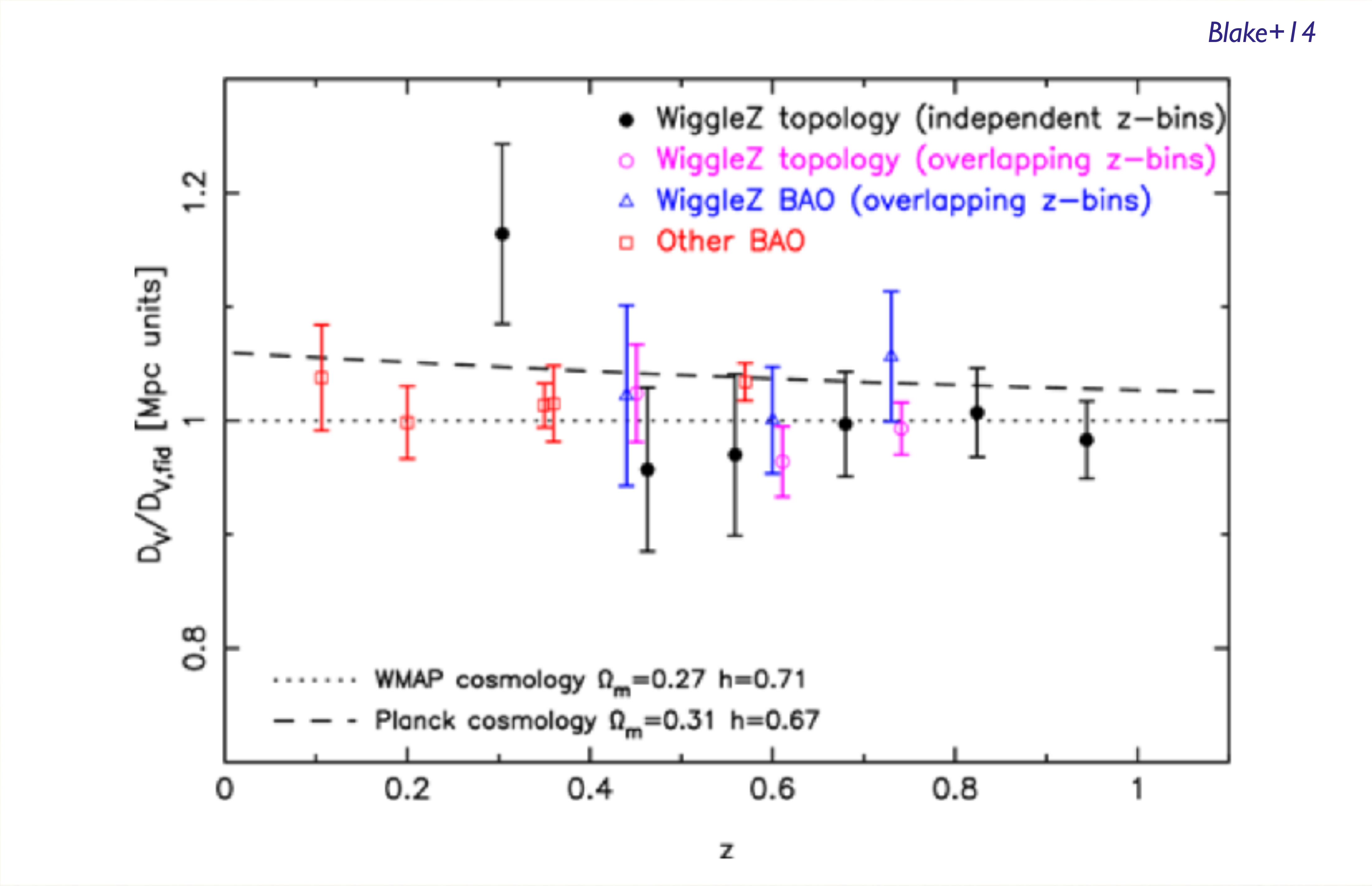


with:

$$\sigma_{1\|}=\sqrt{\frac{1}{3}+\frac{2\beta}{5}+\frac{\beta^2}{7}}\sigma_1 \qquad \sigma_{1\perp}=\sqrt{\frac{2}{3}+\frac{4\beta}{15}+\frac{2\beta^2}{35}}\sigma_1$$

only depends on angles and β!

Topology=a cosmic standard ruler?



Summary : What can we learn from MFs?

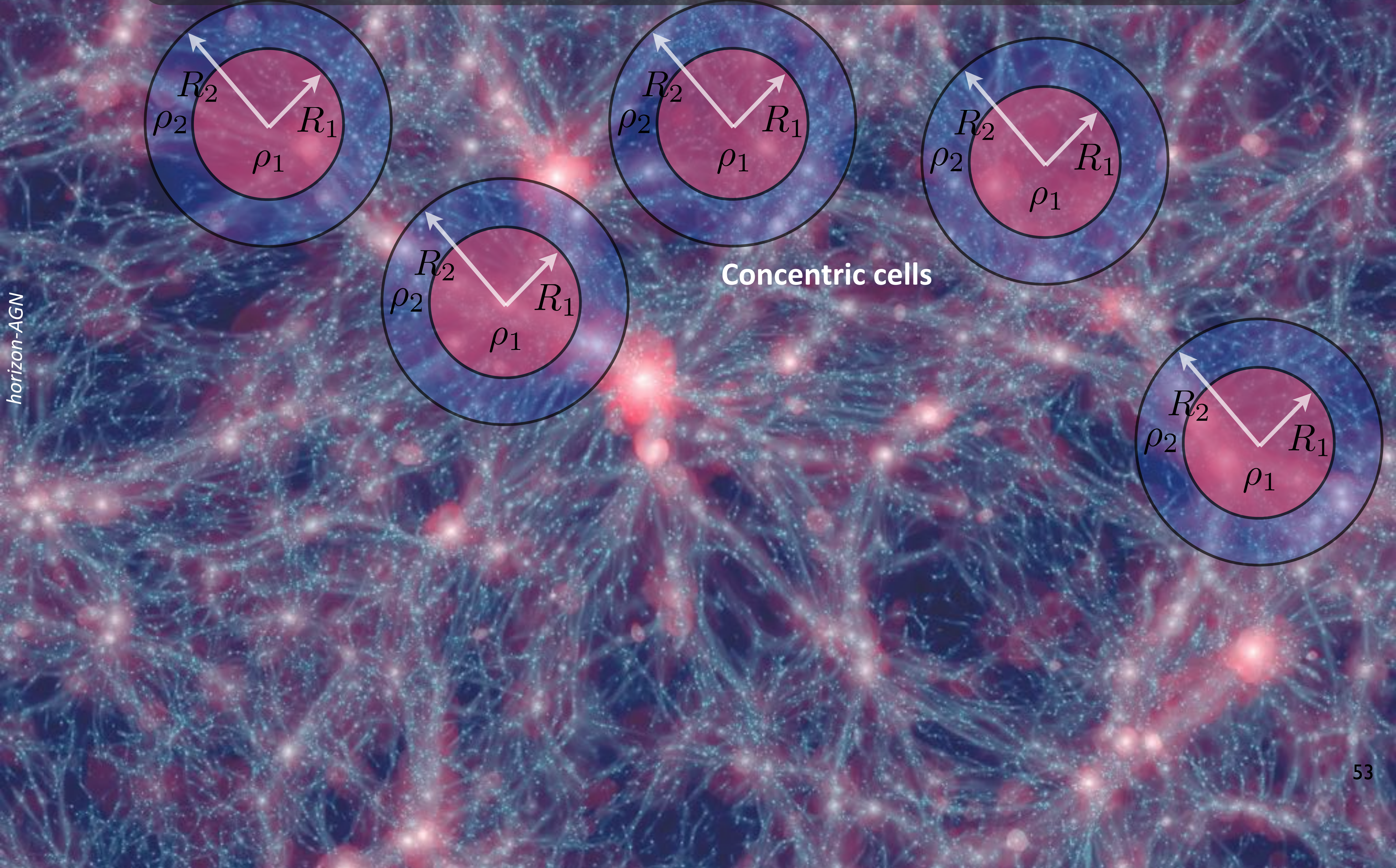
We are able to predict accurately **Minkowski functionals and extrema counts in redshift space** at large enough scale.

These statistics:

- can probe **modification of gravity** as they can give access to $\boldsymbol{\beta = \Omega_m^{\gamma}/b}$, $\gamma \simeq 0.55$(GR) varying the orientation of slices and measuring the amplitude of 2D genus;
- can probe **dark energy** through the measure of $\boldsymbol{\sigma_{DM}} = D(z)\sigma_0$ (times «skewness» which is predicted by theory).

Partie III : comptages de galaxies

Our goal: predict multi-scale densities PDF for $\sigma \sim 1$



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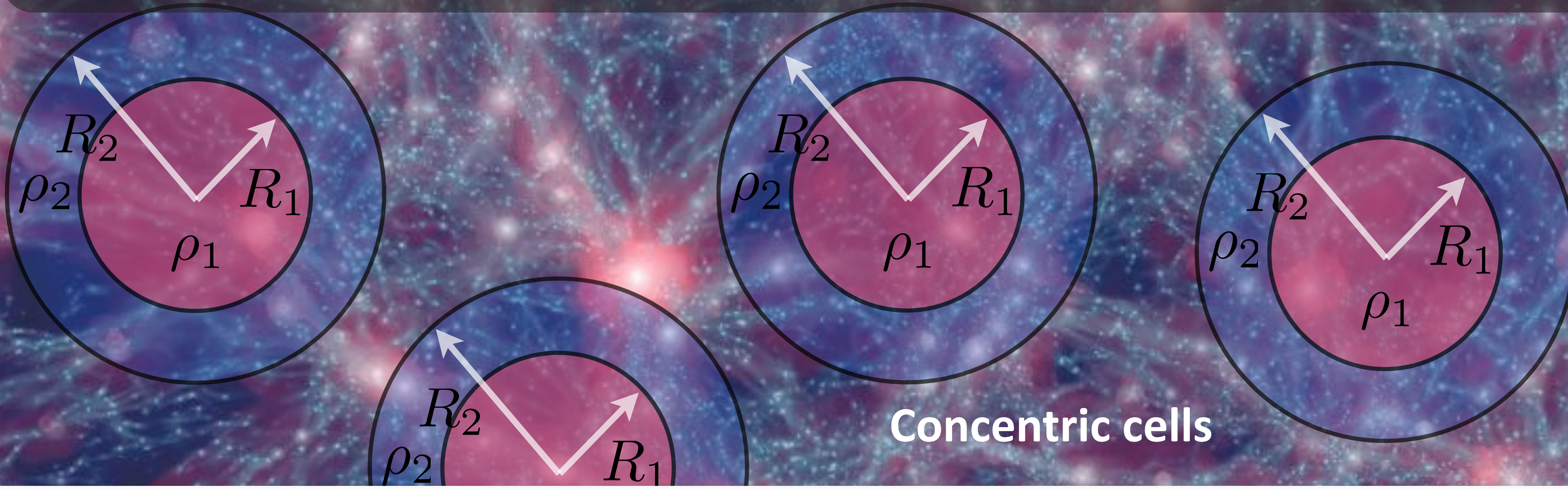
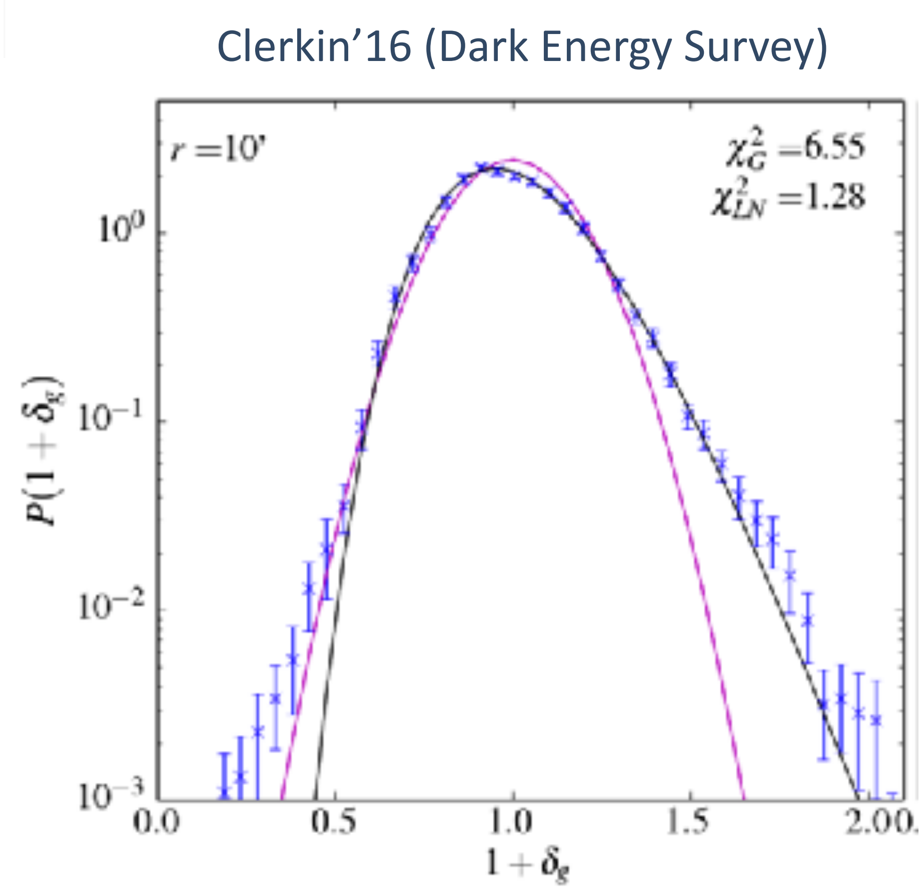
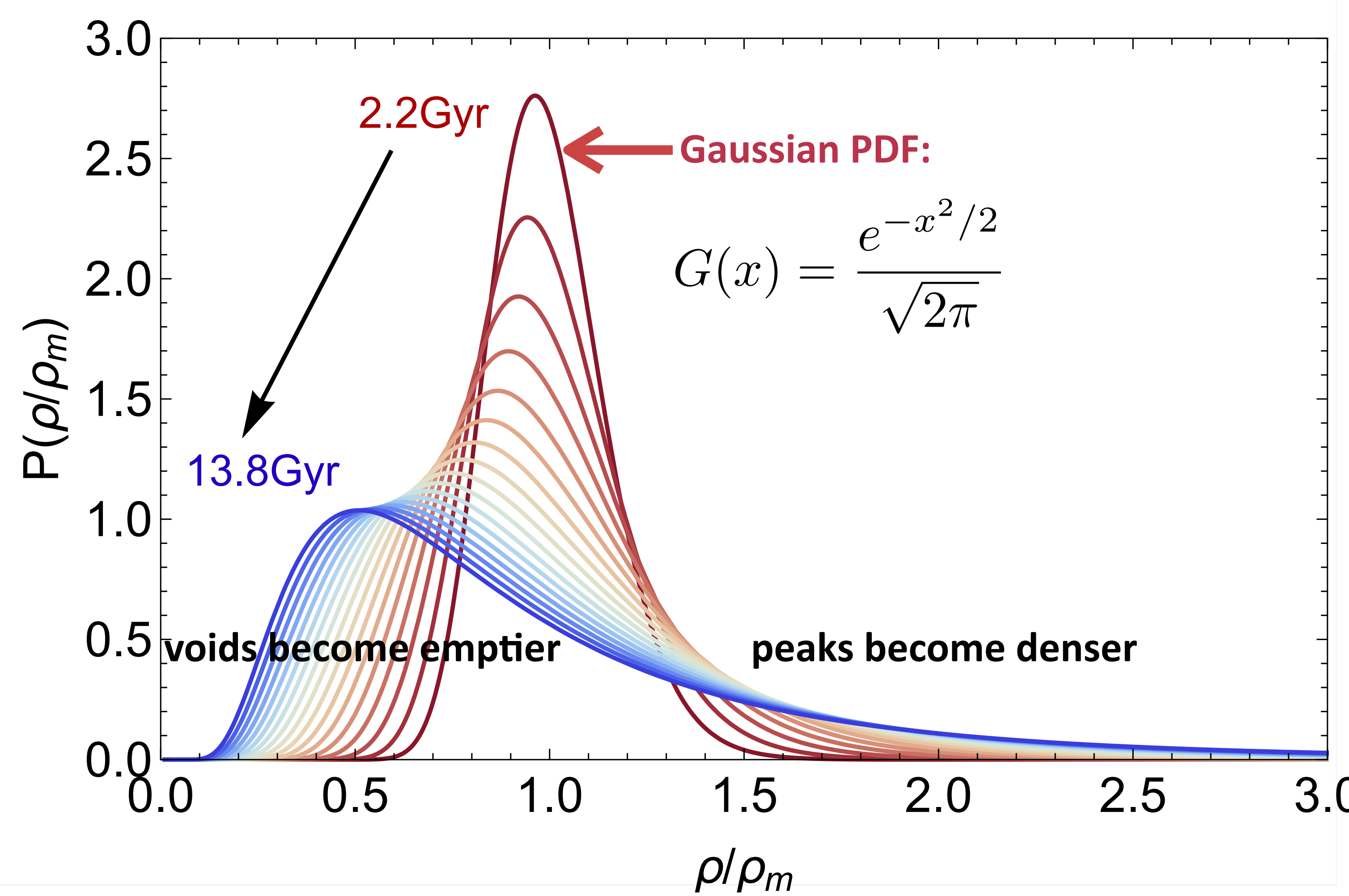


