

Constraining Inflationary Histories, **now** & then

Inflation Now $1+w(a) = \epsilon_s f(a/a_{\Lambda\text{eq}}; a_s/a_{\Lambda\text{eq}}; \xi_s)$

goes to $\epsilon(a)_{x3/2} = 3(1+q)/2 \sim 1$ good e-fold. only ~ 2 params

$\epsilon = -d \ln H / d \ln a \sim 0$ to 2 to 3/2 to $\sim .4$ now, on its way to 0?

cf. $w(a)$: w_0, w_a, w in z-bins, w in modes, $\epsilon(a)$: in modes, jerk

Inflation Then $\epsilon(k) = (1+q)(a) =$ mode expansion in resolution ($\ln H a \sim \ln k$)
 $\sim r/16$ (Tensor/Scalar Power & gravity waves)

Cosmic Probes Now CMB(Apr08), CFHTLS SN(192), WL(Apr07), LSS/BAO, Ly α

Cosmic Probes Then JDEM-SN + DUNE-WL + Planck1

Zhiqi Huang, Bond & Kofman 08 $\epsilon_s = -0.06 \pm 0.20$ now, inflaton (potential gradient)²

to ± 0.07 then Planck1+JDEM SN+DUNE WL, weak $a_s < 0.3$ now < 0.21 then

INFLATION NOW PROBES NOW

➤ **Cosmological
Constant ($w=-1$)**

➤ **Quintessence
($-1 \leq w \leq 1$)**

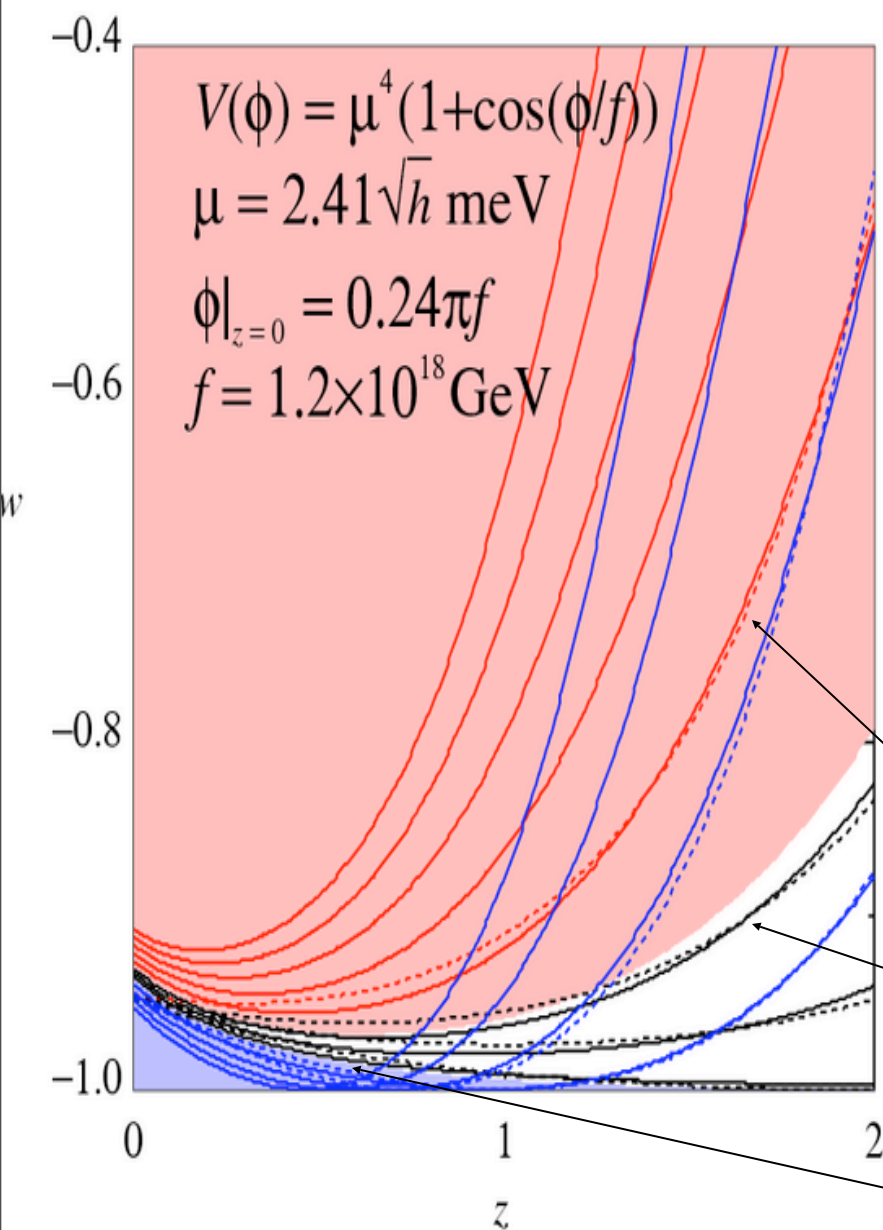
➤ **Phantom field
($w \leq -1$)**

➤ **Tachyon fields
($-1 \leq w \leq 0$)**

➤ **K-essence**

(no prior on w)

w-trajectories for $V(\phi)$: pNGB example e.g.sorbo et07



For a given quintessence potential $V(\phi)$, we set the “initial conditions” at $z=0$ and evolve backward in time.

w-trajectories for $\Omega_m(z=0) = 0.27$ and $(V'/V)^2/(16\pi G)(z=0) = 0.25$, \sim the 1-sigma limit, varying the initial kinetic energy $w_0 = w(z=0)$

Dashed lines: 2-param approximation

Wild rise solutions

Slow-to-medium-roll solutions

Complicated scenarios:
roll-up then roll-down

Approximating Quintessence for Phenomenology

Zhiqi Huang, Bond & Kofman 08

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad + \quad \text{Friedmann Equations + DM+B}$$

quintessence

$$\begin{cases} d\theta/dN = \sqrt{3\epsilon_V \Omega_\phi} \cos \theta - \frac{3}{2} \sin 2\theta, \\ d\Omega_\phi/dN = 3\Omega_\phi(1 - \Omega_\phi) \cos 2\theta, \\ d\epsilon_V/dN = -\sqrt{12\epsilon_V \Omega_\phi} (\eta_V - 2\epsilon_V) \sin \theta, \end{cases}$$

$$1+w=2\sin^2 \theta$$

phantom

$$\begin{cases} d\theta/dN = \sqrt{-3\epsilon_V \Omega_\phi} \cosh \theta - \frac{3}{2} \sinh 2\theta, \\ d\Omega_\phi/dN = 3\Omega_\phi(1 - \Omega_\phi) \cosh 2\theta, \\ d\epsilon_V/dN = \sqrt{-12\epsilon_V \Omega_\phi} (\eta_V - 2\epsilon_V) \sinh \theta, \end{cases}$$