horizon-AGN





Our goal: predict multi-scale densities PDF for $\sigma \sim 1$

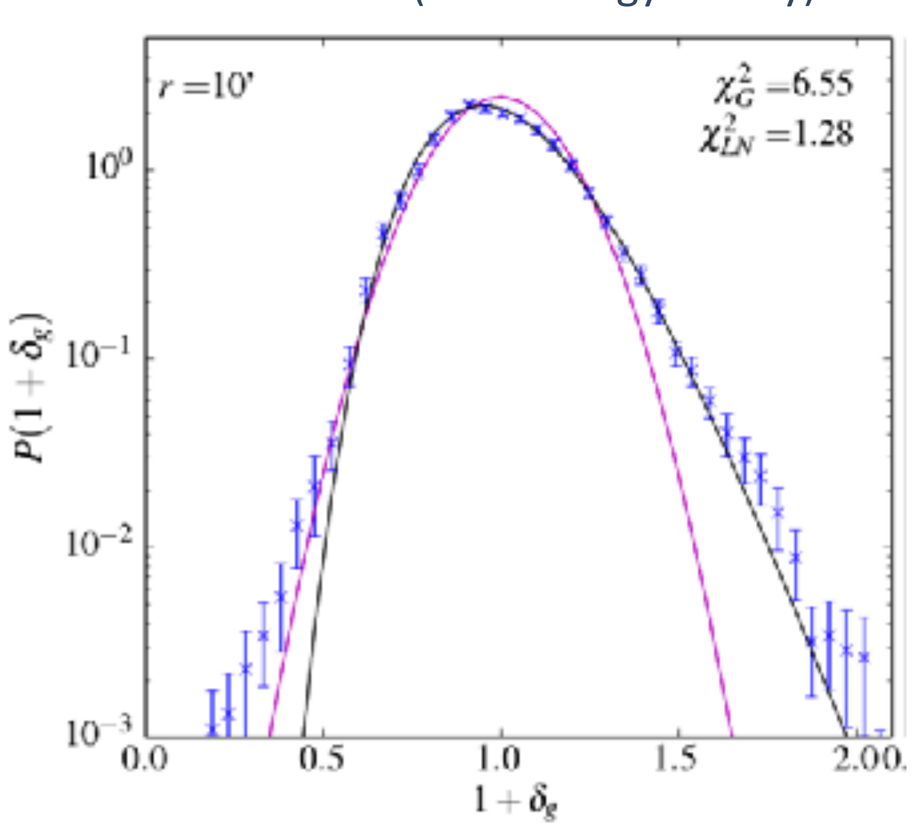
Upshot: large deviations theory

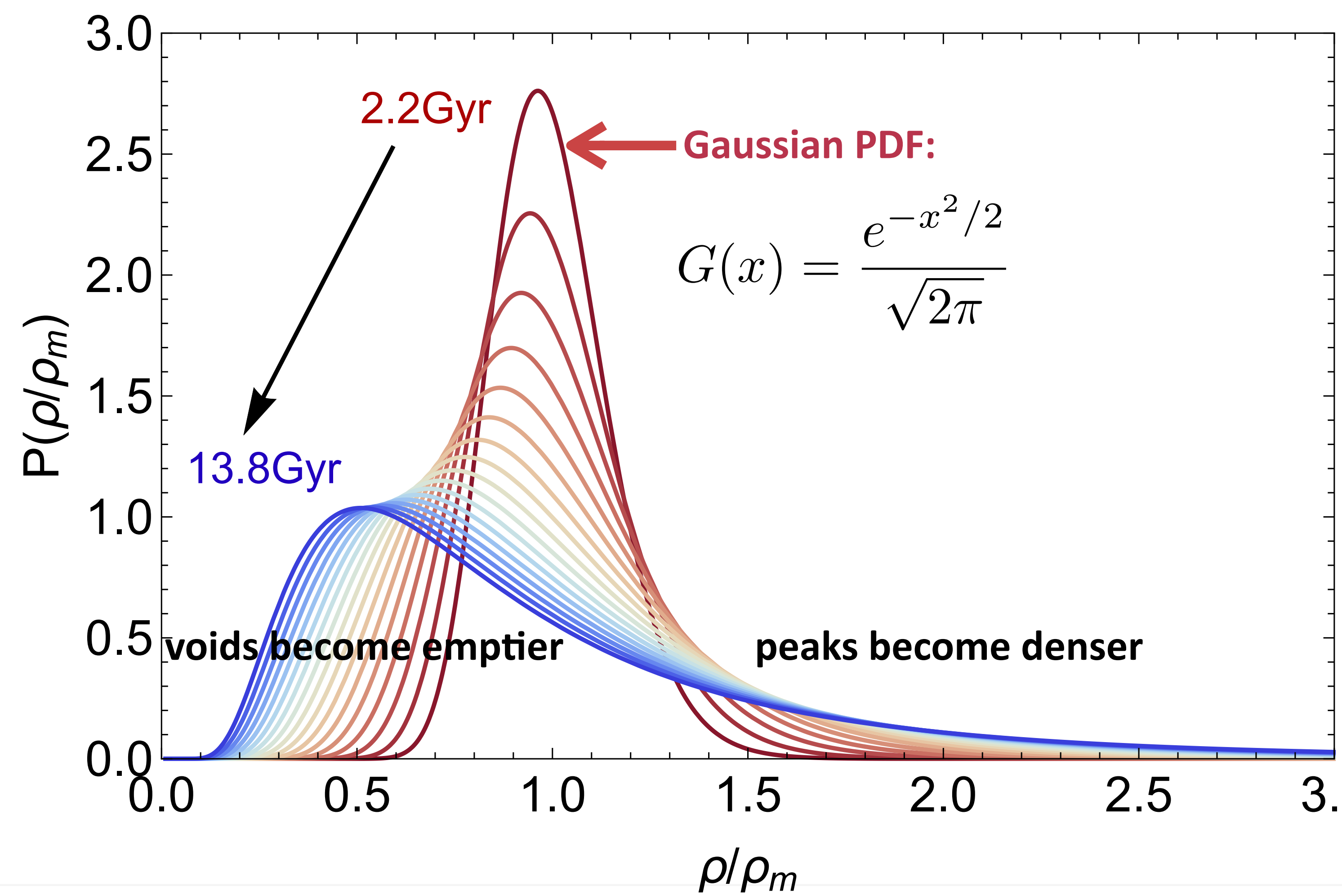
« an unlikely fluctuation is brought about by the least unlikely of all unlikely paths»

Concentric cells

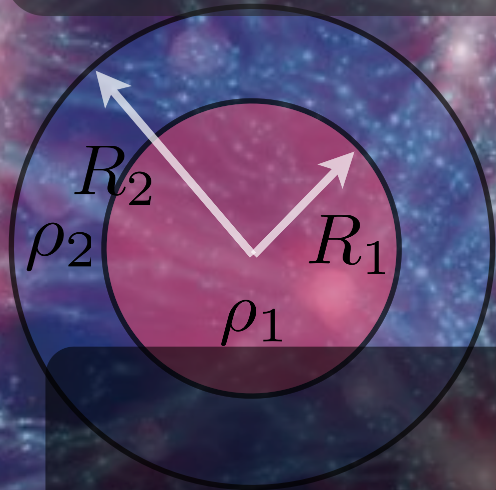
horizon-AGN

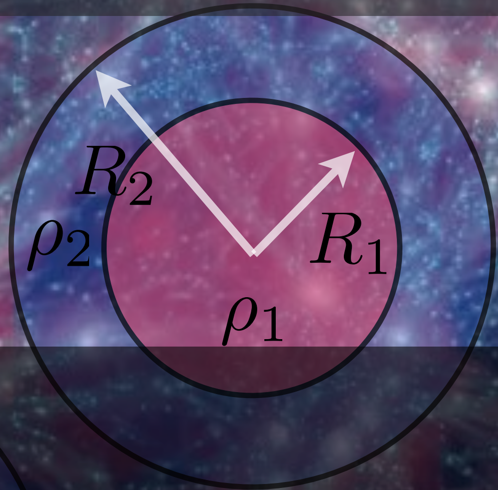
Clerkin'16 (Dark Energy Survey)

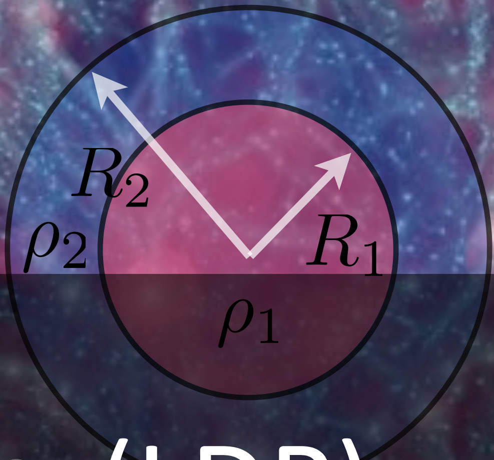




Outline





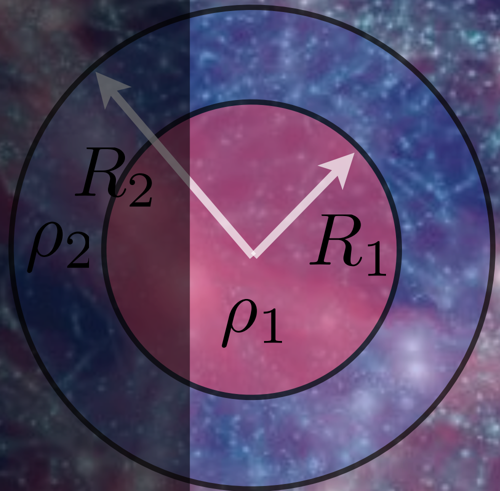


Concentric cells

3.1. Large deviation principle (LDP)

3.2. Cosmic PDFs

3.3. A new cosmological probe?



horizon-AGN

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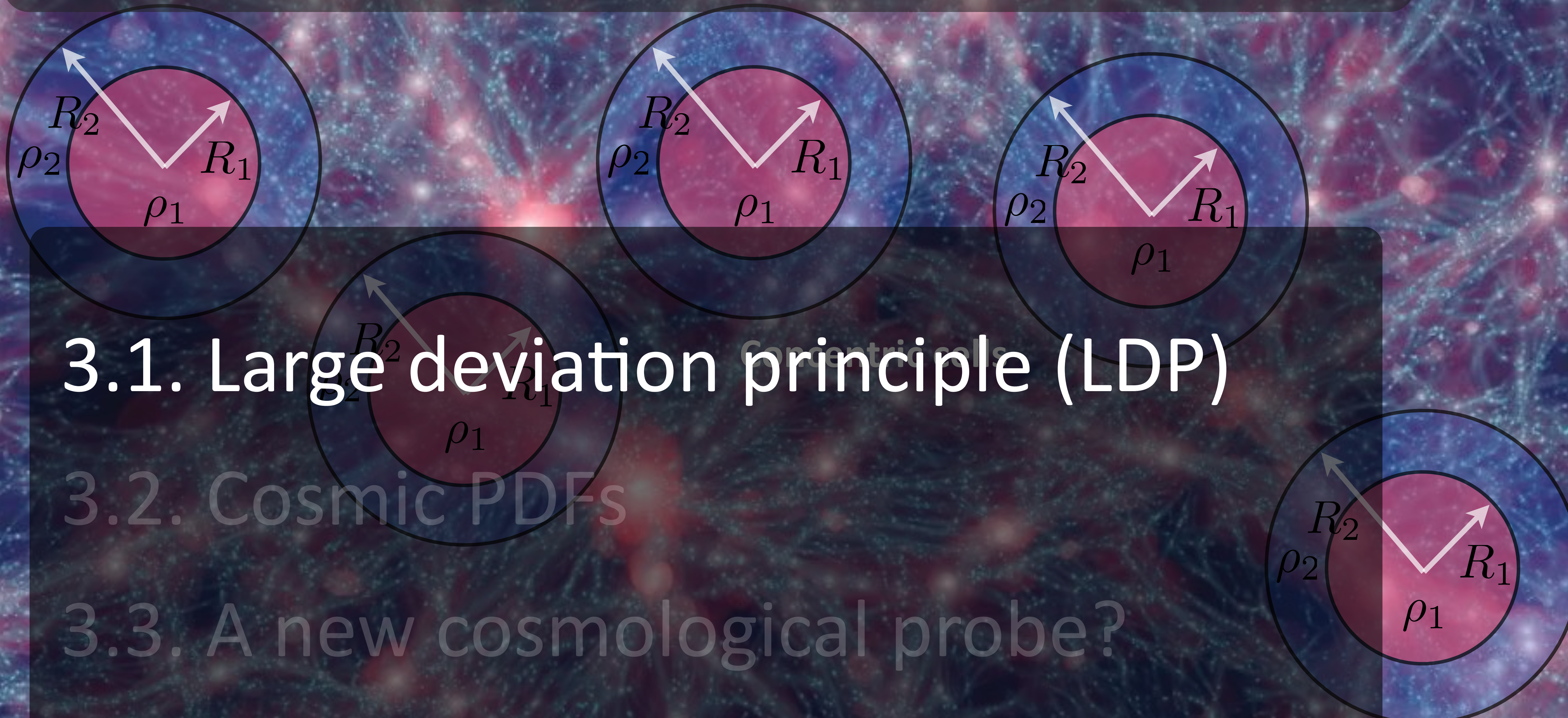

3.1. Large deviation principle (LDP)

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3.3. A new cosmological probe?

horizon-AGN

Large-deviation Theory

Varadhan 1984
Touchette 2009
Ellis 2009

what is the most likely way for an unlikely event to happen?

Exponential decay of the probability of **rare events** in some random systems.
Central Limit theorem : convergence towards a Gaussian… what about the tails?

1. A canonic example: coin tossing 2. Properties 3. LDP @ LSS

Events y_1=tails=0, y_2=heads=1 occur with probability $p_1 = p_2 = 1/2$.
Let's repeat n times this experiment: $w = (w_1, \ldots, w_n) \in \{0,1\}^n$ and consider the average number of heads: $X = \sum_{i=1}^n w_i/n$

When n goes to infinity, X is expected to tend to 1/2.
In mathematical terms, for all non-zero epsilon,

$$\lim_{n\to\infty} \mathcal{P}(||X - 1/2|| < \epsilon)) = 1$$

$$\lim_{n\to\infty} \mathcal{P}(||X - x|| < \epsilon)) = 0\,, \forall x : ||x - 1/2|| > \epsilon$$

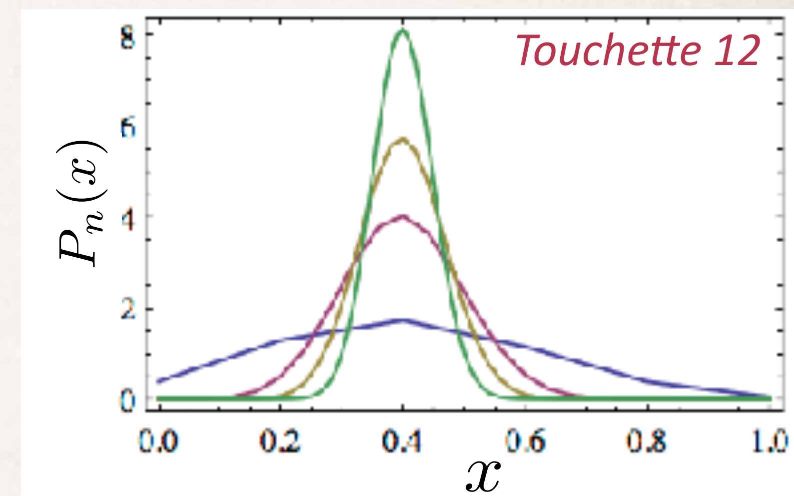
This decay can be shown to be exponential:

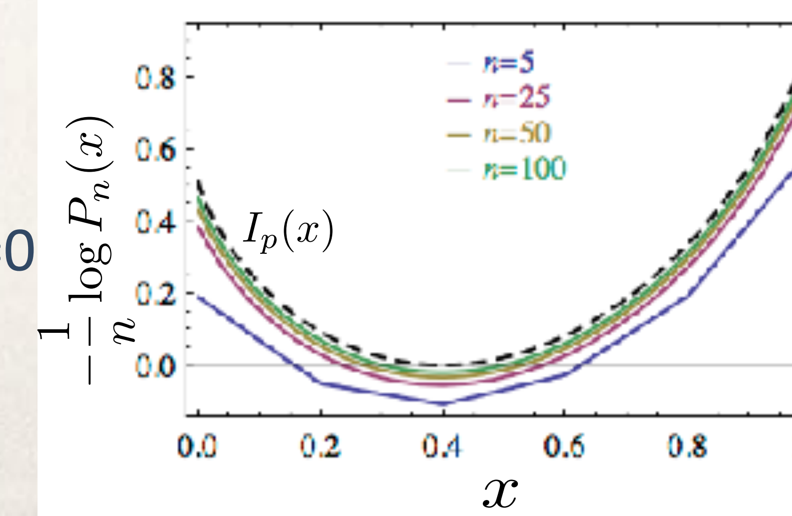
$$\mathcal{P}(||X - x|| < \epsilon)) \underset{n\to\infty}{\approx} \exp(-nI_p(x))$$

where the **rate function** controls the rate of **exponential decay**:

$$I_p(x) = x\ln x + (1-x)\ln(1-x) + \ln 2$$

In particular, the rate function is strictly positive except in 1/2 where I=0 so that we observe a concentration around the mean for large n.





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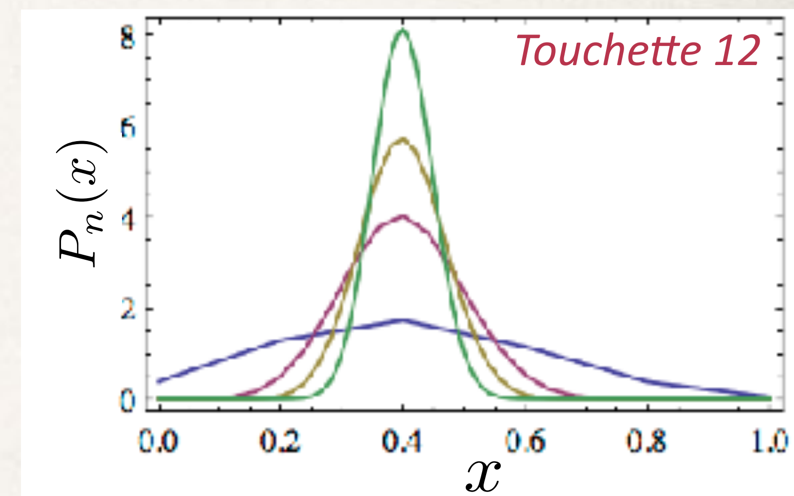

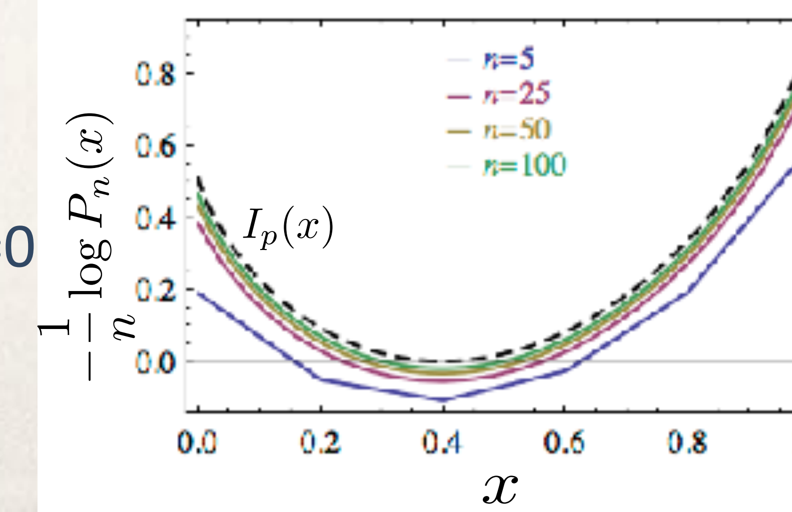

X satisfies a Large-deviation Principle

Large-deviation Theory

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a. The rate function is the Legendre-Fenchel transform of the (scaled) **cumulant generating function**

Varadhan's theorem

$$\varphi(\lambda) = \sup_{\lambda}(\lambda x - I(x)) \quad \text{where} \quad \varphi(\lambda) = \lim_{n\to\infty} \frac{K(n\lambda)}{n}$$

This property comes from a saddle-point (or Laplace) approximation of

$$\exp(n\varphi(\lambda)) \equiv \langle \exp(n\lambda x)\rangle_x = \int P_n \exp(n\lambda x) \approx \int \exp(-nI(x) + n\lambda x)$$

scaled

In the large n limit, the behaviour away from the saddle point does not matter!

Large-deviation Theory

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scaled

In the large n limit, the behaviour away from the saddle point does not matter!

b. The rate function of any mapping of x is

Contraction principle

$$I(y) = \inf_{x, x\to y} I(x)$$

The rate function for y is the smallest rate function (=most likely) of the values x that lead to y.

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-know the rate function of the initial conditions of the Universe e.g (Gaussian):

$$I(\tau(R_0)) = \sigma^2(R_p) \times 1/2\tau(R_0)^2/\sigma^2(R_0)$$

Large-deviation Theory

Bernardeau&Reimberg 16

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-deduce the rate function of the final densities from the *Contraction Principle*

$$I(\rho) = I(\tau = \zeta^{-1}(\rho))$$

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-provided we can identify the most likely initial density contrast that leads to a given final density

initial contrast

$$\rho = \zeta(\tau)$$

final density

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-deduce the rate function of the final densities from the *Contraction Principle*

$$I(\rho) = I(\tau = \zeta^{-1}(\rho))$$

-provided we can identify the most likely initial density contrast that leads to a given final density

initial contrast

$$\rho = \zeta(\tau)$$

final density

-compute the scaled cumulant generating function (SCGF) via *Varadhan's theorem*

$$\varphi(\lambda) = \sup_{\lambda}(\lambda\rho - I(\rho))$$

Large-deviation Theory

what is the most likely way for an unlikely event to happen?

Exponential decay of the probability of **rare events** in some random systems.
Central Limit theorem : convergence towards a Gaussian... what about the tails?

1. A canonic example: coin tossing   2. Properties   **3. LDP @ LSS**

The parameter that drives the exponential decrease of the probabilities is the variance: $n \leftrightarrow 1/\sigma^2$

-know the rate function of the initial conditions of the Universe e.g (Gaussian):

$$I(\tau(R_0)) = \sigma^2(R_p) \times 1/2\tau(R_0)^2/\sigma^2(R_0)$$

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final density

-compute the scaled cumulant generating function (SCGF) via *Varadhan's theorem*

$$\varphi(\lambda) = \sup_\lambda(\lambda\rho - I(\rho))$$

-compute the density PDF via an inverse Laplace transform of the SCGF

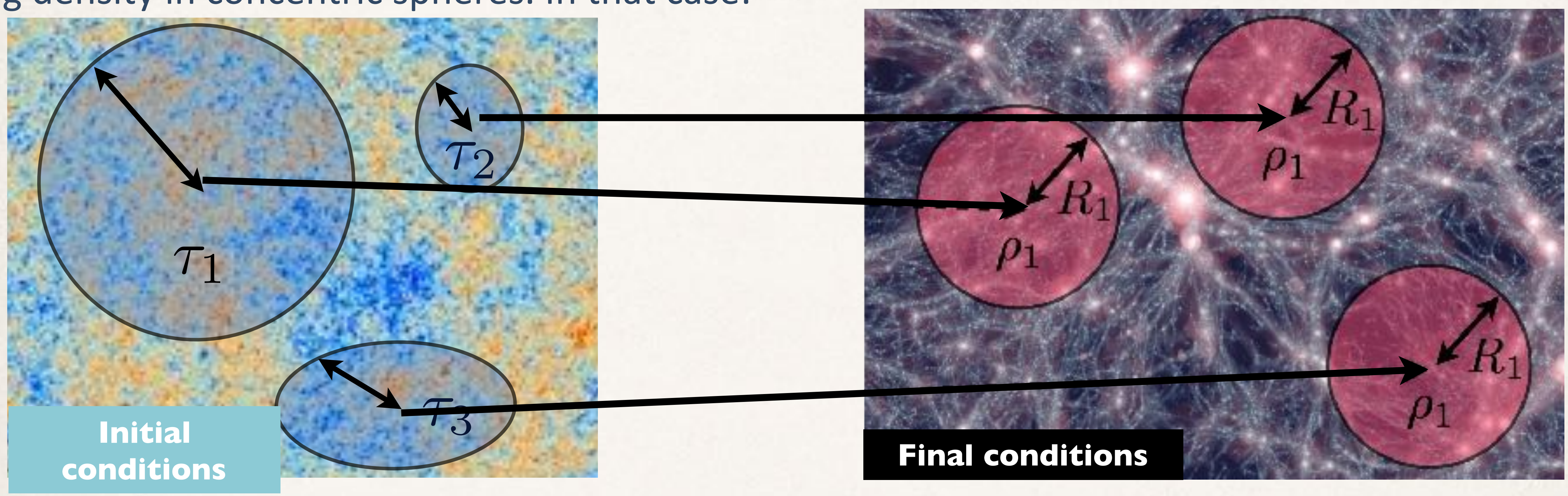
$$\exp\,\varphi(\lambda) = \int P(\rho)\exp(\lambda\rho) \leftrightarrow P(\rho) = \int_{-\imath\infty}^{\imath\infty}\frac{\mathrm{d}\lambda}{2\imath\pi}\exp(\lambda\rho - \varphi(\lambda))$$

Large-deviation Theory

Bernardeau' 94
Bernardeau & Valageas '00
Valageas '02

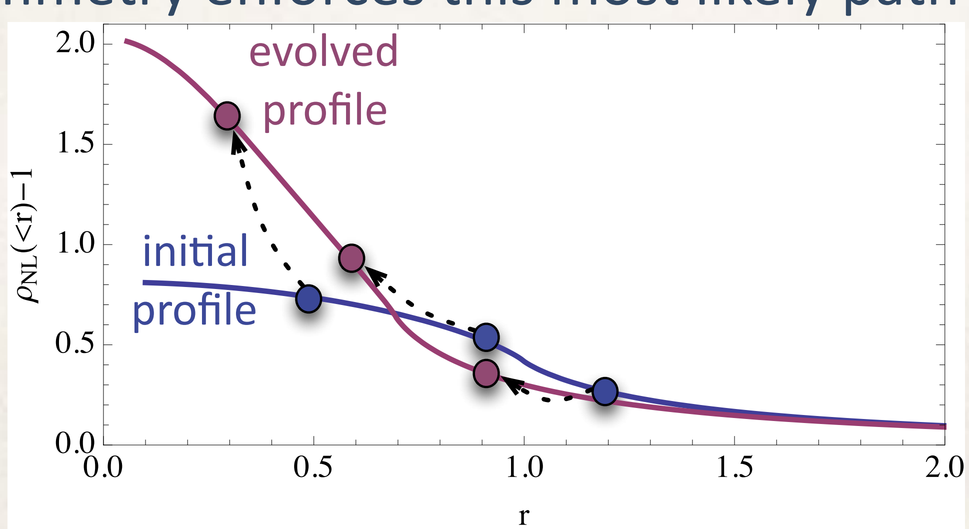
what is the most likely initial configuration a final density originates from?

This most likely path can be found for very specific configurations with sufficient degree of symmetry e.g density in concentric spheres. In that case:



Different initial configurations can lead to the same final state! What is the most likely one?

Spherical symmetry enforces this most likely path to be the so-called **Spherical Collapse dynamics**:



$$\tau \rightarrow \rho = \zeta_{\rm SC}(\tau)$$
$$r_0 \rightarrow r = r_0\rho^{-1/3}$$

Large-deviation Theory

in a nutshell

LDP tells us how to compute the **cumulant generating function** from the initial conditions using the spherical collapse as the « mean dynamics »:

$$\varphi(\{\lambda_k\}) = \sup(\lambda_i \rho_i - I(\rho_i))$$

Varadhan's theorem

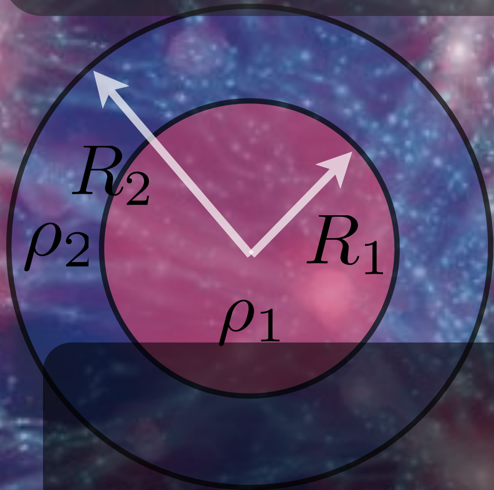
The density **PDF** is then obtained via an inverse Laplace transform of the CGF

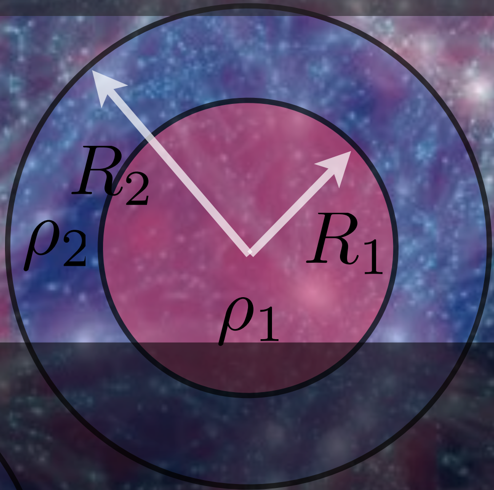
$$\exp \varphi(\lambda) = \int P(\rho) \exp(\lambda\rho) \leftrightarrow P(\rho) = \int_{-\imath\infty}^{\imath\infty} \frac{\mathrm{d}\lambda}{2\imath\pi} \exp(\lambda\rho - \varphi(\lambda))$$

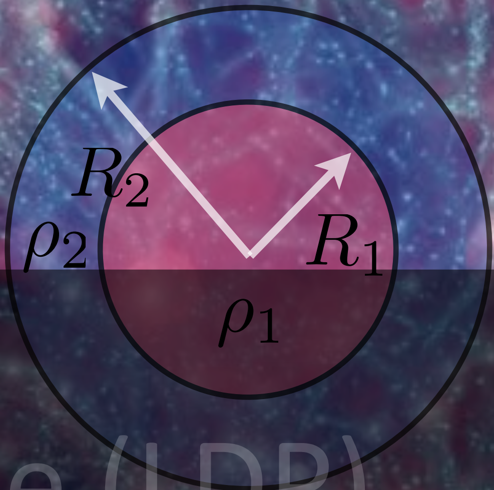
Parameter-free theory which depends on cosmology through : the spherical collapse dynamics and the linear power spectrum.

Predictions are fully **analytical** if one applies the LDP to the log. *(Uhlemann, SC' 16)*

Outline



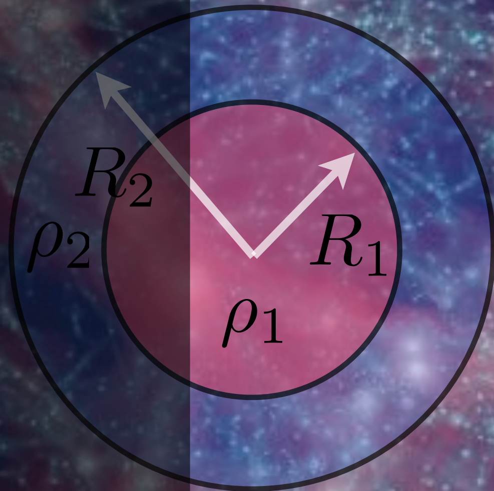




3.1. Large deviation principle (LDP)

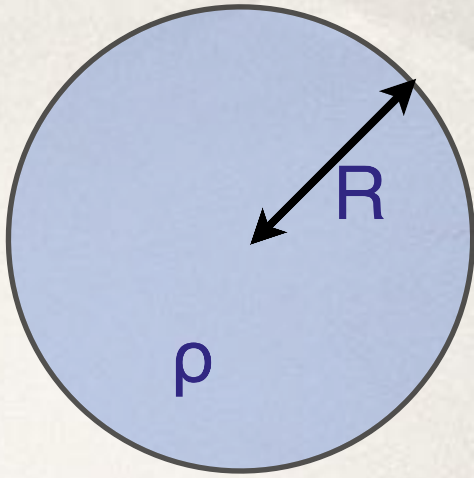
3.2. Cosmic PDFs

3.3. A new cosmological probe?



horizon-AGN

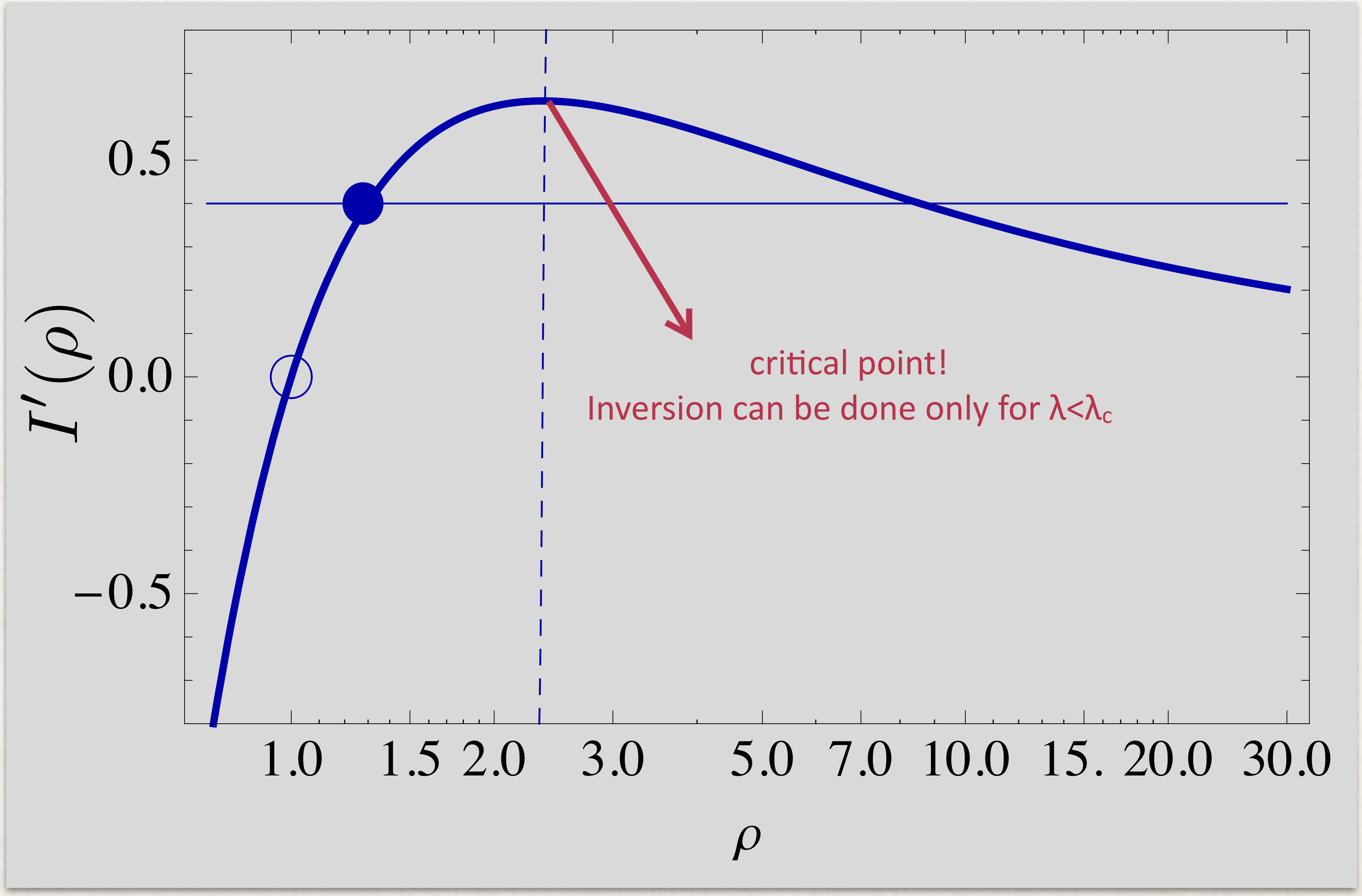
A. One-cell density PDF



1st step: compute the cumulant generating function $\varphi(\lambda) = \sup_\lambda(\lambda\rho - I(\rho))$

or equivalently $\varphi(\lambda) = \lambda\rho - I(\rho)$ with stationary condition $\lambda = I'(\rho)$

⚠ inverting the stationary condition is not possible for all λ!