Include a $w < -1$ phantom field, via a negative kinetic energy term

$$1+w=-2\sinh^2 \theta$$

$$\theta \equiv \begin{cases} \sin^{-1} \frac{\dot{\phi}}{\sqrt{2\rho_\phi}} & \text{for quintessence,} \\ \sinh^{-1} \frac{\dot{\phi}}{\sqrt{2\rho_\phi}} & \text{for phantom.} \end{cases}$$

$$\Omega_\phi \equiv \frac{8\pi G \rho_\phi}{3H^2}, \epsilon_V \equiv \frac{\beta}{16\pi G} \left(\frac{V'}{V}\right)^2, \eta_V \equiv \frac{\beta}{8\pi G} \frac{V''}{V}$$

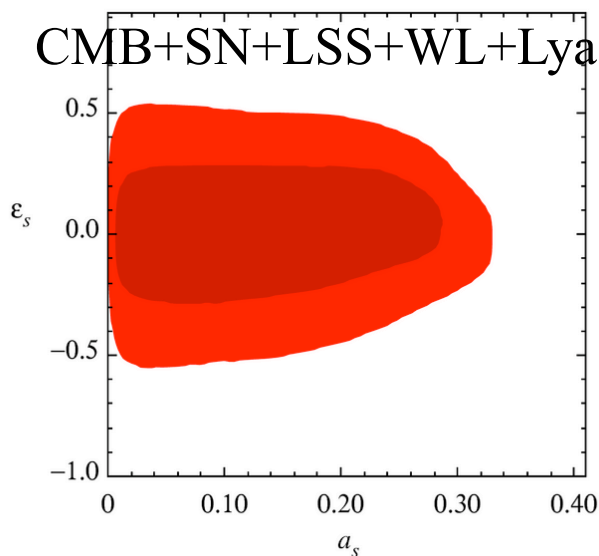
slow-to-moderate roll conditions

$$\begin{cases} \frac{1}{2}\dot{\phi}^2 \ll V(\phi), \\ \varepsilon_s \lesssim 1, \\ \eta_s \lesssim 1, \end{cases} \quad \text{at } 0 < z < 2.$$

$1+w < 0.3$ (for $0 < z < 10$) gives a **2-parameter** model (a_s and ε_s):

$$a_{eq} \equiv \left(\frac{\Omega_{m0}}{\Omega_{\Lambda 0}}\right)^{\frac{1}{3}} \sim 0.7,$$

$$w(a) = -1 + \frac{2\varepsilon_s}{3} \left\{ \frac{1}{\sqrt{|\varepsilon_s|}} \left(\frac{a_s}{a}\right)^3 + \sqrt{1 + \left(\frac{a_{eq}}{a}\right)^3} - \left(\frac{a_{eq}}{a}\right)^3 \ln\left[\left(\frac{a}{a_{eq}}\right)^{\frac{3}{2}} + \sqrt{1 + \left(\frac{a}{a_{eq}}\right)^3}\right] \right\}^2 + O(\theta^3),$$



Early-Exit Scenario: scaling regime info is lost by Hubble damping, i.e. small a_s

$1+w < 0.2$ (for $0 < z < 10$) and gives a **1-parameter** model ($a_s \ll 1$):

$$w(a) = -1 + \frac{2\varepsilon_s}{3} \left\{ \sqrt{1 + \left(\frac{a_{eq}}{a}\right)^3} - \left(\frac{a_{eq}}{a}\right)^3 \ln\left[\left(\frac{a}{a_{eq}}\right)^{\frac{3}{2}} + \sqrt{1 + \left(\frac{a}{a_{eq}}\right)^3}\right] \right\}^2.$$

3-parameter parameterization

next order corrections:

$\Omega_m(a)$ (depends on ϵ_s redefines a_{eq})

$\epsilon_V = \epsilon_s(a)$ (adds new ζ_s parameter)

enforce asymptotic kinetic-dominance $w=1$ (add a_s power suppression)

refine the fit to encompass even baroque trajectories.

$$\sqrt{|\epsilon_V|} = \sqrt{|\epsilon_s|} \left[1 + \zeta_s \left(\left(\frac{a}{a_{eq}} \right)^{\frac{3}{2}} - 1 \right) \right]$$

this choice is analytic. the correction on w is only ~ 0.01

3-parameter parameterization

$$\begin{aligned}
 w(a) = & -1 + \frac{2\epsilon_s}{3} \left\{ \frac{\left(\frac{a_s}{a}\right)^{3-3.6a_s|\epsilon_s|(1-\Omega_{m0})}}{\sqrt{1 + \frac{\epsilon_s}{3|\epsilon_s|} \left(\frac{a_s}{a}\right)^{6-7.2a_s|\epsilon_s|(1-\Omega_{m0})}}} \frac{1}{\sqrt{|\epsilon_s|}} \right. \\
 & + \left[\sqrt{1 + \left(\frac{a_{eq}}{a}\right)^3} - \left(\frac{a_{eq}}{a}\right)^3 \ln\left(\left(\frac{a}{a_{eq}}\right)^{\frac{3}{2}} + \sqrt{1 + \left(\frac{a}{a_{eq}}\right)^3}\right) \right] (1 - \zeta_s) \\
 & + 0.36\epsilon_s(1 - \Omega_{m0}) \frac{\left(\frac{a}{a_{eq}}\right)^2}{1 + \left(\frac{a}{a_{eq}}\right)^4} \left[0.9 - 0.7\frac{a}{a_{eq}} - 0.045\left(\frac{a}{a_{eq}}\right)^2 \right] \\
 & \left. + \frac{2\zeta_s}{3} \left[\sqrt{1 + \left(\frac{a}{a_{eq}}\right)^3} - 2\left(\frac{a_{eq}}{a}\right)^3 \left(\sqrt{1 + \left(\frac{a}{a_{eq}}\right)^3} - 1 \right) \right] \right\}^2
 \end{aligned}$$

where

$$a_{eq} \equiv \left(\frac{\Omega_{m0}}{1 - \Omega_{m0}} \right)^{\frac{1}{3[1-0.36\epsilon_s(1-\Omega_{m0})]}}$$