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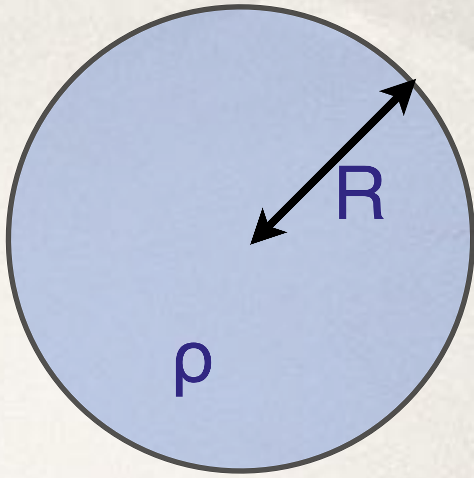

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or equivalently $\varphi(\lambda) = \lambda\rho - I(\rho)$ with stationary condition $\lambda = I'(\rho)$

2nd step: compute the PDF

The inverse Laplace transform requires integration into the complex plane:

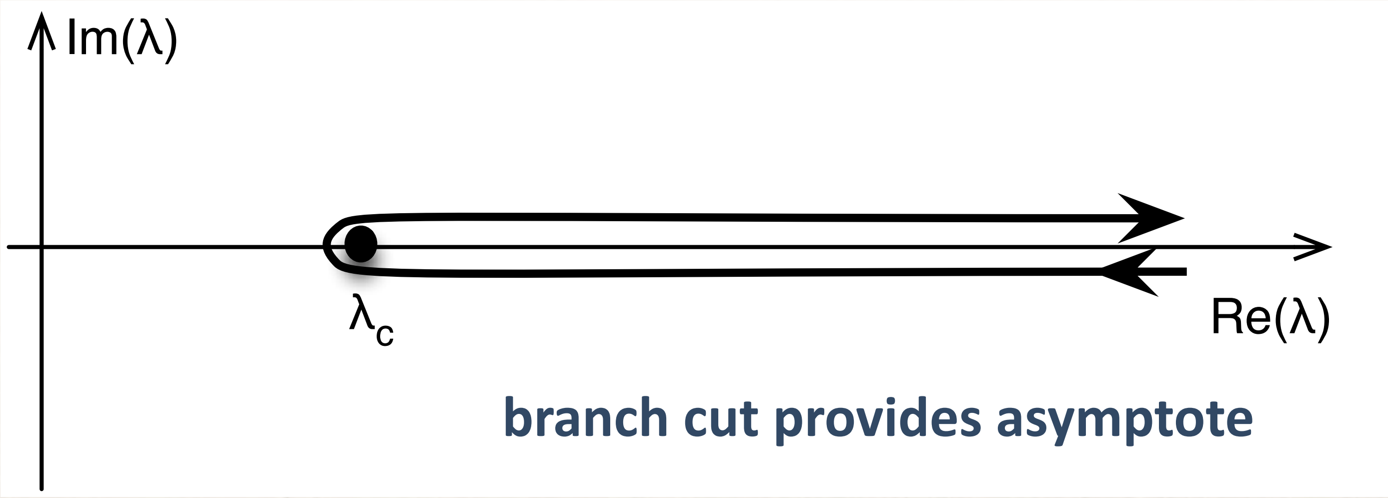
$$\mathcal{P}(\rho) = \int_{-\imath\infty}^{+\imath\infty} \frac{\mathrm{d}\lambda}{2\imath\pi} \exp(-\lambda\rho + \varphi(\lambda))$$

Numerical integration AND **analytical** approximations at low and large densities:

$$\mathcal{P}(\rho) = \sqrt{\frac{I''(\rho)}{2\pi}} \exp(-I(\rho))$$

at low density

$$\mathcal{P}(\rho) = \frac{3a_{3/2}}{4\sqrt{\pi}} \exp(\varphi_c - \lambda_c\rho) \frac{1}{(\rho + \ldots)^{5/2}}$$



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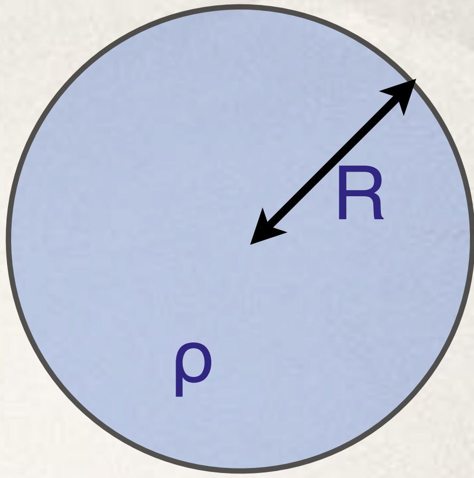

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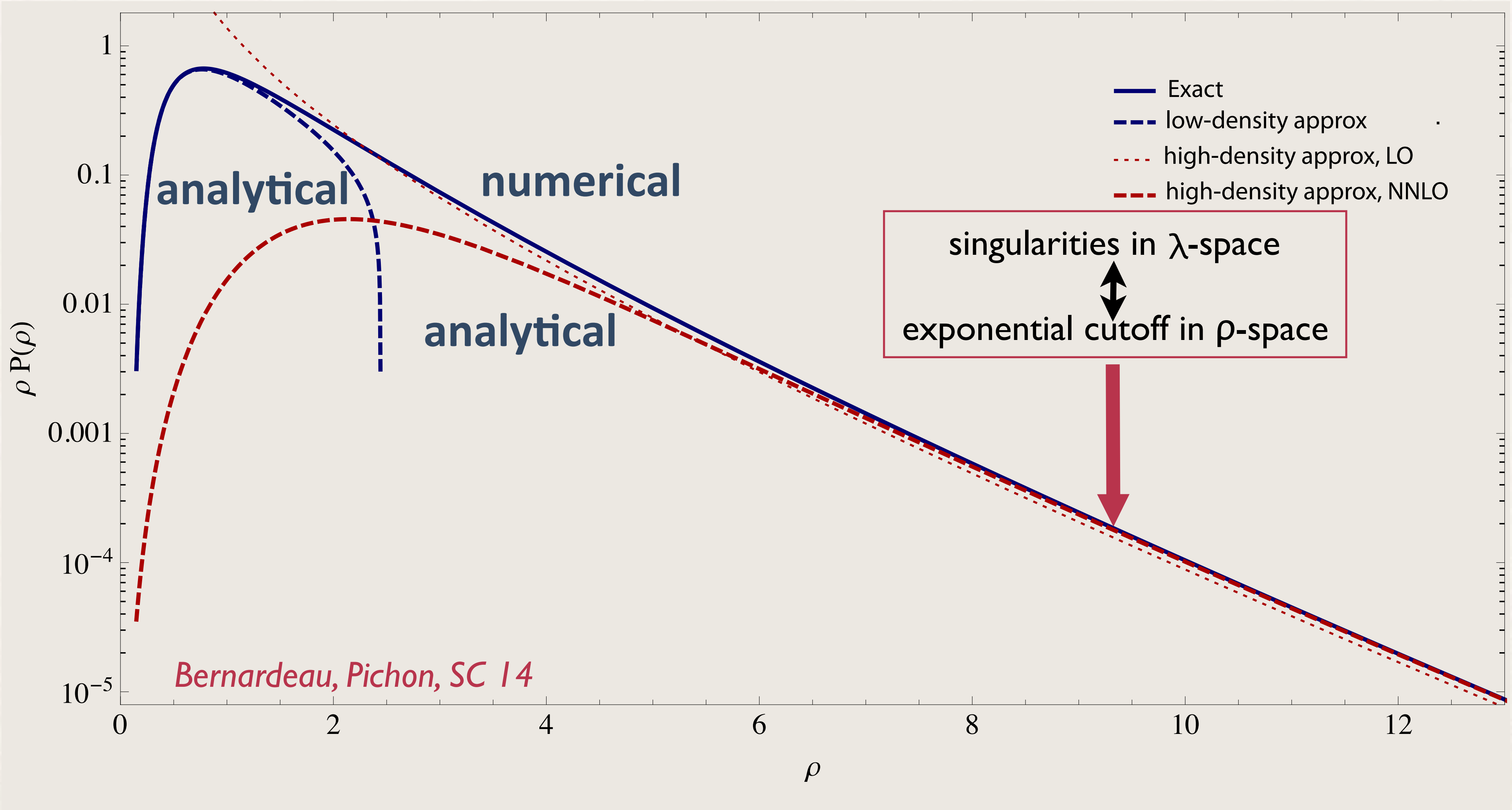
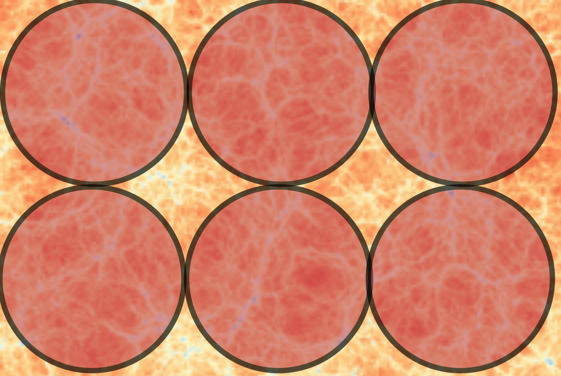


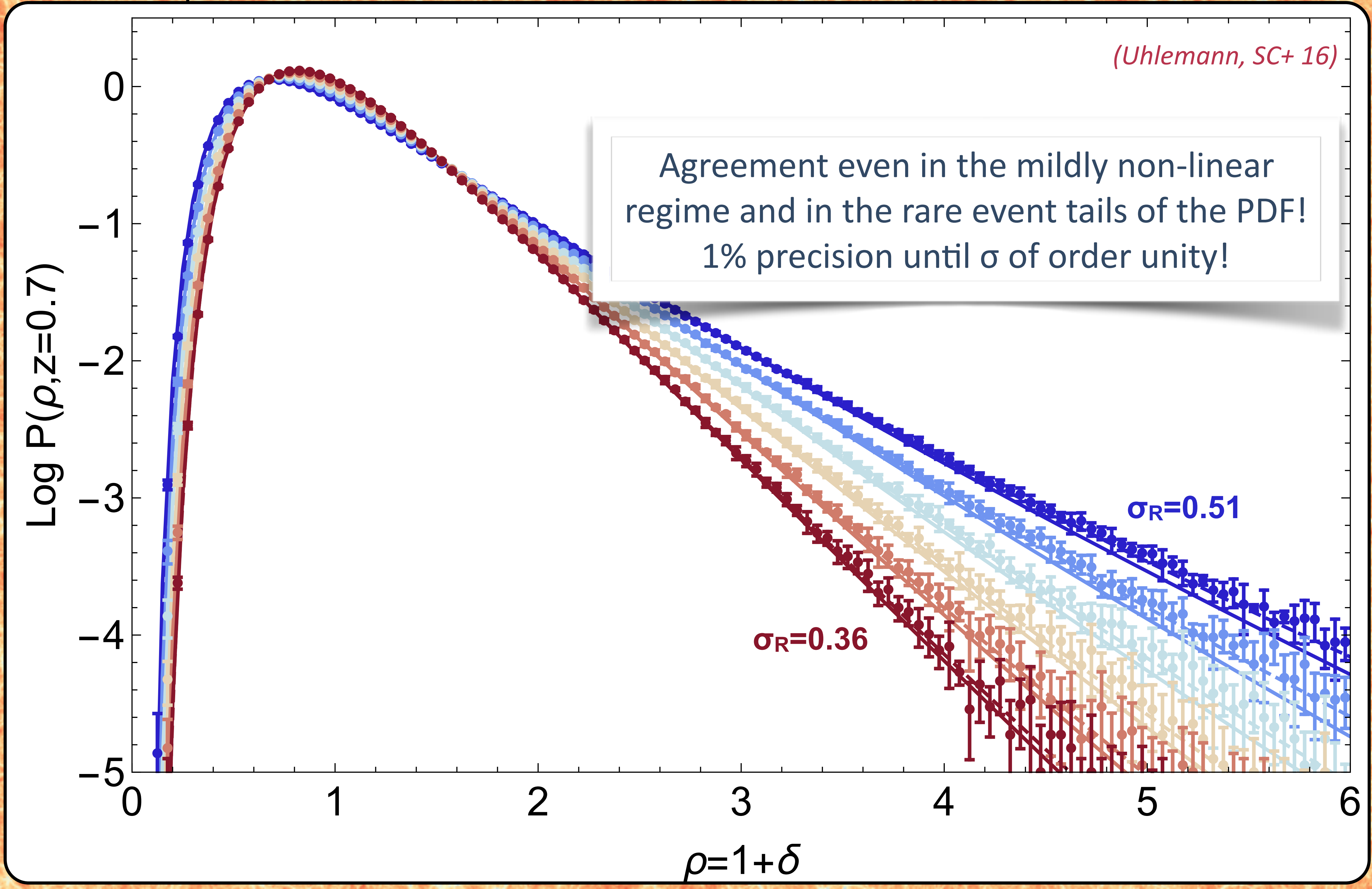
Numerical integration technically done by choosing the path of zero imaginary part

One-cell density PDF



Horizon-Run: 3.1 h^{-1} Gpc

R = 10…15 h^{-1} Mpc

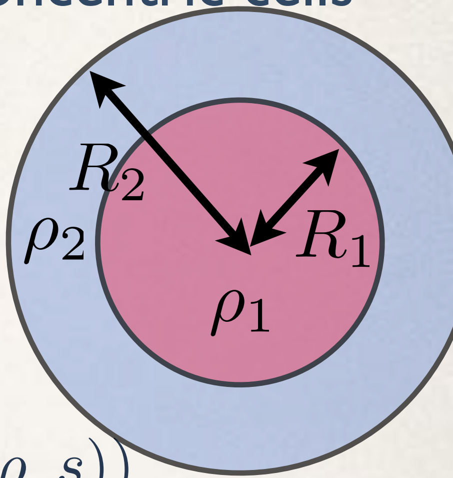


B. Two-cell PDF

Bernardeau, Pichon, SC 14

Same formalism can be used to compute the statistics of cosmic densities in N>1 concentric cells

Introduce slope = possible proxy for peaks & voids



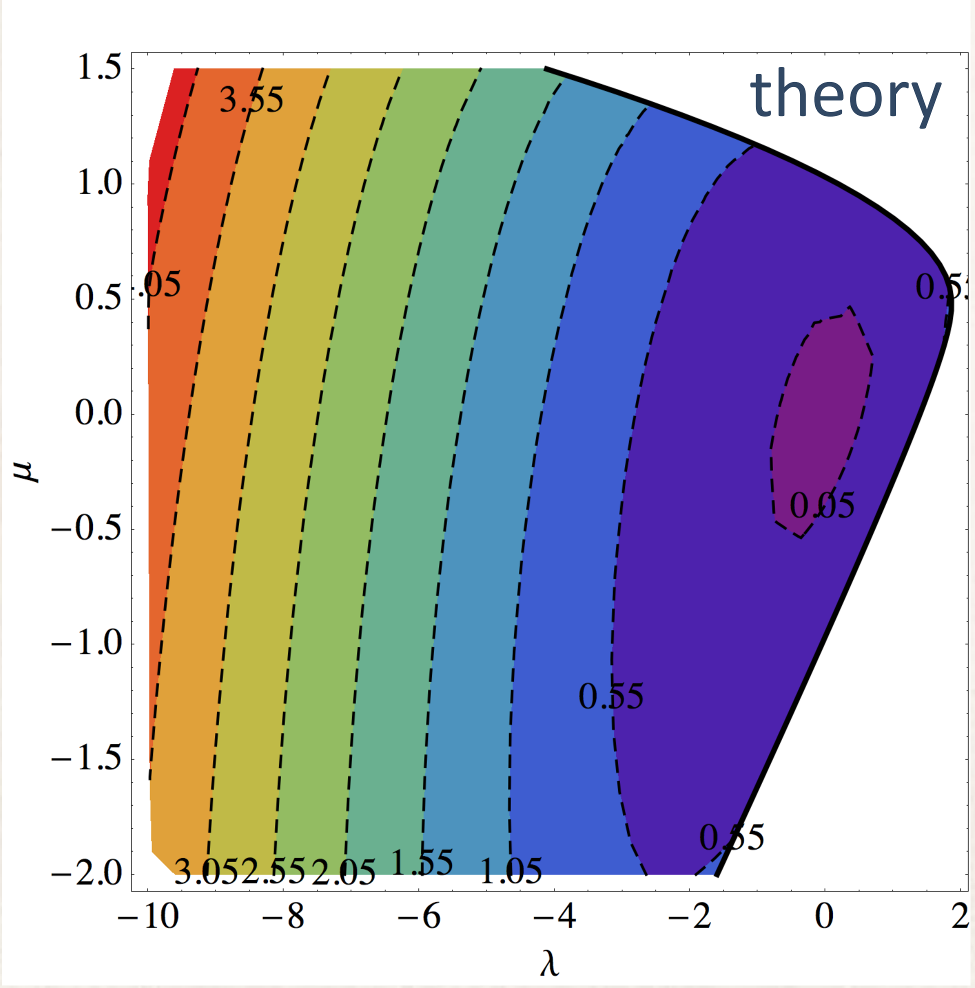
$$P(\rho_1, \rho_2)\mathrm{d}\rho_1\mathrm{d}\rho_2 \xleftrightarrow{\rho = \rho_1} P(\rho, s)\mathrm{d}\rho\,\mathrm{d}s$$

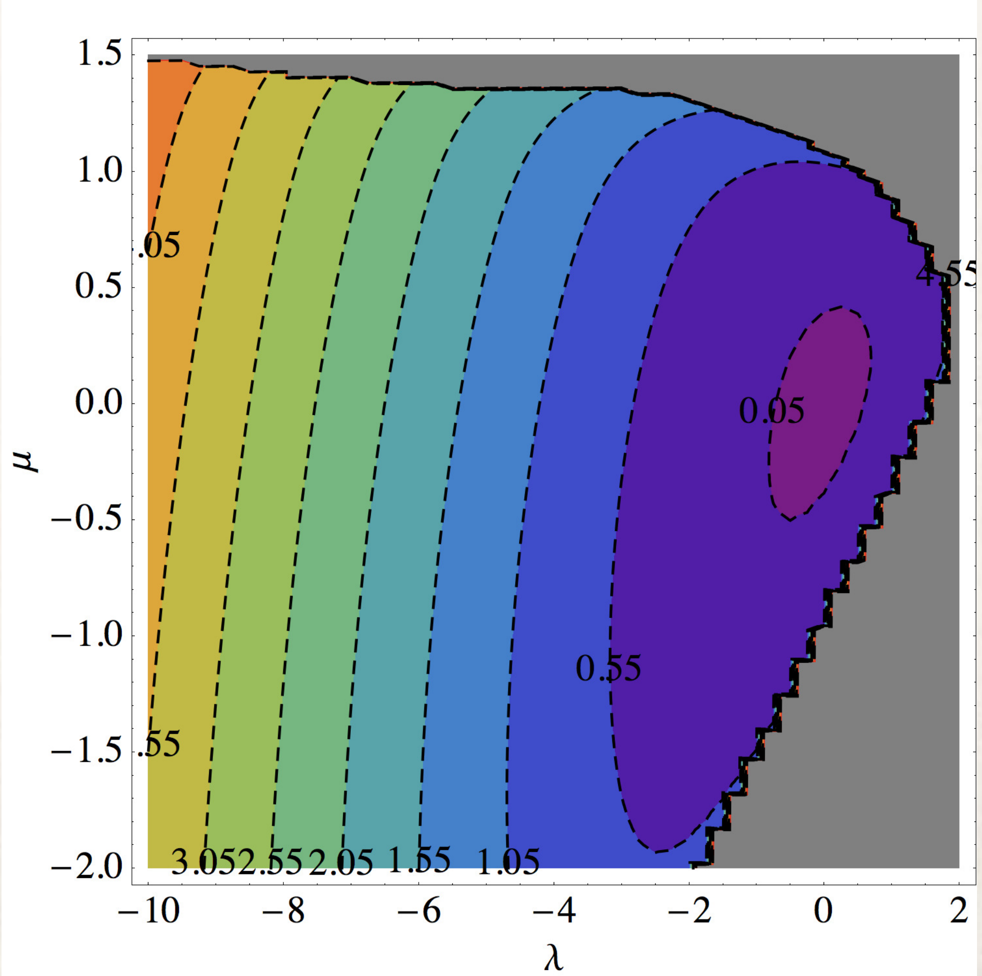
$$s = R_1 \frac{\rho_2 - \rho_1}{R_2 - R_1} \quad \textbf{density slope}$$

1st step: compute the cumulant generating function $\varphi(\lambda, \mu) = \sup_{\lambda,\mu}(\lambda\rho + \mu s - I(\rho, s))$

or equivalently $\varphi(\lambda, \mu) = \lambda\rho + \mu s - I(\rho, s))$ with stationary condition $\begin{cases} \lambda = \frac{\partial I(\rho,s)}{\partial \rho} \\ \mu = \frac{\partial I(\rho,s)}{\partial s} \end{cases}$

⚠️ There is a critical line where the stationary condition is singular.





simulation for σ = 0.51

B. Two-cell PDF

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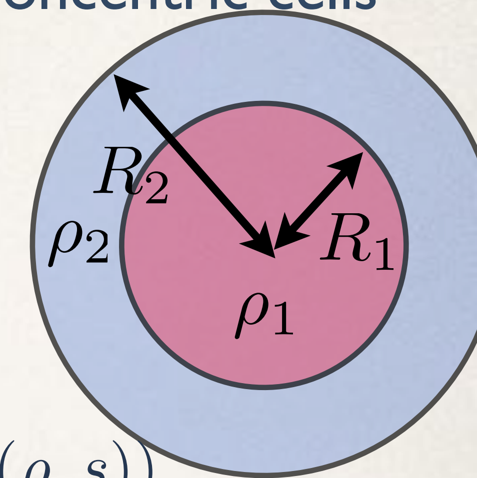

$$P(\rho_1,\rho_2)\mathrm{d}\rho_1\mathrm{d}\rho_2 \xleftrightarrow{\rho=\rho_1} P(\rho,s)\mathrm{d}\rho\,\mathrm{d}s$$

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2nd step: compute the PDF via 2D Inverse Laplace Transform

$$P(\rho,s)=\int_{-i\infty}^{i\infty}\mathrm{d}\lambda\int_{-i\infty}^{i\infty}\mathrm{d}\mu\exp(-\lambda\rho-\mu s+\varphi(\lambda,\mu))$$

This is difficult because we need to choose a 2D path in 4D space with lots of the oscillations and analytical approximations have a poor range of validity.

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Bernardeau, Pichon, SC 14

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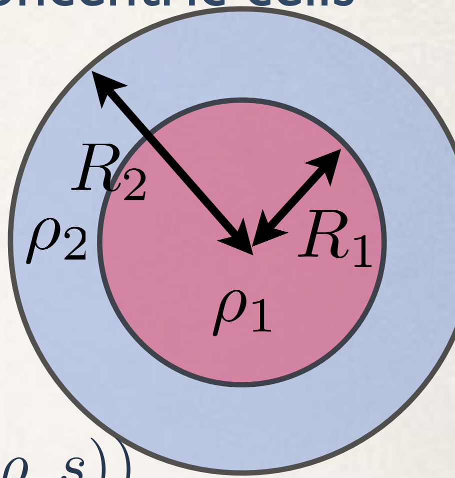

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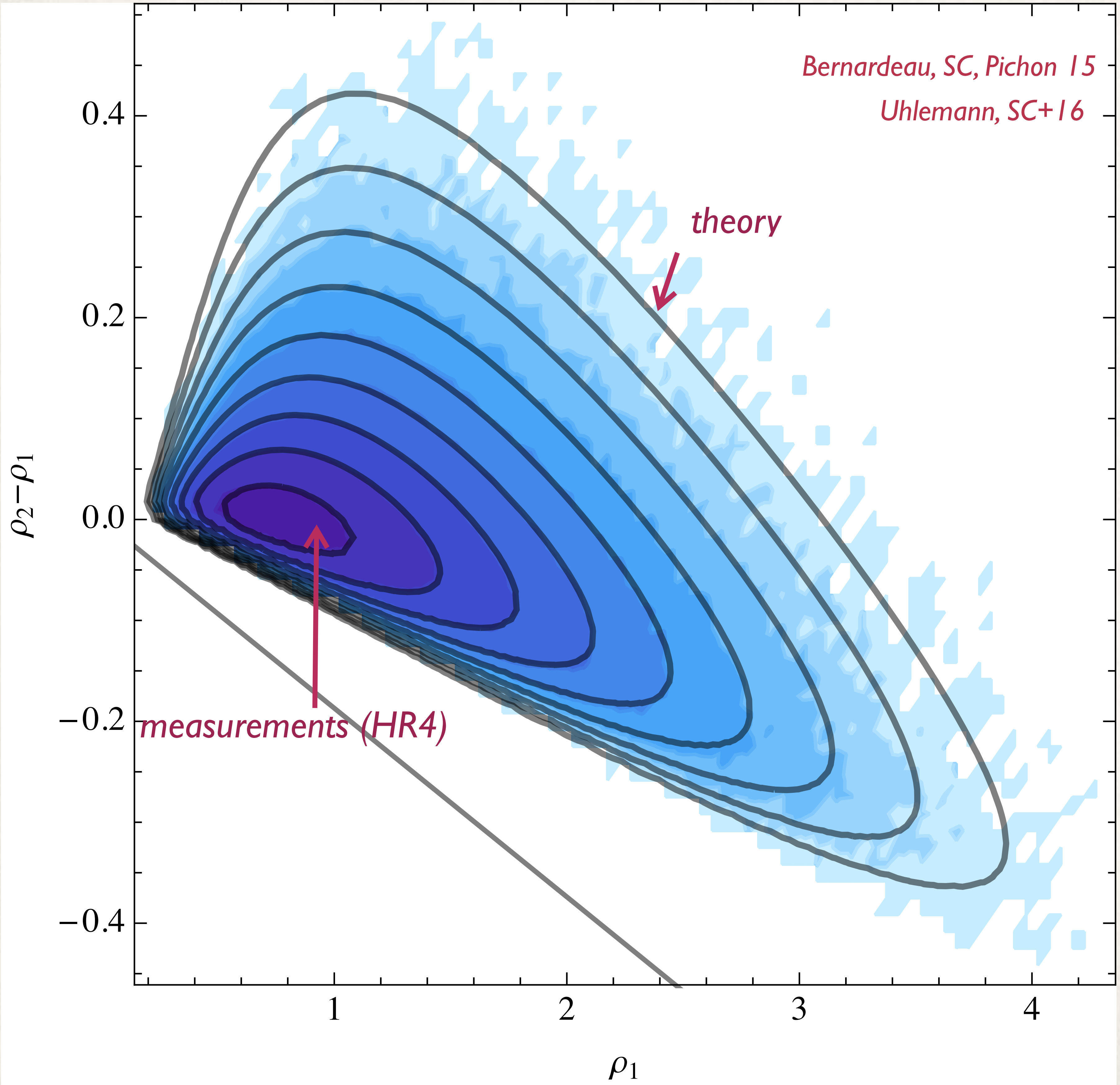
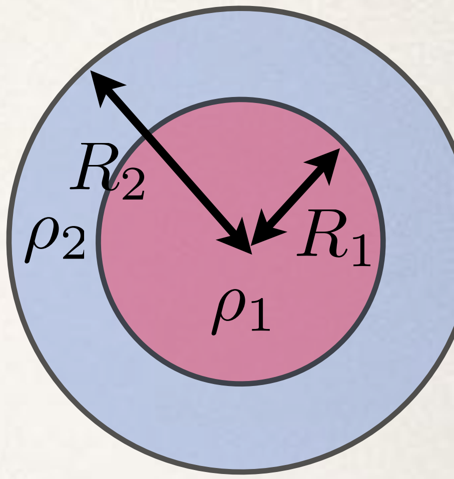
This is difficult because we need to choose a 2D path in 4D space with lots of the oscillations and analytical approximations have a poor range of validity.

Apply the large-deviation principle to the log of the density!
This is a simple change of variable but it removes the singularities and provides very accurate analytical approximations (almost indistinguishable from the numerical integration)!

Two-cell PDF

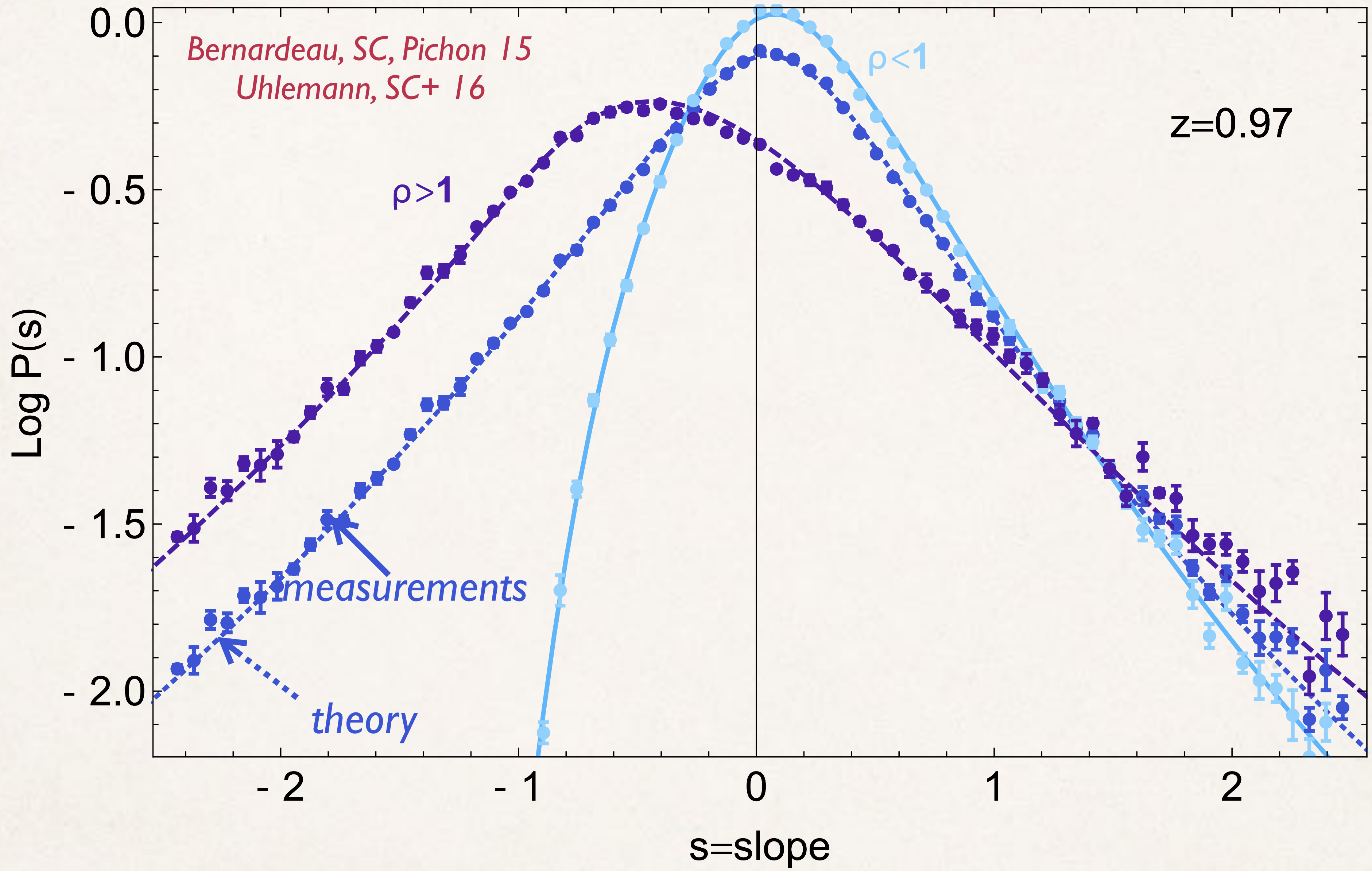


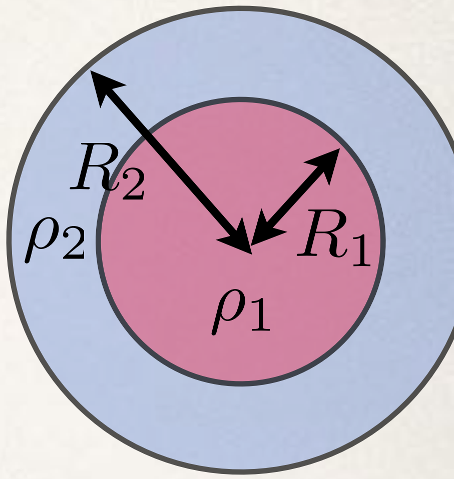
Bernardeau, SC, Pichon 15
Uhlemann, SC+16
theory
measurements (HR4)
0.4
0.2
0.0
−0.2
−0.4
$\rho_2-\rho_1$
1
2
3
4
ρ_1



R_2
R_1
ρ_2
ρ_1

Two-cell PDF: statistics of the slope





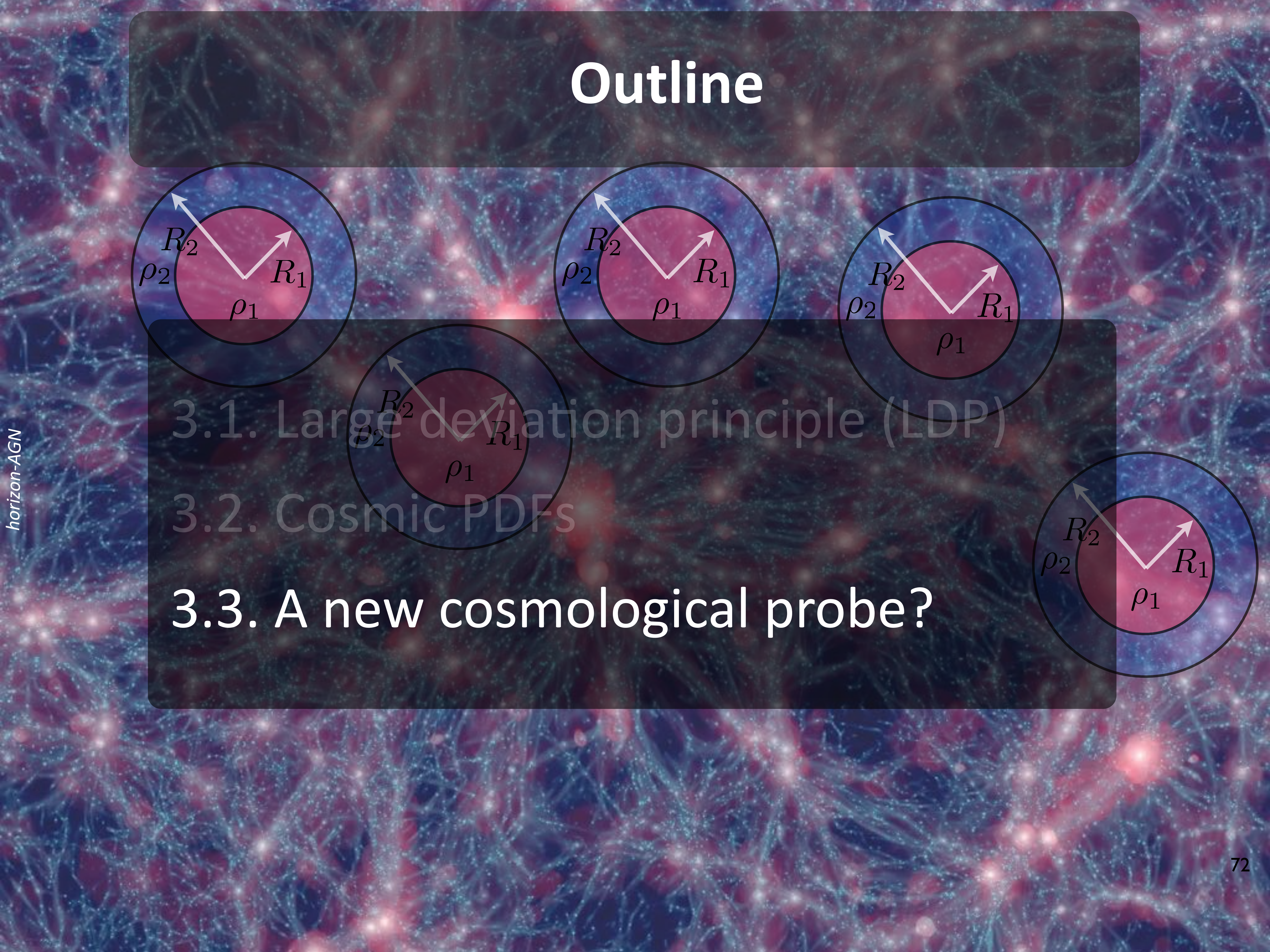
Higher density environments have more negative slopes (peaks!).

Outline

3.1. Large deviation principle (LDP)

3.2. Cosmic PDFs

3.3. A new cosmological probe?



Where is the cosmology dependence?