$$a_s \geq 0$$

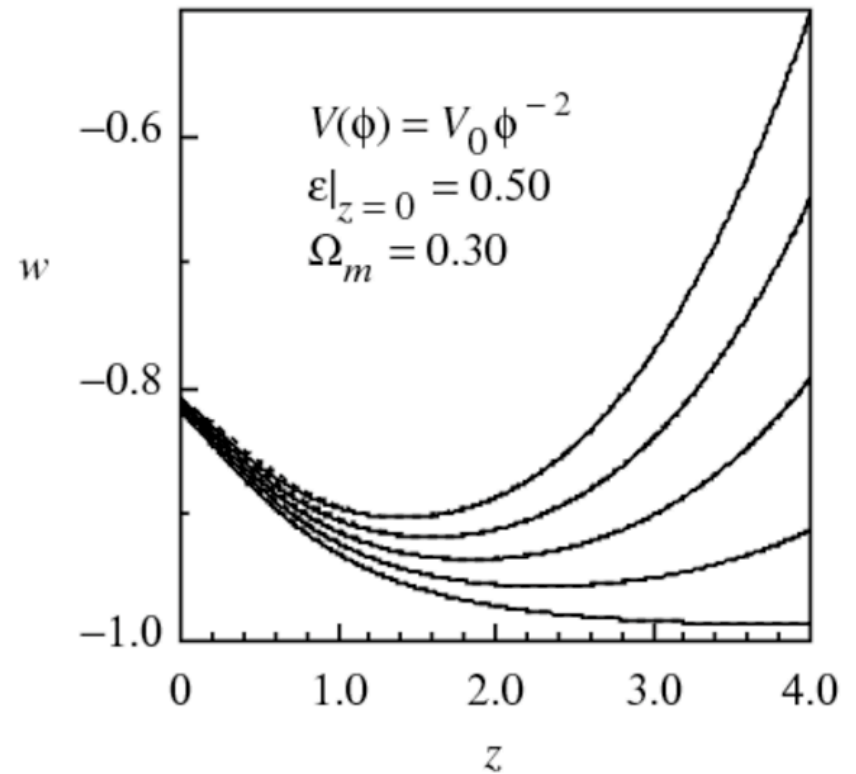
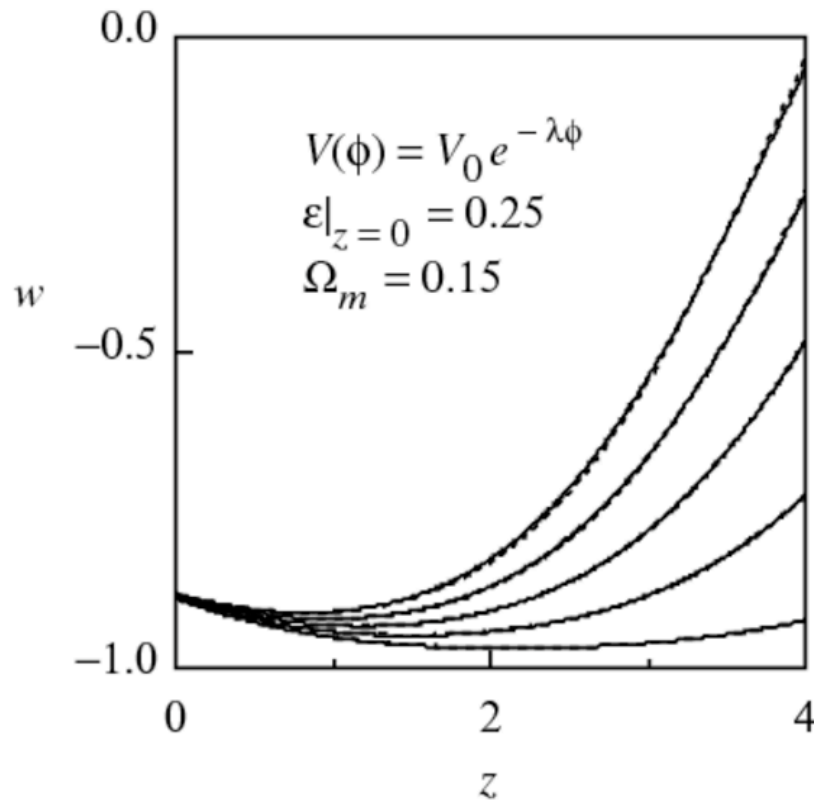
$$-1 < \zeta_s < 1$$