To get one-cell PDF, one has to:

1) know the rate function of the initial conditions e.g (Gaussian):

$$I(\tau(R_0)) = \sigma^2(R_p) \times 1/2\tau(R_0)^2/\sigma^2(R_0)$$

where the initial variance is a function of the **linear power spectrum**

$$\sigma^2(R) = \frac{1}{(2\pi)^3} \int d^3\mathbf{k}\, P_{\rm lin}(k) W_{\rm TH}^2(kR)$$

2) deduce the rate function of the final densities from the Contraction Principle

$$I(\rho) = I(\tau = \zeta^{-1}(\rho))$$

spherical collapse dynamics

3) compute CGF and then PDF

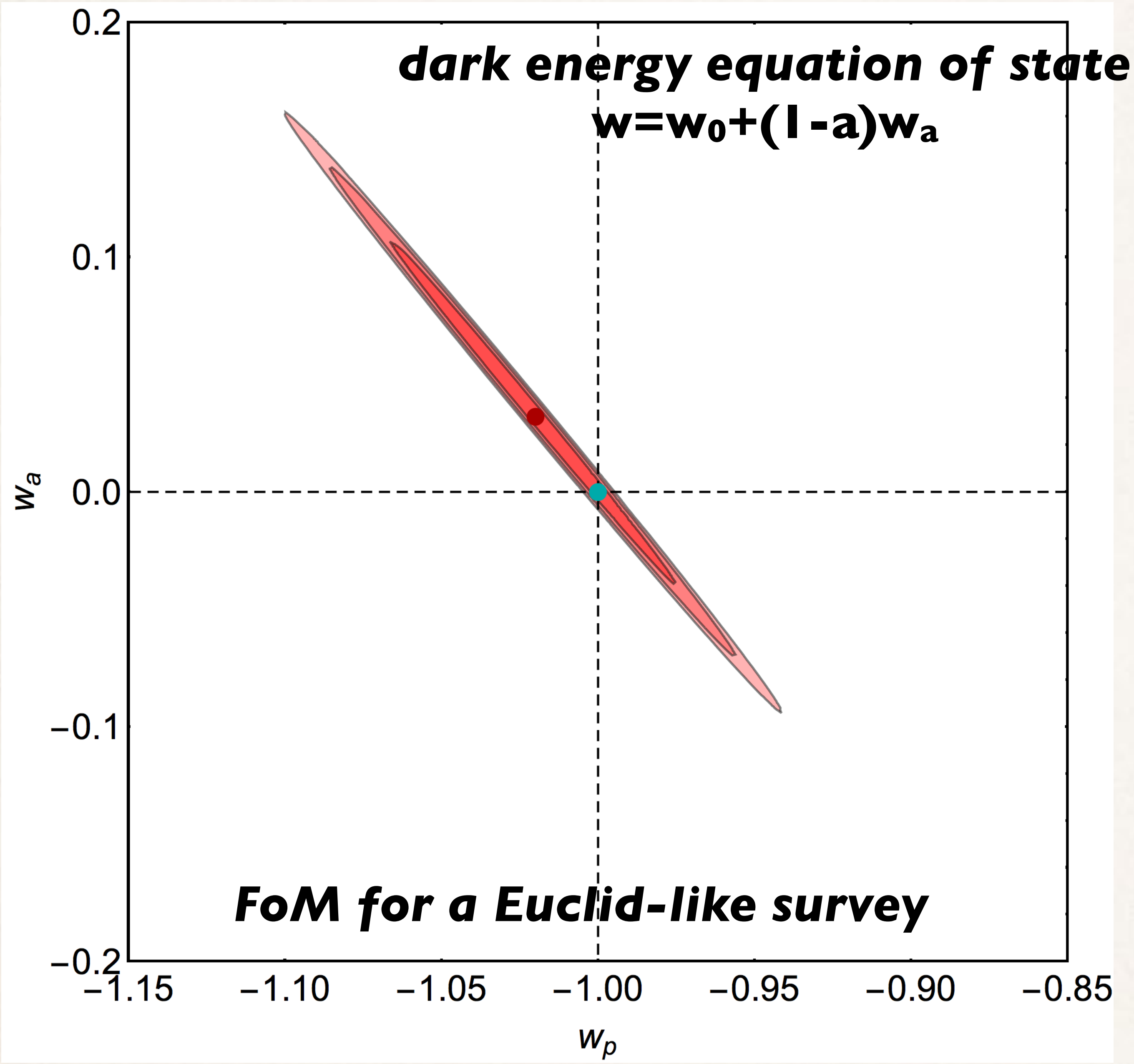
$$\boxed{P(\rho|\nu, P_{\rm lin}, \sigma_{\rm NL}(R, z))}$$