3-parameter fitting

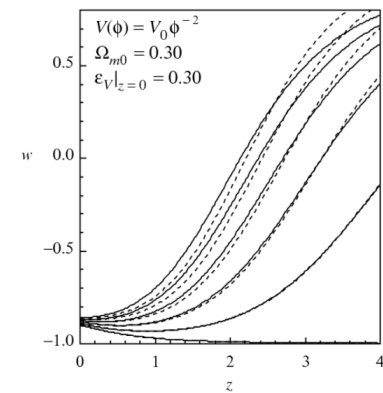
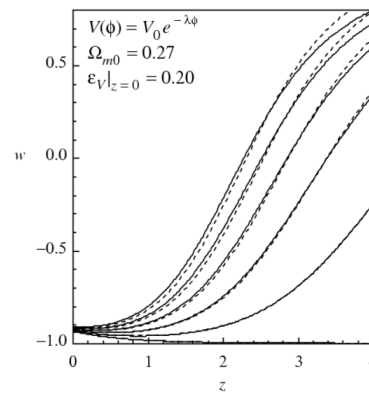
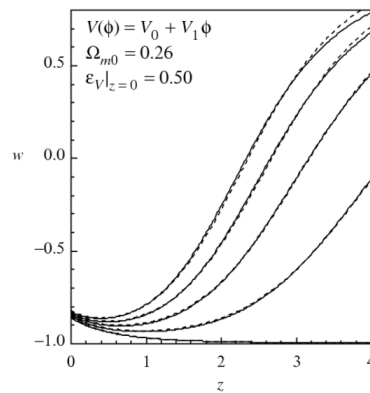
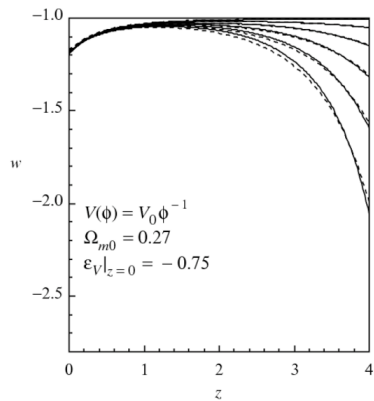
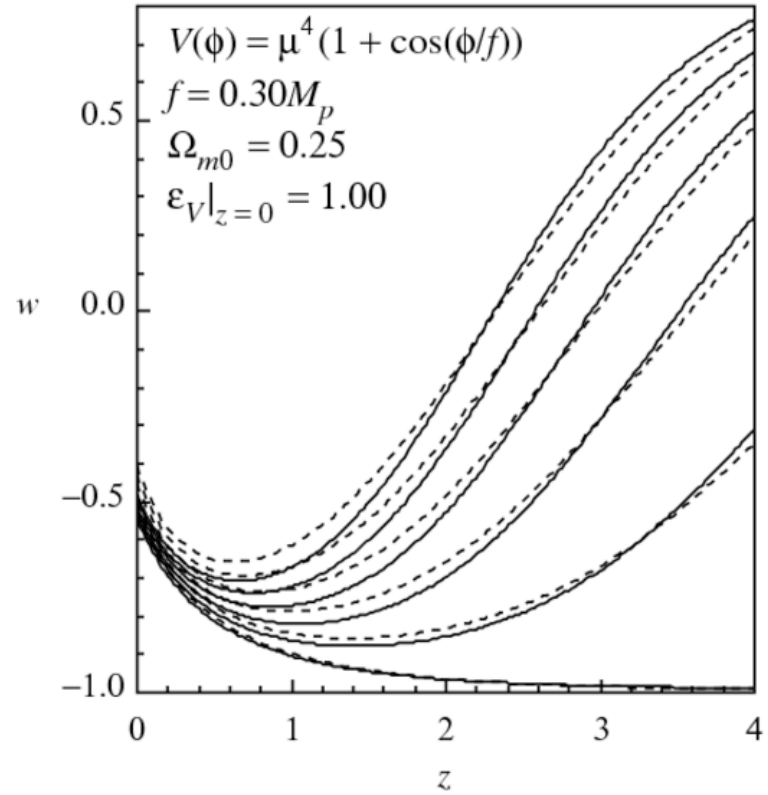
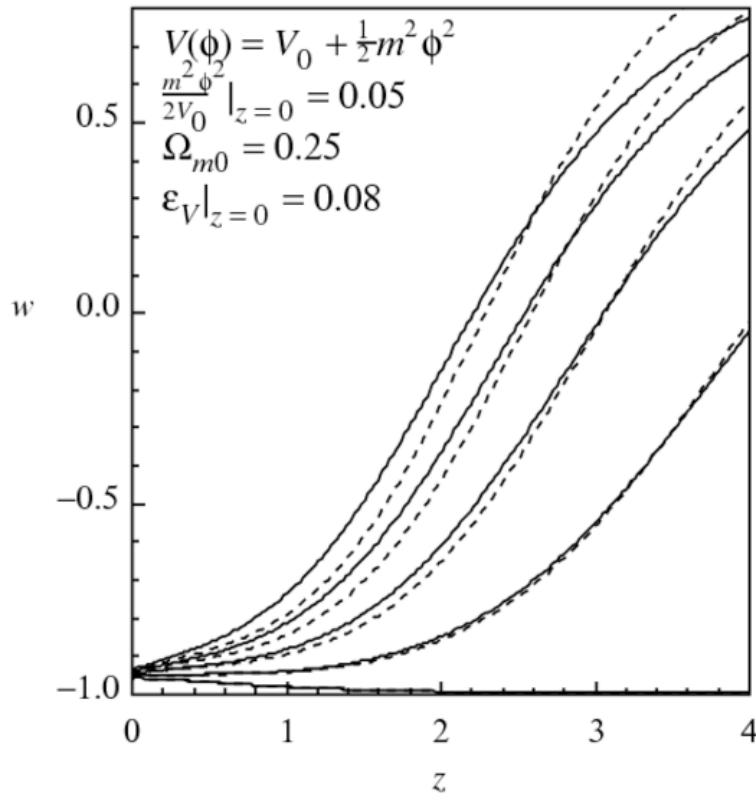
ε_s & ζ_s calculated from ε trajectory (linear least square)

a_s is χ^2 -fit

- **Perfectly fits slow-to-moderate roll**



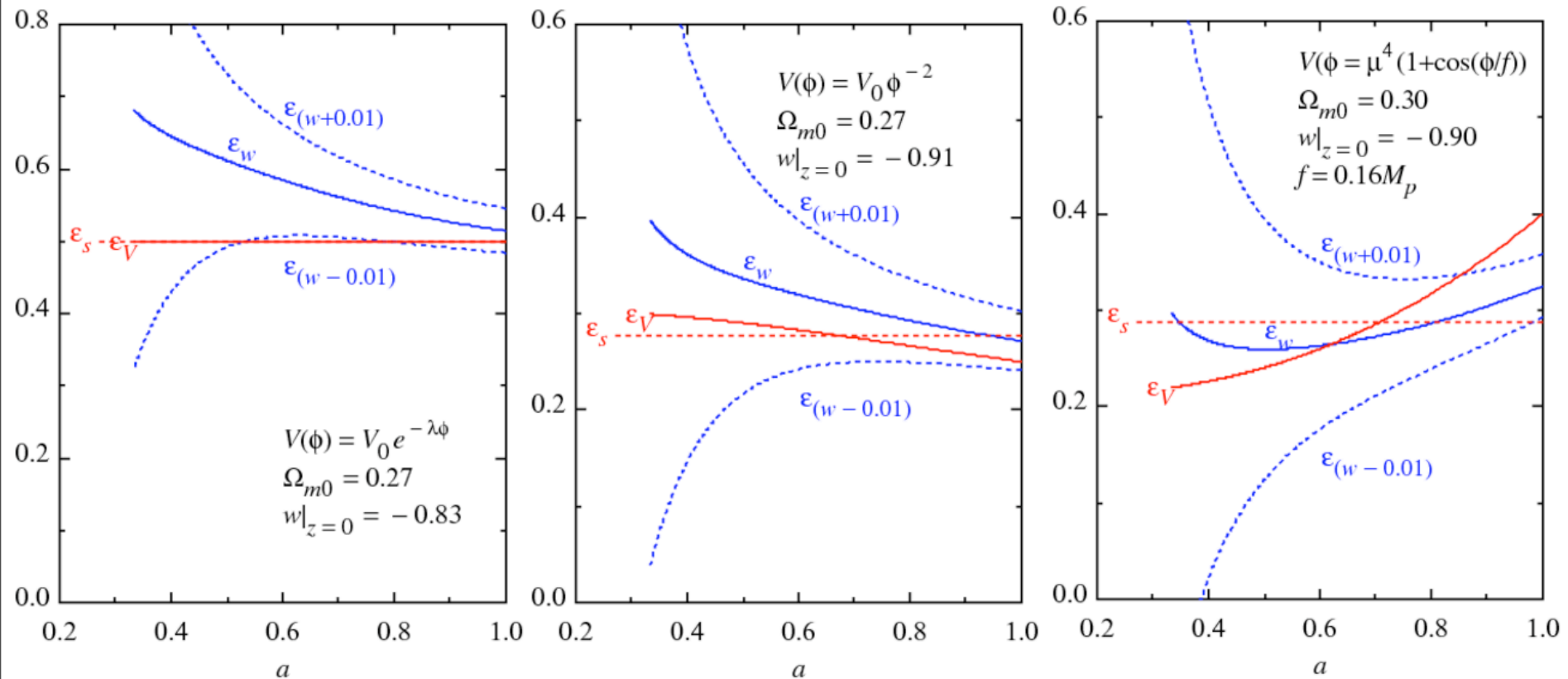
fits wild rising trajectories



ϵ_v trajectories are slowly varying: why the fits are good

Dynamical $\epsilon_w = (1+w)(a)/f(a)$ cf. shape $\epsilon_V = (V'/V)^2 (a) / (16\pi G)$