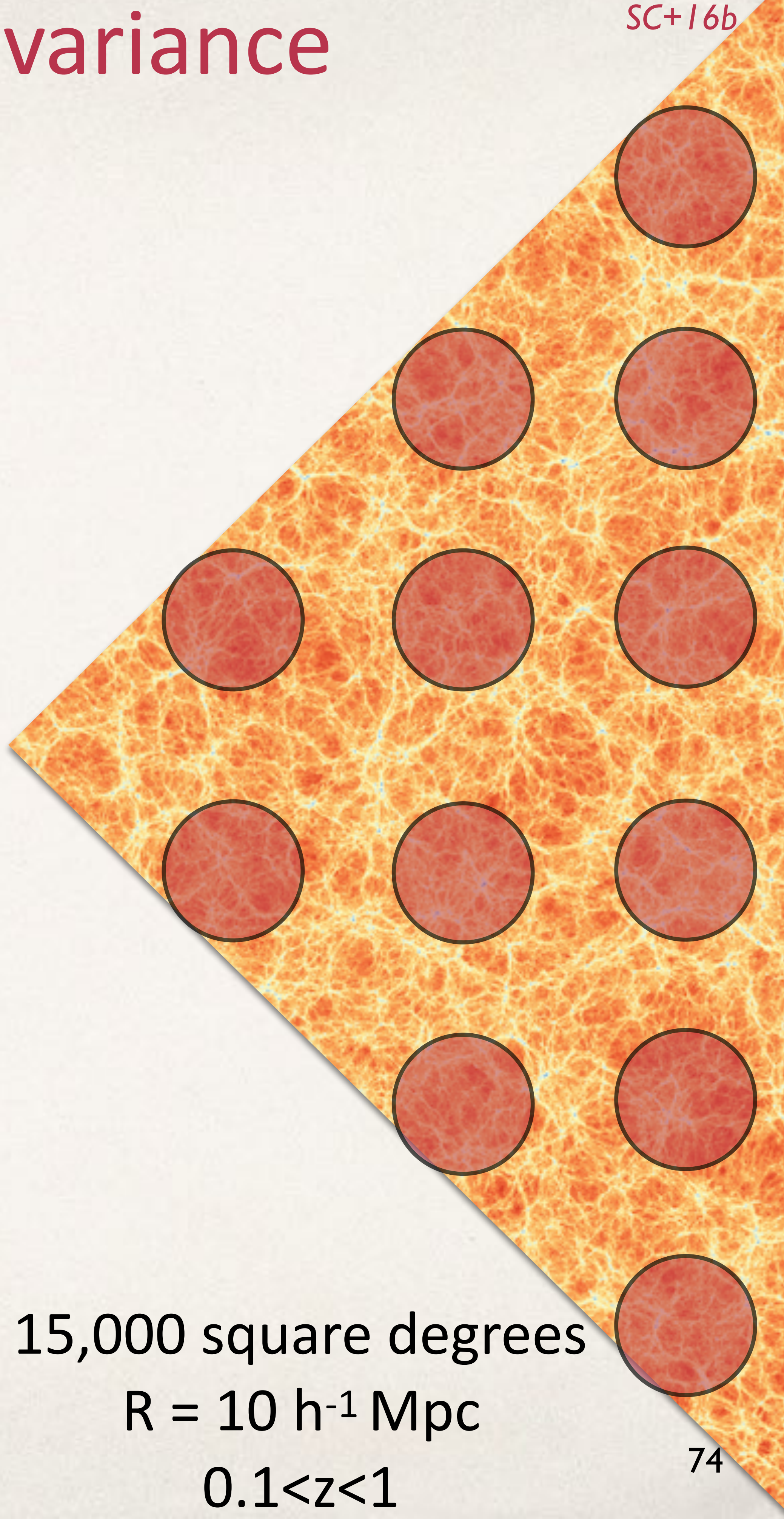
modification of gravity

initial statistics primordial non-Gaussianities

growth of structure dark energy

ML estimator for the variance

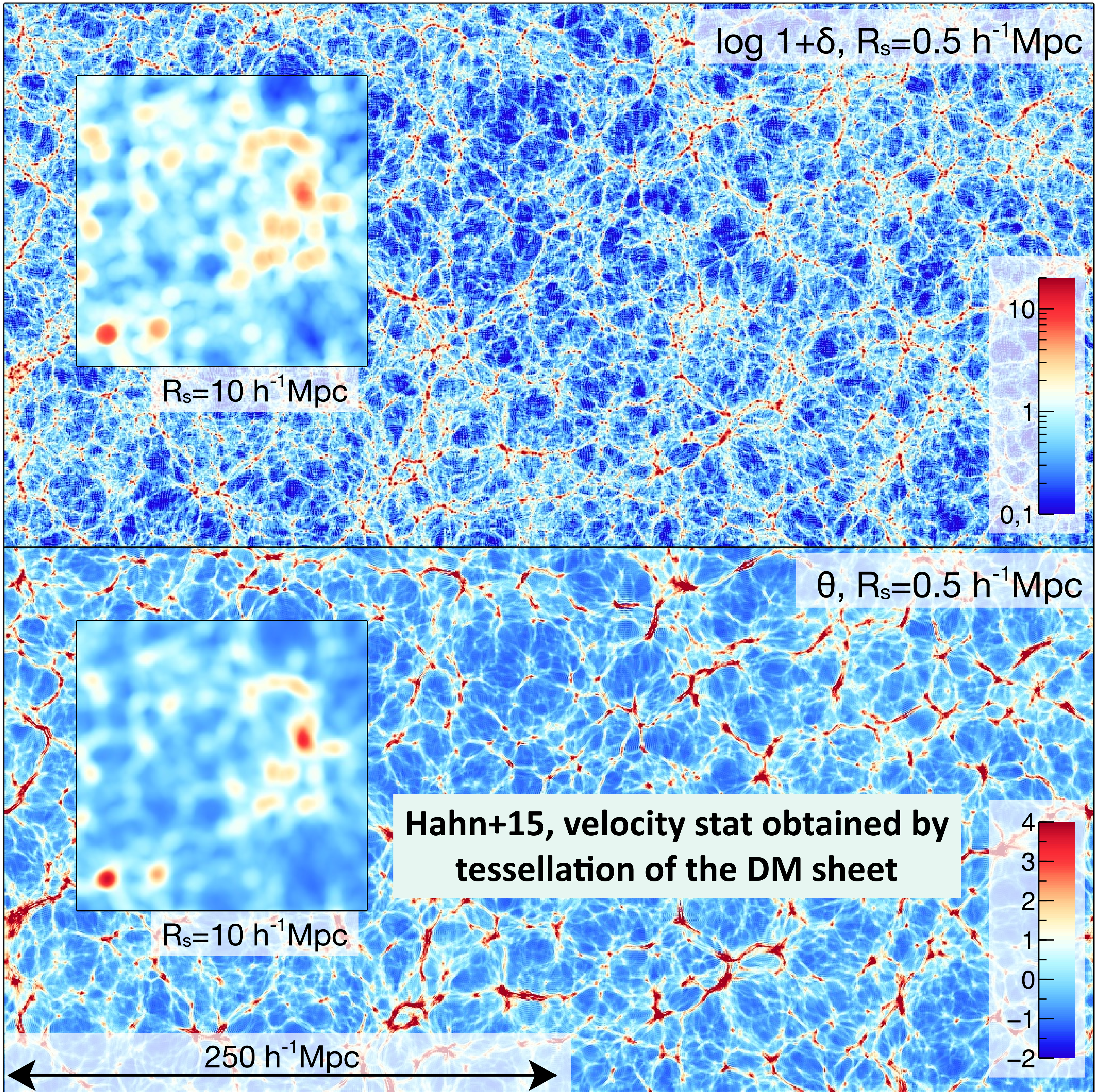




15,000 square degrees
R = 10 h^{-1} Mpc
0.1<z<1

One-cell velocity divergence PDF

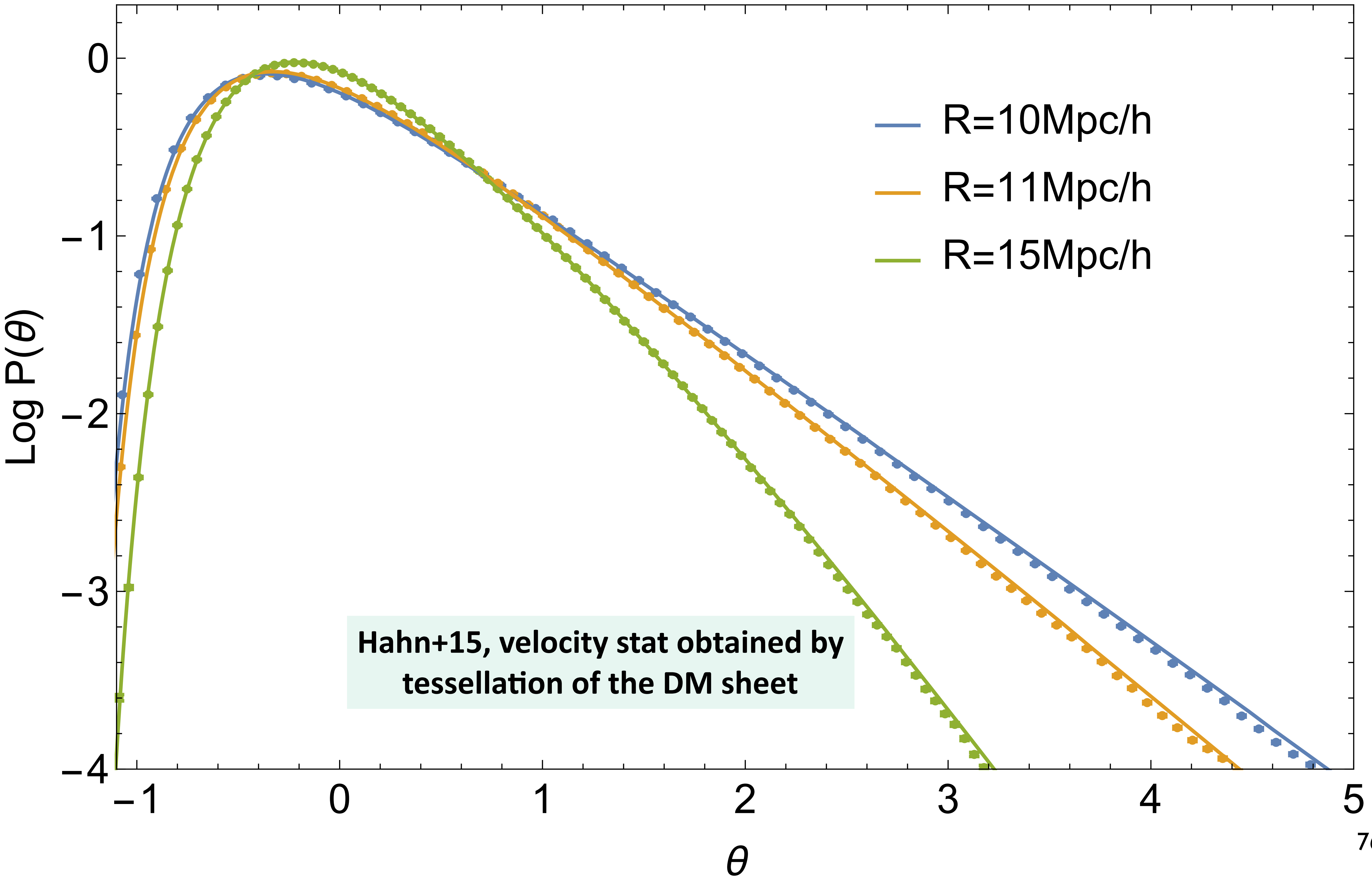
Uhlemann, SC+ 17



Hahn+15, velocity stat obtained by tessellation of the DM sheet

One-cell velocity divergence PDF

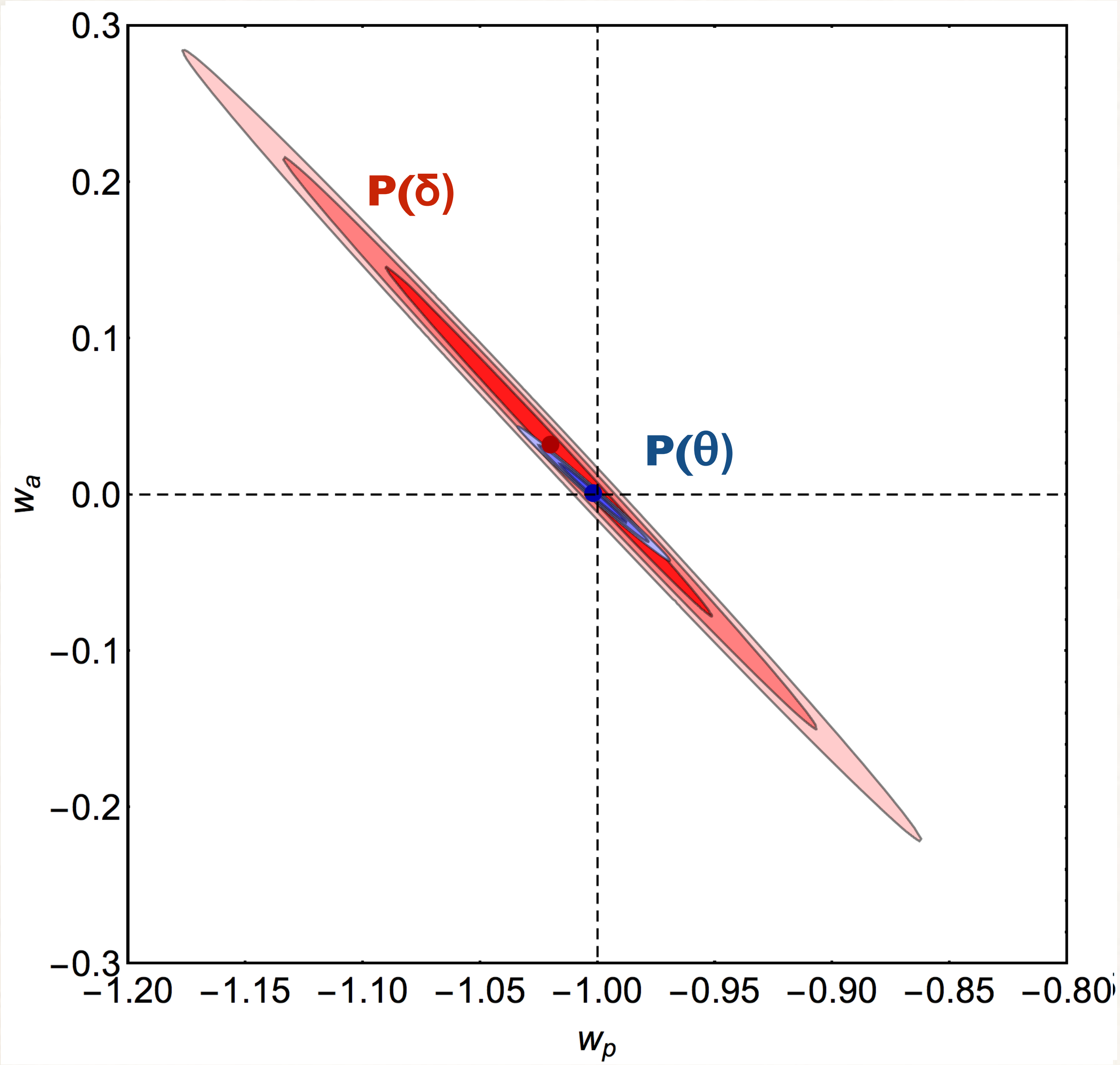
Uhlemann, SC+ 17



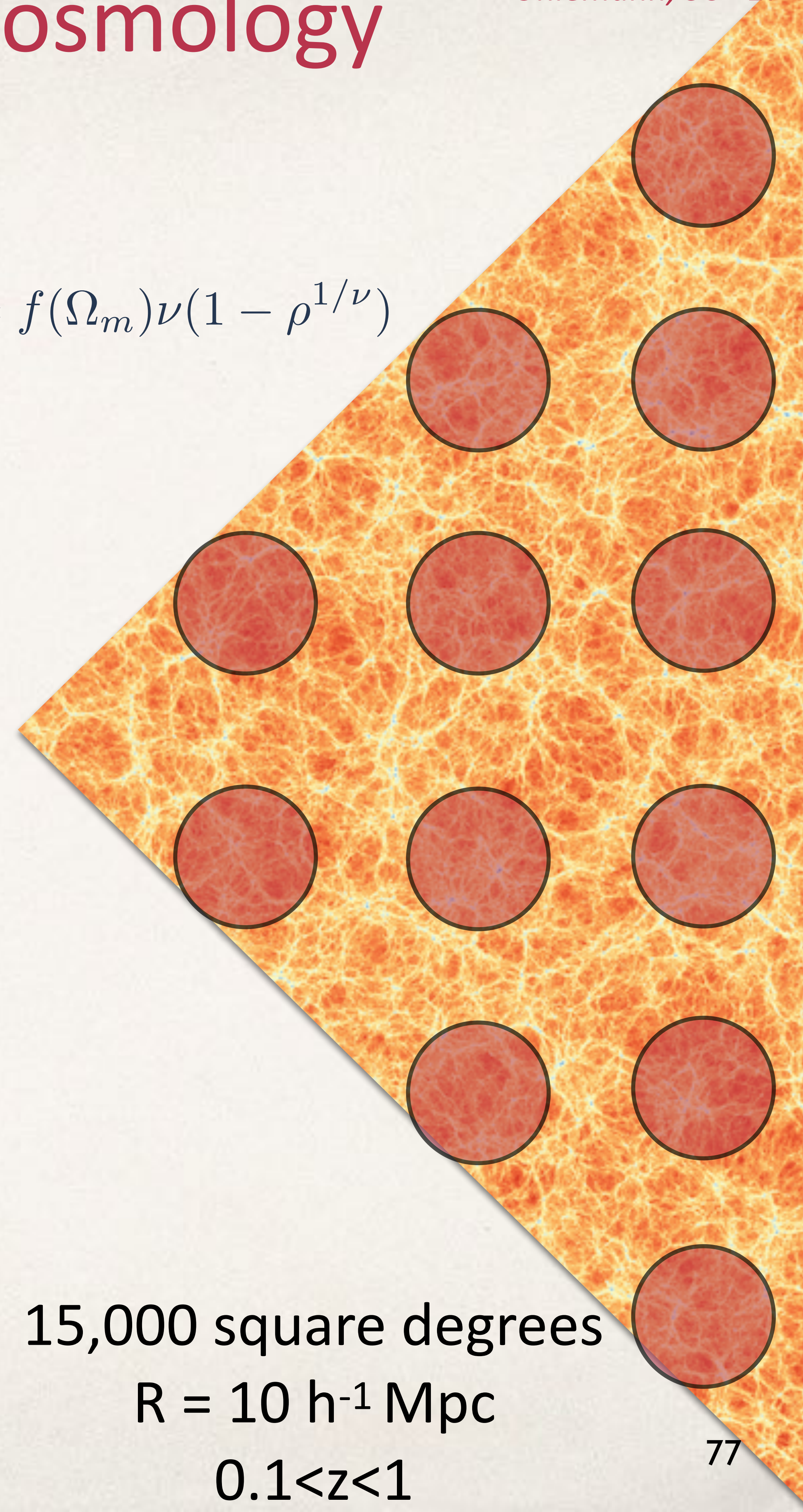
Hahn+15, velocity stat obtained by tessellation of the DM sheet

Uhlemann, SC+ 17

Use **velocity** PDF for cosmology



$$\theta_{\rm SC} = f(\Omega_m)\nu(1-\rho^{1/\nu})$$



Here the rest of the cosmology is fixed…

15,000 square degrees

R = 10 h^{-1} Mpc

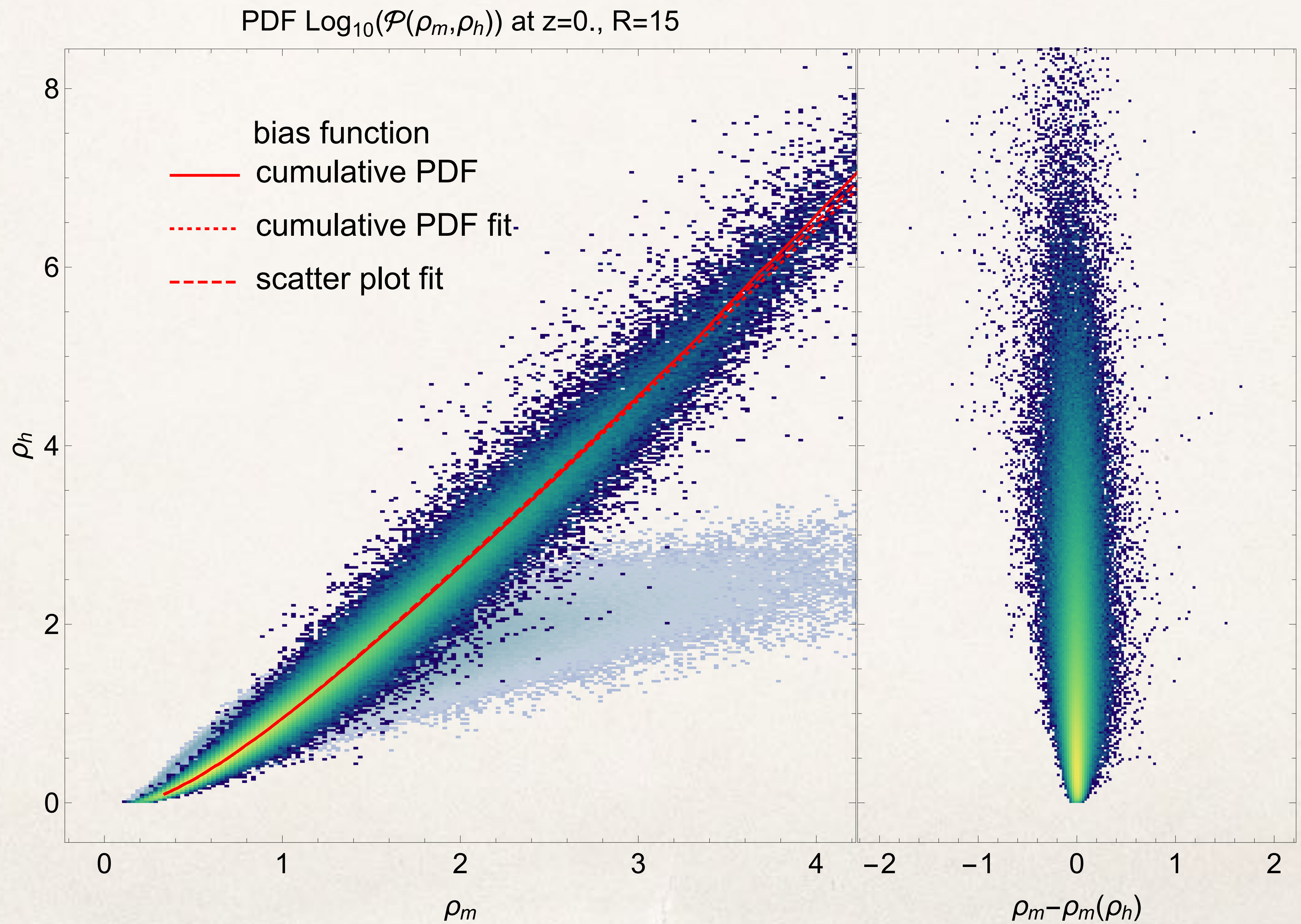
0.1<z<1

How to deal with biased tracers?

Uhlemann, Feix, SC+,17

Halo bias can be accounted for and marginalised over for cosmological experiments…
We use a quadratic log bias model:

$$\log \rho_m = b_0 + \beta_1 \sigma \log \rho_h + \beta_2 \sigma \log^2 \rho_h$$



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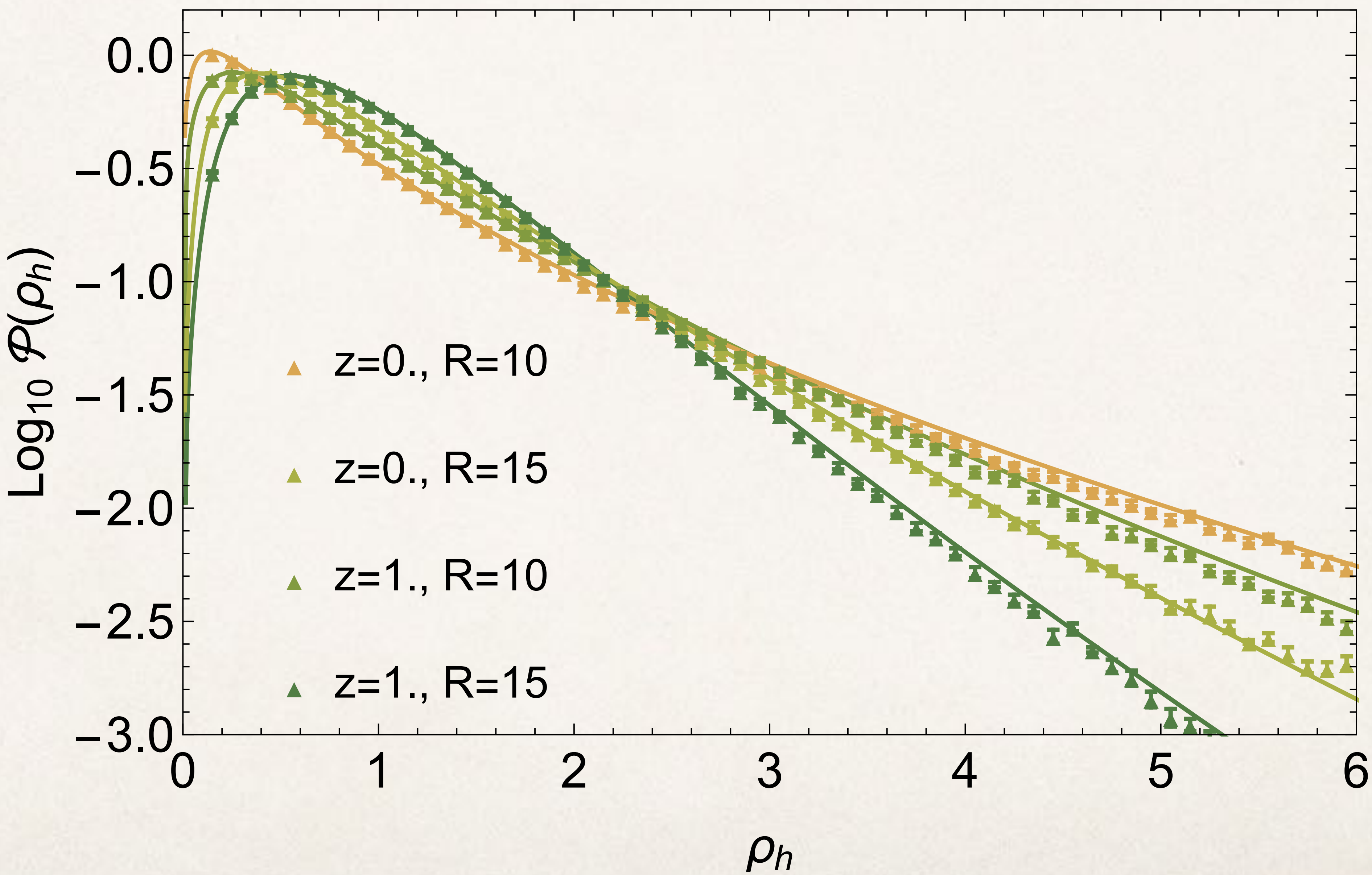

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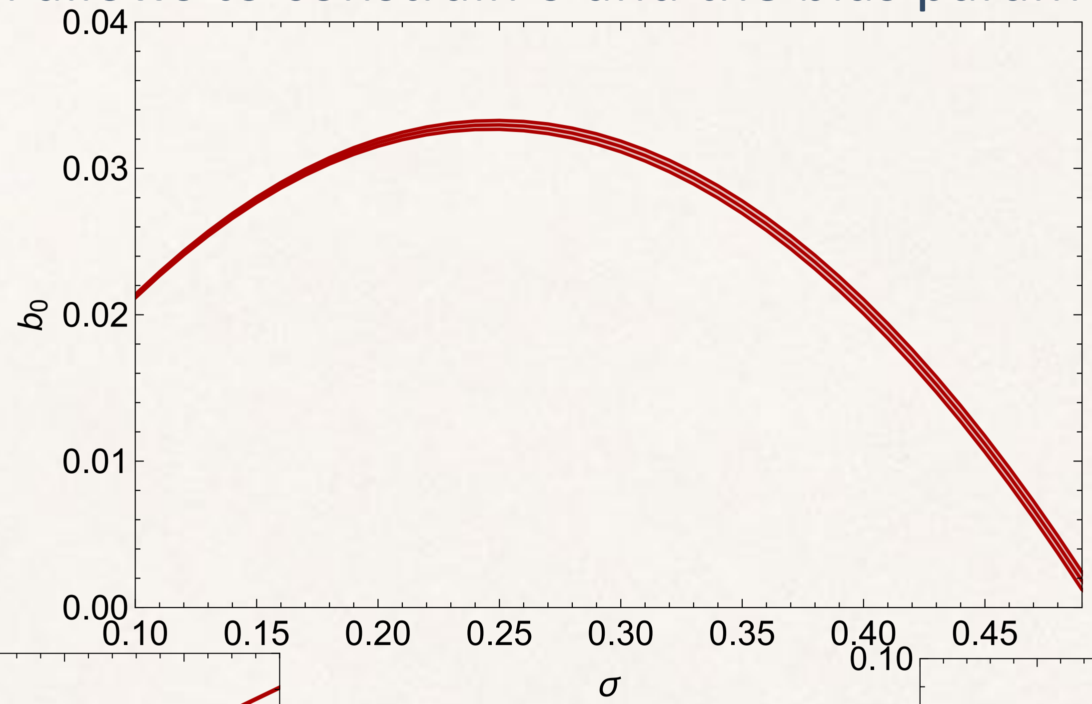
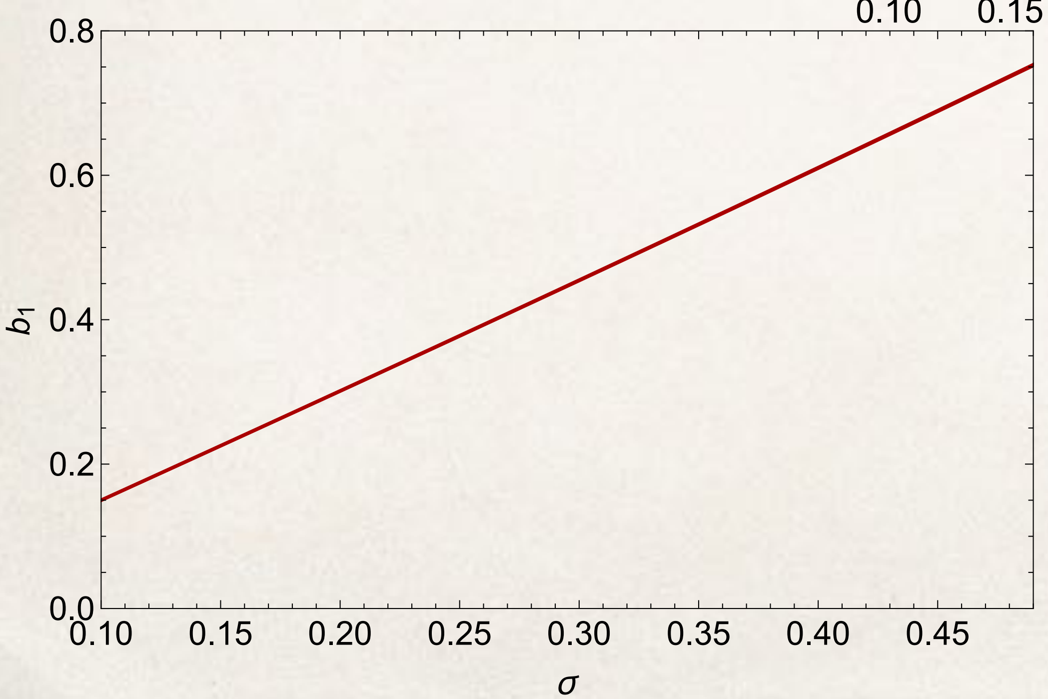
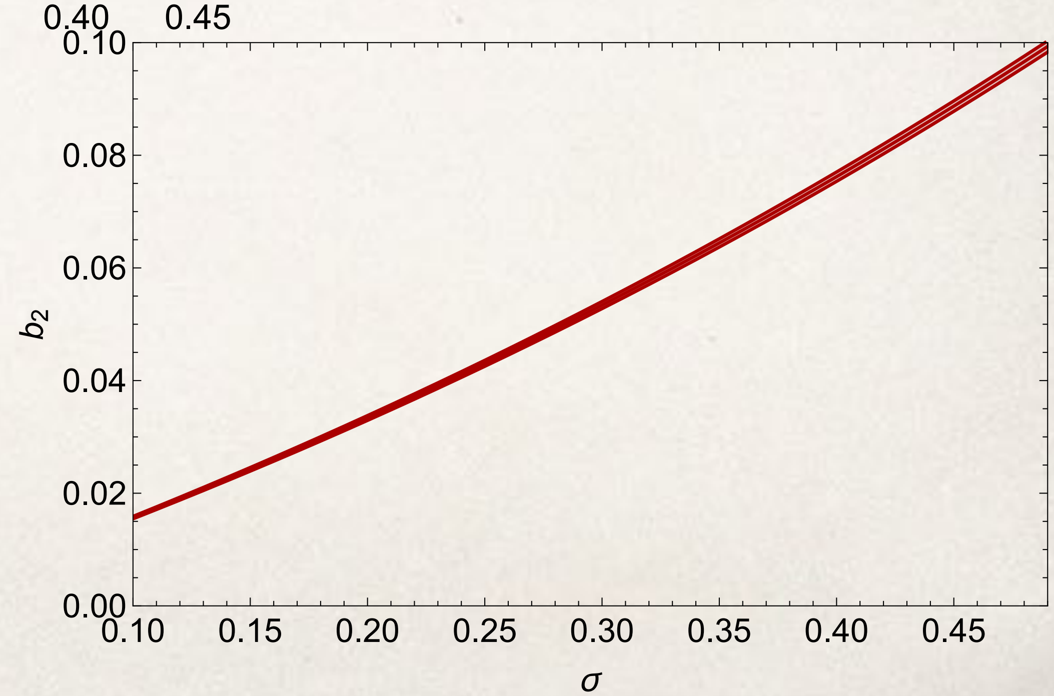
Measuring the PDF then allows to constrain σ and the bias parameters:





DEGENERATE!