$\epsilon_s = \epsilon_v$ uniformly averaged over $0 < z < 2$ in a.

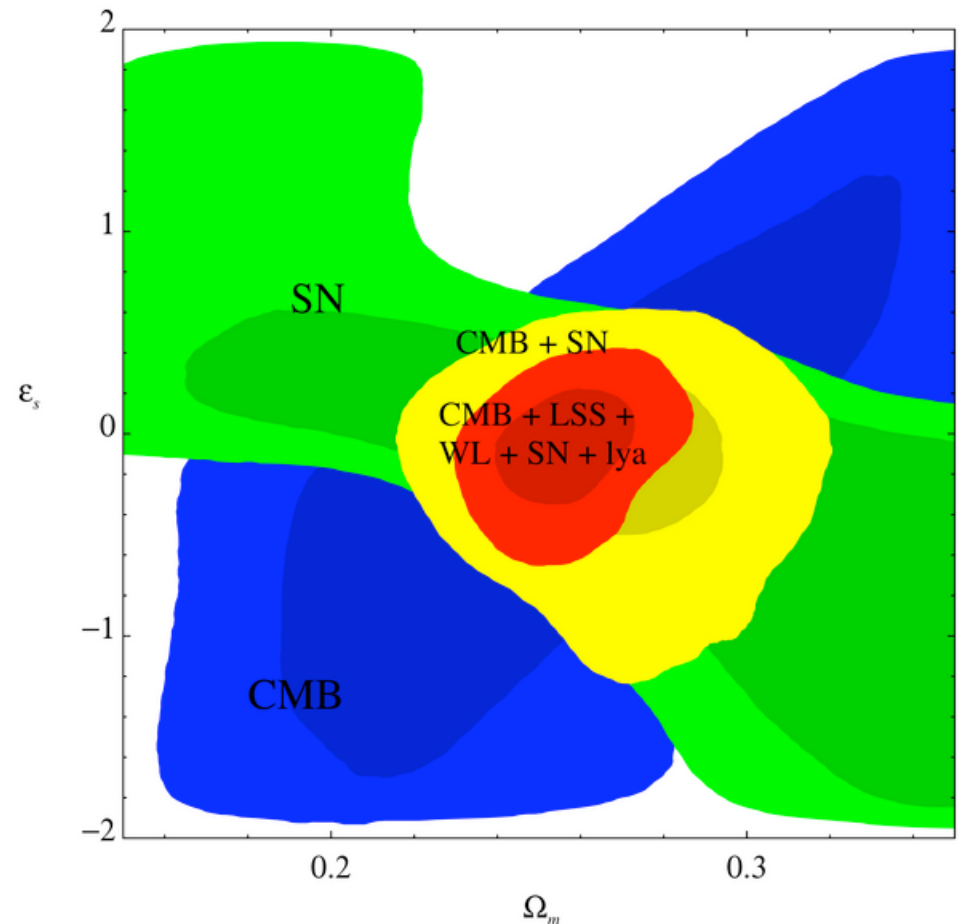
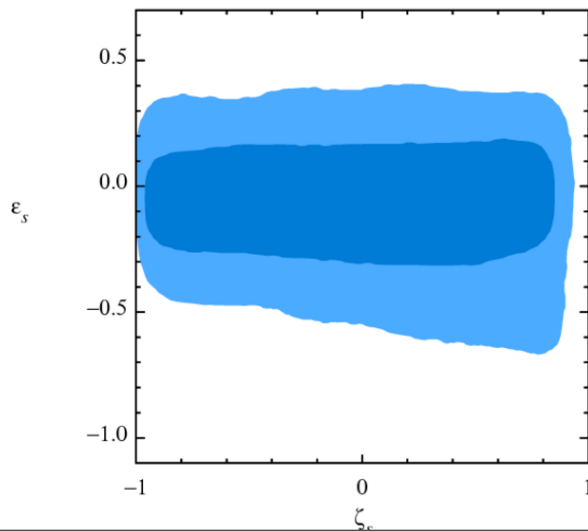
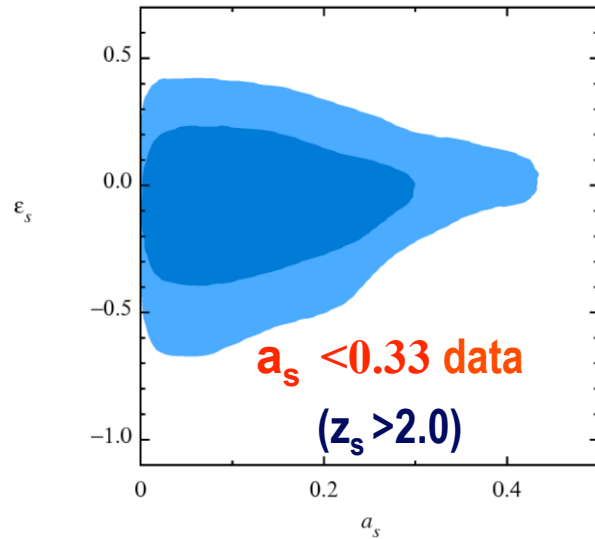


Measuring the 3 parameters with Apr08 data

- Use 3-parameter formula over $0 < z < 4$ & $w(z > 4) = w_h$

$$\epsilon_s = -0.00 \quad +0.20 \quad -0.20 \quad \text{1params}$$

$$\epsilon_s = -0.06 \quad +0.19 \quad -0.21 \quad \text{3params}$$



Standard Parameters of Cosmic Structure Formation

$$\begin{array}{ccccccc}
 \Omega_k & \Omega_b h^2 & \Omega_{dm} h^2 & \Omega_\Lambda & \tau_c & \ln A_s & n_s & r = A_t / A_s \\
 1+w_0, w_a & & & & & & dn_s / d \ln k & n_t \\
 \theta \sim l_s^{-1} & & & & & & & \sim \ln \sigma_8^2
 \end{array}$$

New Parameters of Cosmic Structure Formation

$$\begin{array}{ccc}
 1+w(a) & \epsilon(k), \quad k \approx Ha & \ln H(k_p) \\
 \epsilon_s f(a/a_{\Lambda eq}, a_s/a_{\Lambda eq}, \xi_s) & \ln P_s(k) & \ln P_t(k)
 \end{array}$$

+ subdominant isocurvature/cosmic string/ tSZ

CMB/LSS Phenomenology [CITA/CifAR there](#)

[CITA/CifAR here](#)

- Bond
- Contaldi
- Lewis
- Sievers
- Pen

• McDonald

• Majumdar

• Nolta

• Iliev

• Kofman

• Vaudrevange

• Huang

Prokushkin

- Dalal
- Dore
- Kesden
- MacTavish
- Pfrommer
- Shirokov

[& Exptal/Analysis/Phenomenology
Teams here & there](#)

- Boomerang03 (98)
- Cosmic Background Imager1/2
- Acbar07
- WMAP (Nolta, Dore)
- CFHTLS – WeakLens
- CFHTLS - Supernovae
- RCS2 (RCS1; Virmos-Descart)

[UofT here](#)

- Netterfield
- Carlberg
- Yee

- Mivelle-Deschenes (IAS)
- Pogosyan (U of Alberta)
- Myers (NRAO)
- Holder (McGill)
- Hoekstra (UVictoria)
- van Waerbeke (UBC)

Parameter data now: **CMBall_pol**

SDSS P(k), BAO, 2dF P(k)

**Weak lens (Virgos/RCS1, CFHTLS,
RCS2) ~100sqdeg Benjamin etal. aph/
0703570v1**

Lya forest (SDSS)

**SN1a “gold”(192,15 z>1 to 242+)
CFHTLS**

**then: ACT (SZ), Spider, Planck,
21(1+z)cm GMRT,SKA**

**COSMIC
PARAMETERS
NOW & THEN**

The Parameters of Cosmic Structure Formation

Cosmic Numerology: april08 cmb +LSS/WL/SN

wmap5

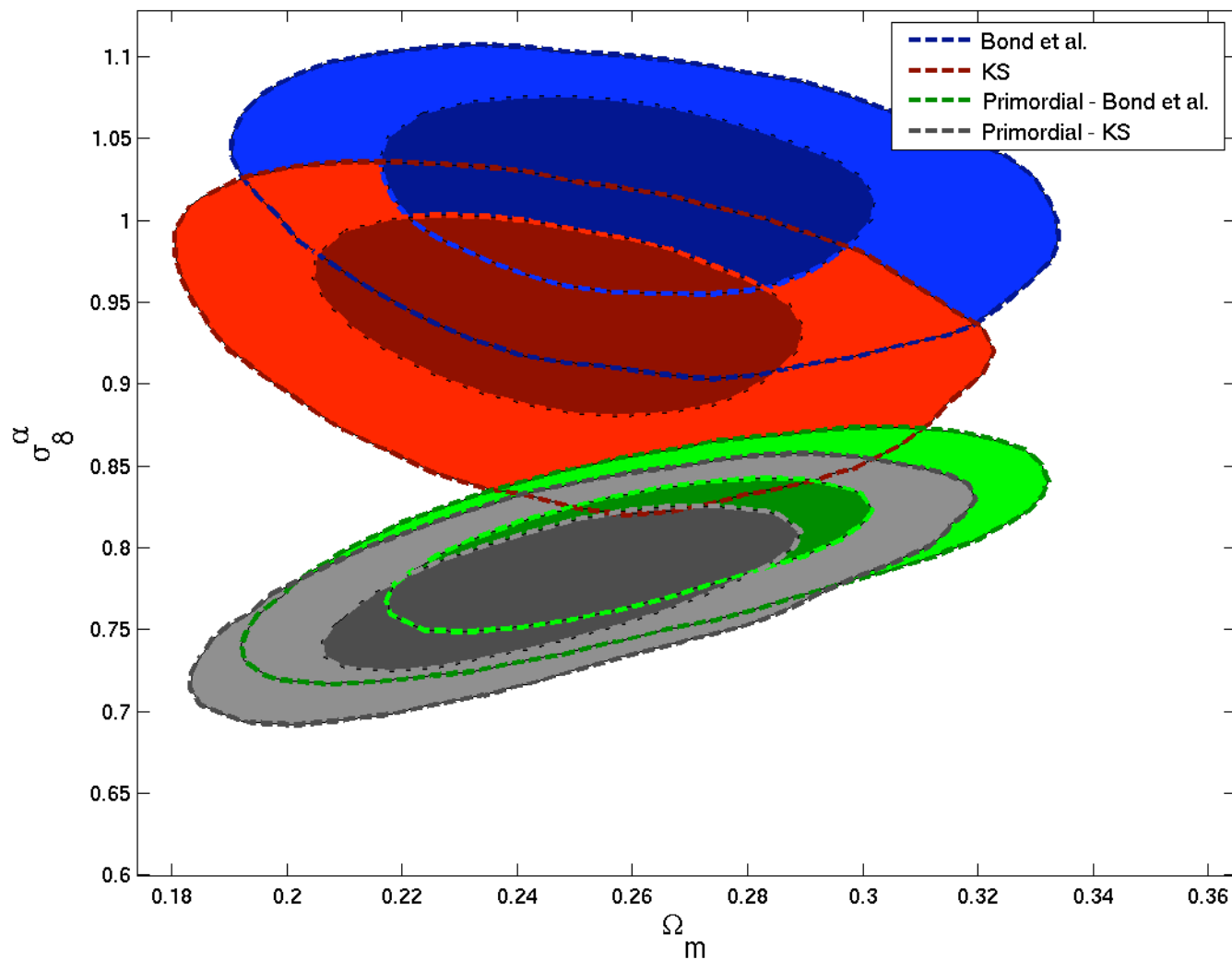
$$n_s = .976 \pm .011 \text{ (+-.005 Planck1)}$$