Uhlemann, Feix, SC+,17

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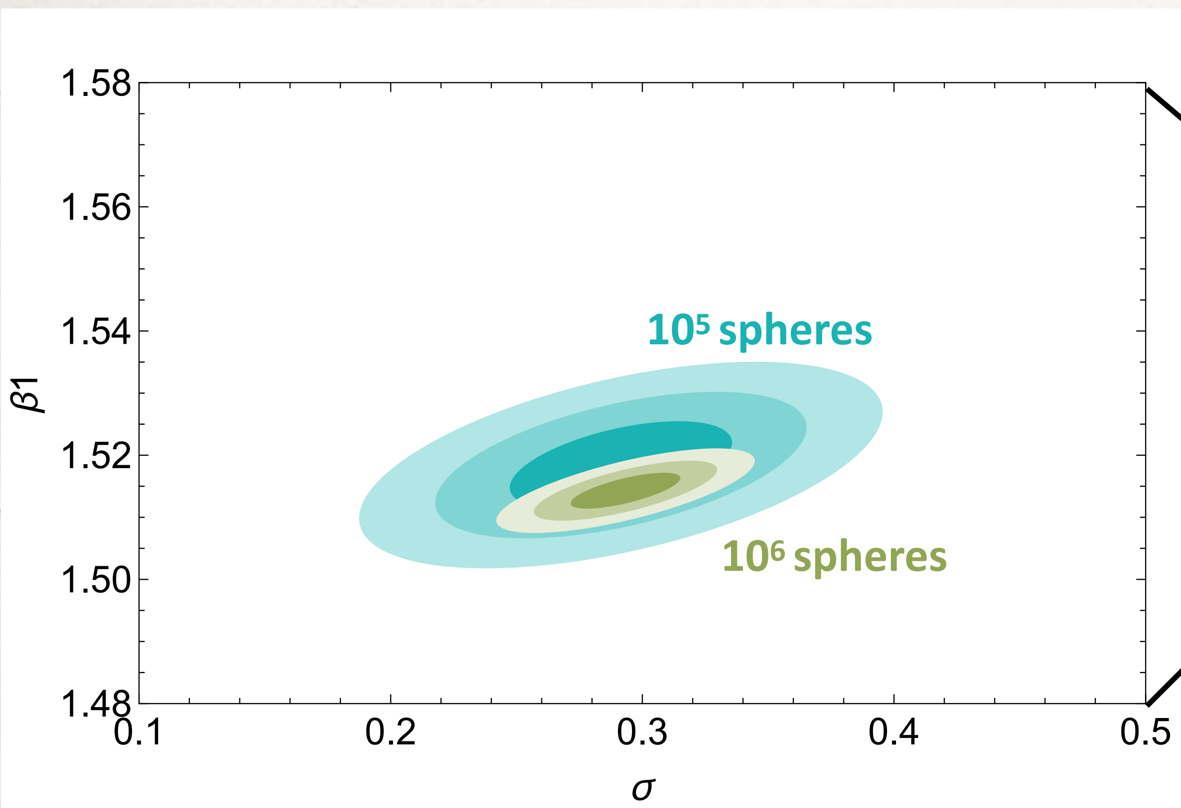
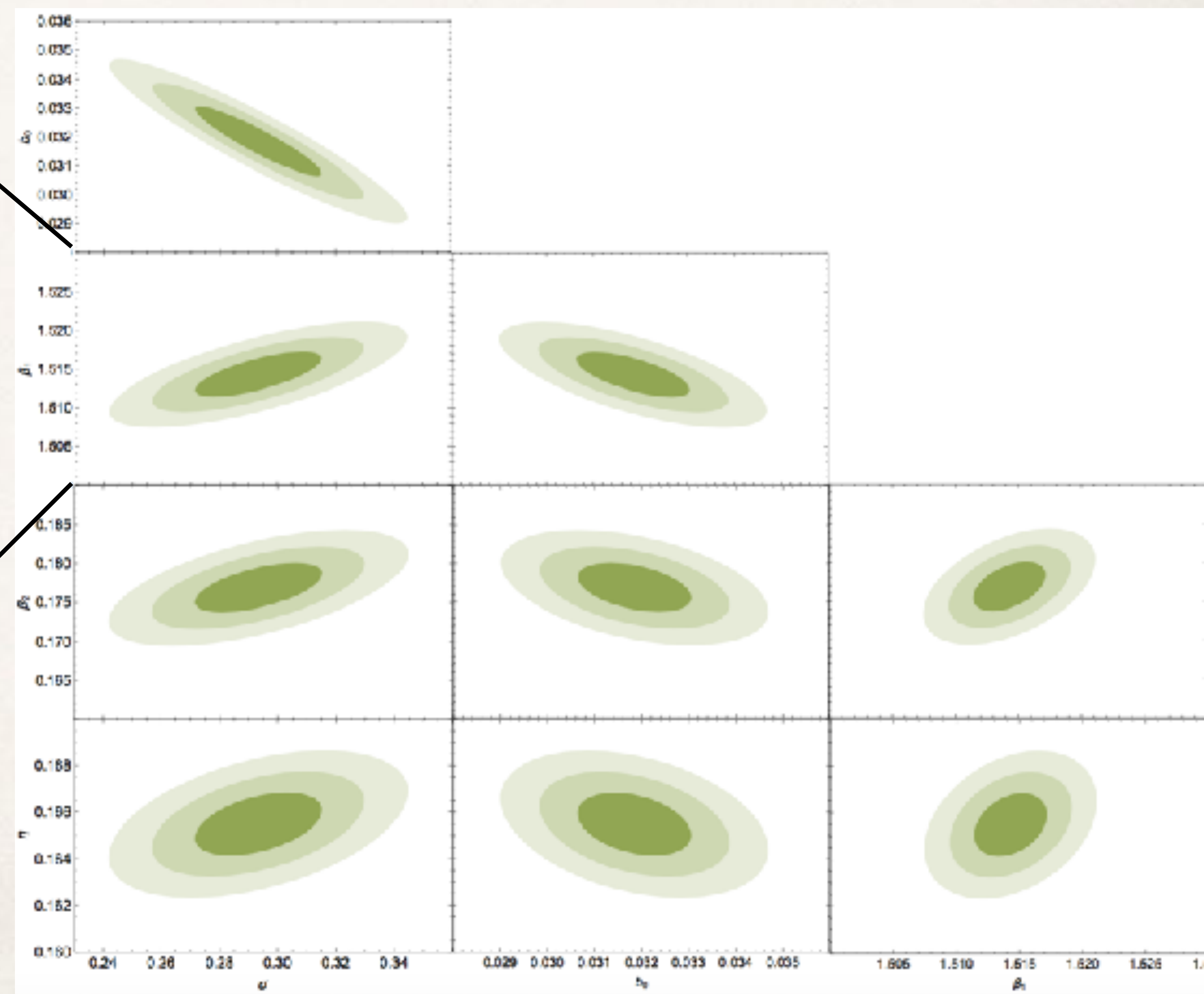
Measuring the PDF then allows to constrain σ and the bias parameters:

+ 2pt PDF



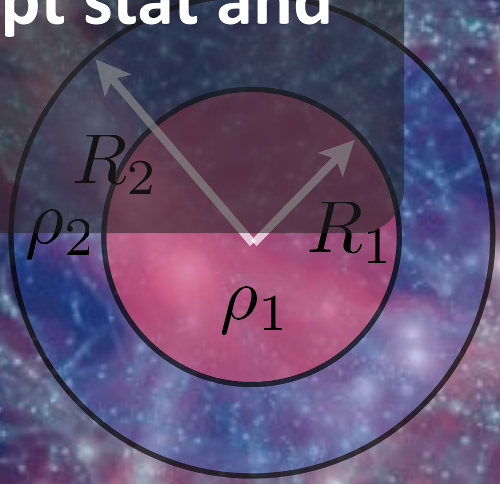


Conclusion:

Multi-scale densities PDF can be predicted in the mildly non-linear regime with surprising accuracy e.g <1% on P(ρ) for σ=O(1) even in the rare event tails, thanks to large deviations theory.

Predictions are fully analytical and explicitly cosmology-dependent!

We can have a model for biased tracers of the density, velocities, 2pt stat and (in progress) cosmic shear maps.



horizon-AGN

Large deviation principle:

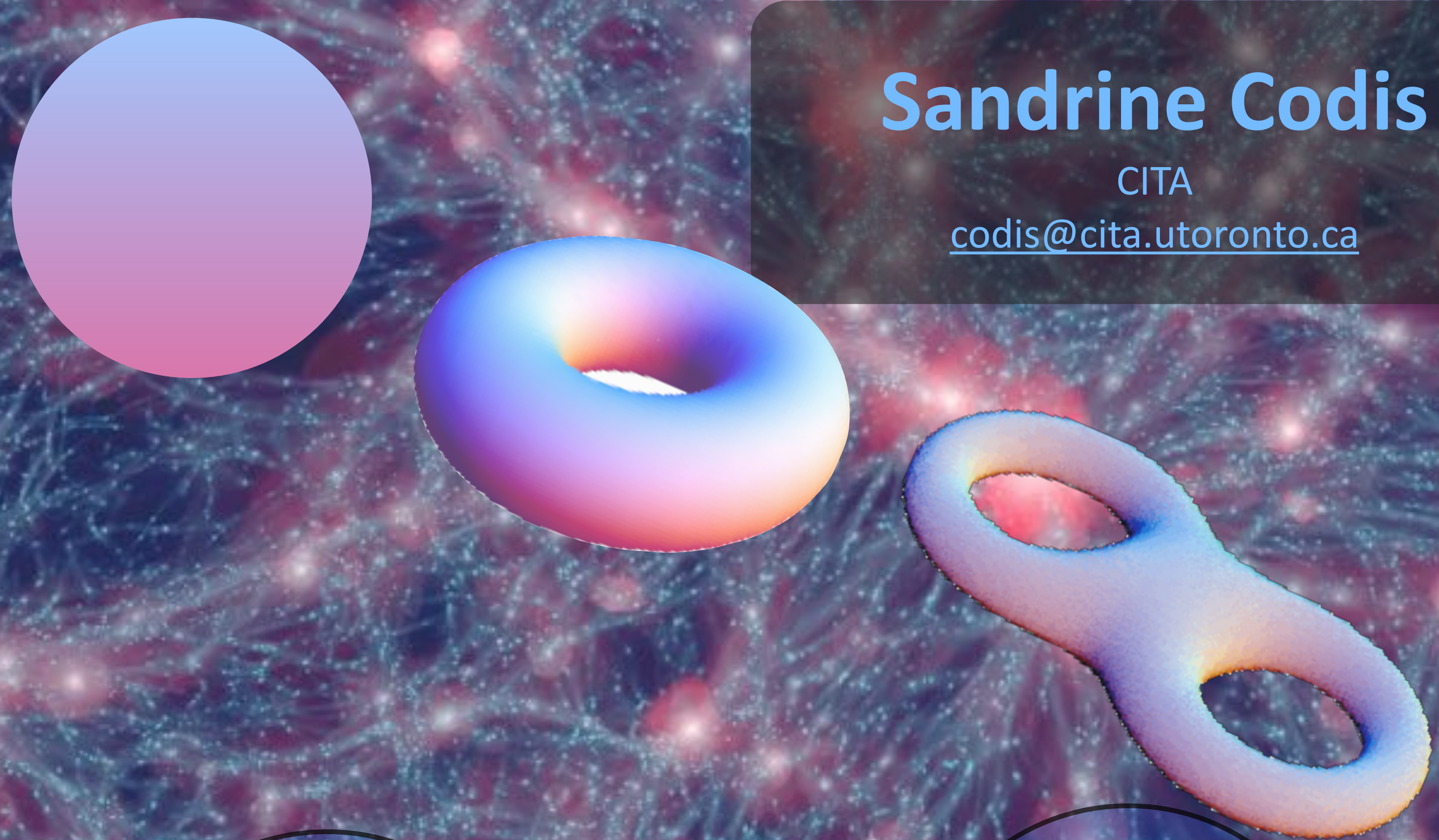
an unlikely fluctuation is brought about by the least unlikely of all unlikely paths.

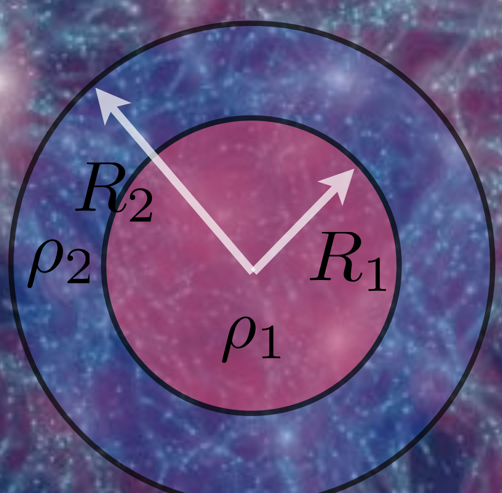
Statistiques d’ordre supérieur
TD

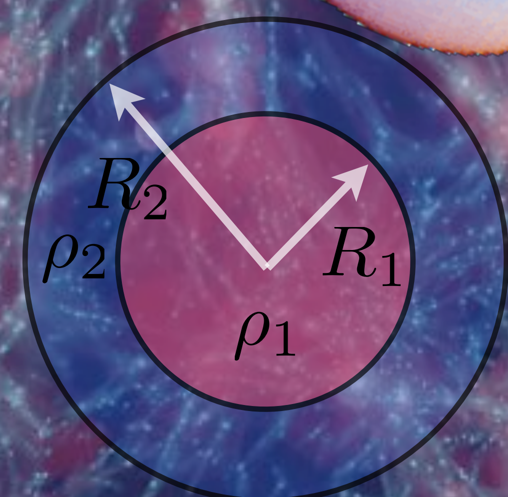
Sandrine Codis

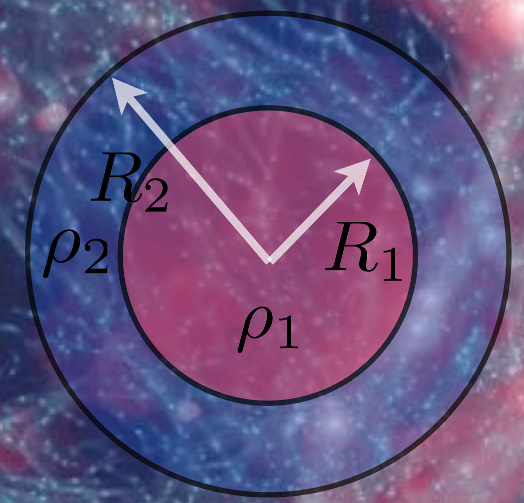
CITA

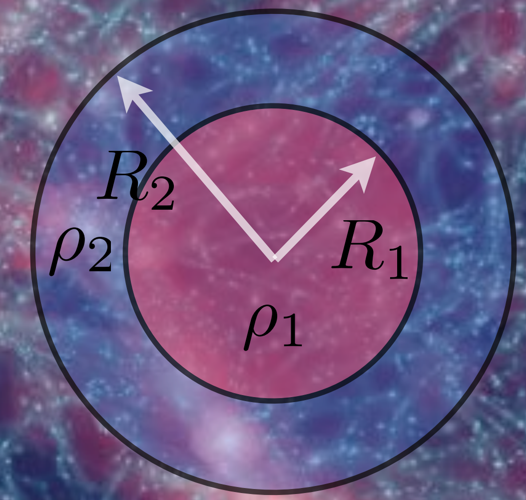
codis@cita.utoronto.ca

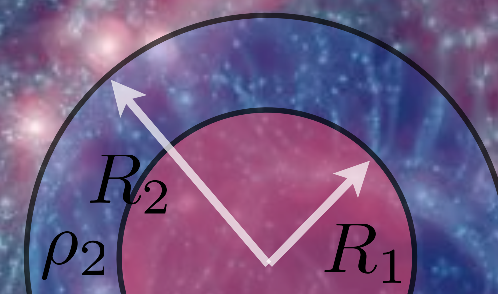












horizon-AGN

Statistiques d'ordre supérieur : TD

Exercice 1: PDF du champ de densité cosmique

-Mesurer la PDF de la densité aux trois redshifts proposés.
-Comparer à une Gaussienne, un développement de Edgeworth tronqué à n=3 puis n=4. On utilisera ici deux méthodes: les cumulants mesurés et les cumulants à l'ordre des arbres
-Utiliser le code LSSFast pour calculer la prédiction dans le régime de grande déviation et comparer le résultat à la mesure et au développement de Edgeworth.

Exercice 2: Topologie

-Ecrire la PDF jointe d'un champ Gaussien aléatoire 2D δ et de ses derivées premières et secondes.
-Trouver quelles combinaisons lineaires des variables sont décorrélées.
-Ecrire le developpement de Gram-Charlier dans ces variables
-Calculer le genus 2D Gaussien
-Calculer sa premiere correction non-Gaussienne