$$r = A_t / A_s < 0.33_{\text{cmb}} \text{ 95\% CL (+-.03 P1)}$$

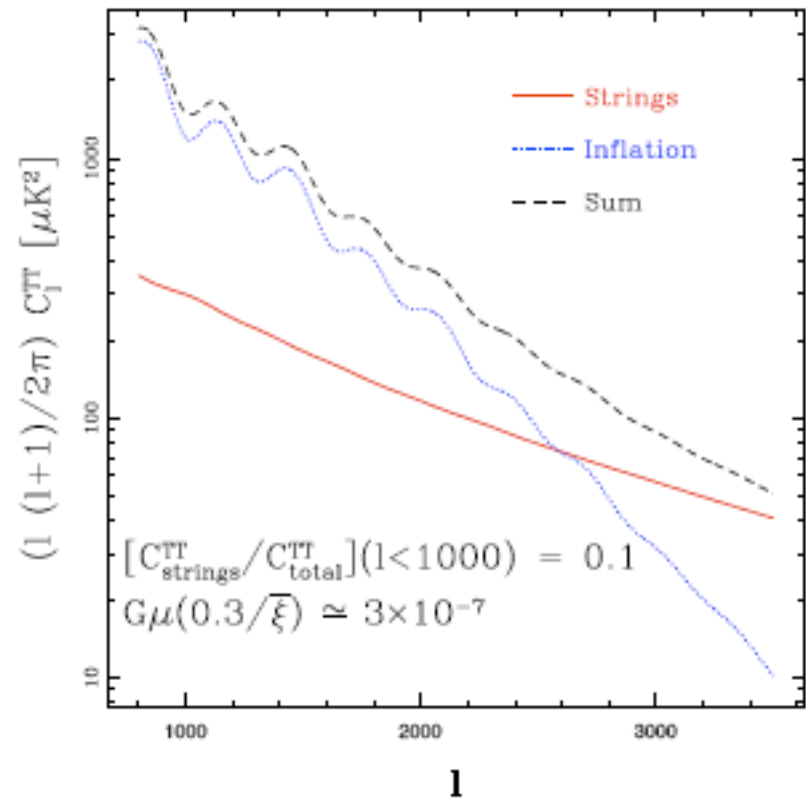
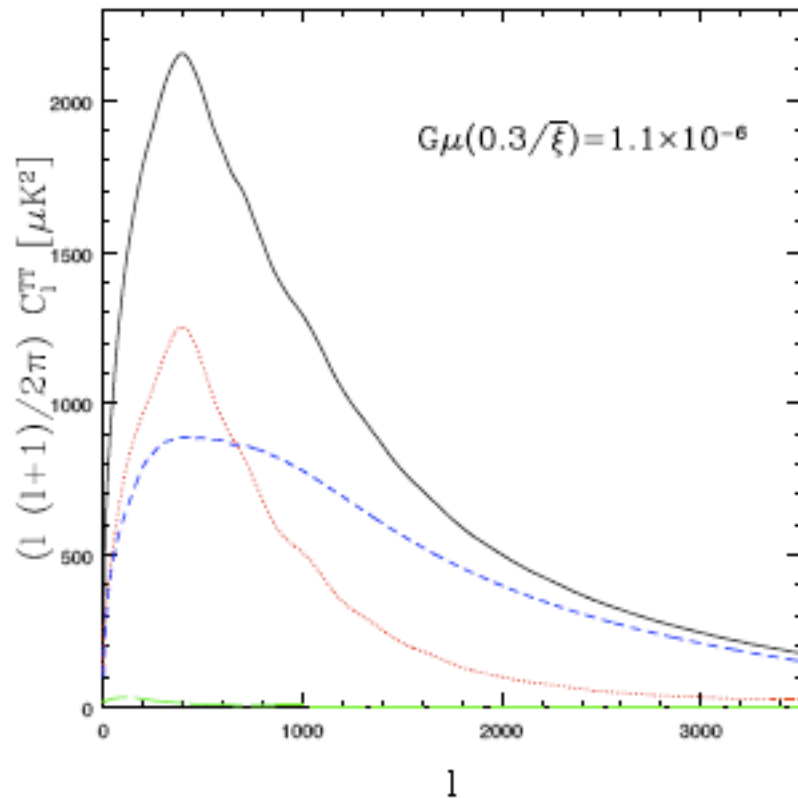
$$-9 < f_{\text{NL}} < 111 \text{ (+-5-10 P1)}$$

CBI/BIMA/Acbar Excess Issue. Thermal SZ explanation requires modified CL templates over that given by adiabatic hydro simulations and by simple semi-analytic calculations

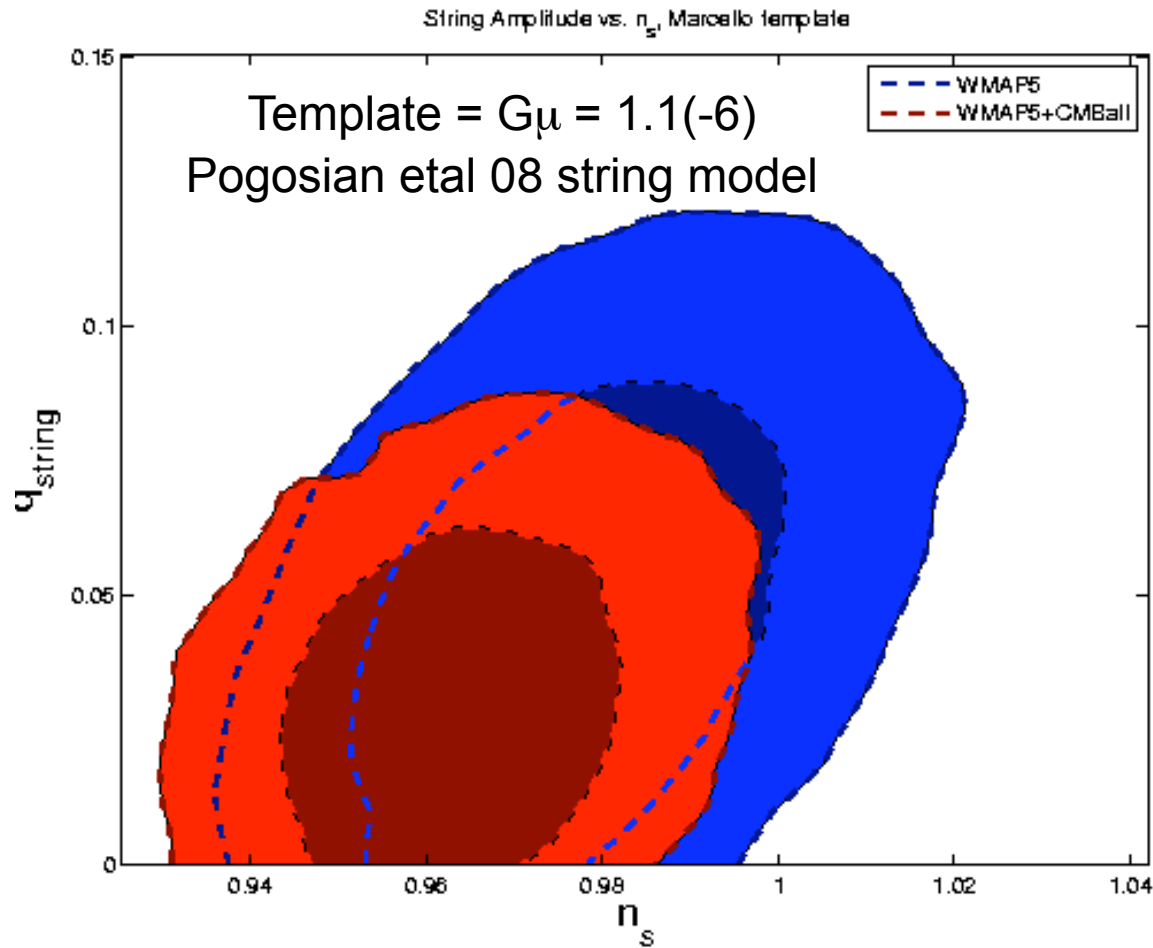
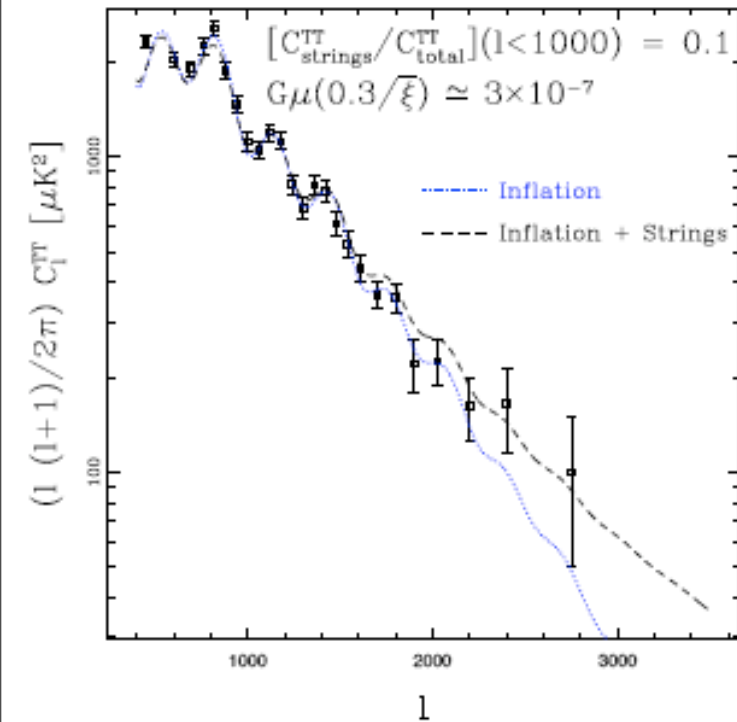
σ_8 From CBI Excess, With Src Marginalization



COSMIC STRING CONSTRAINTS Pogosian etal 08 semi-analytic models (cf. numerical string models Bevis 07)



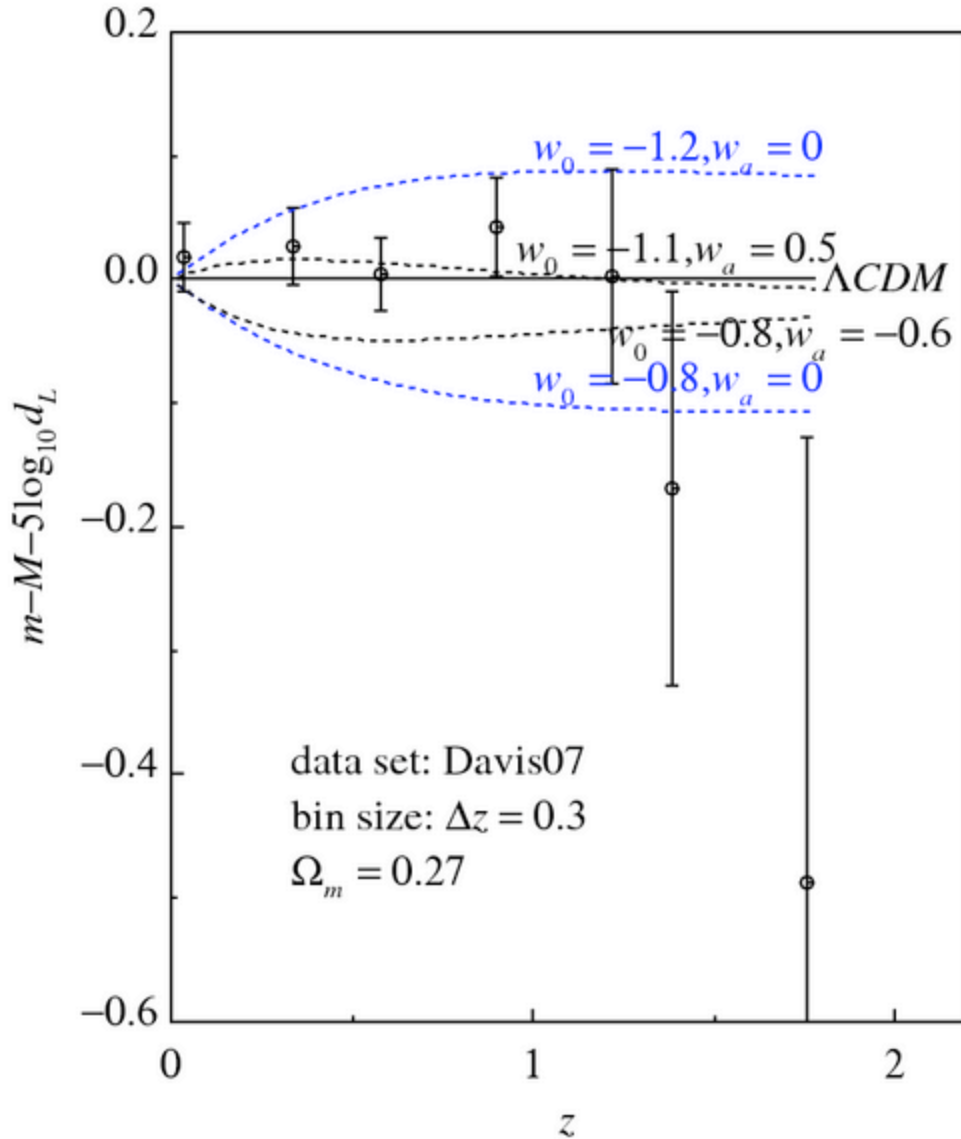
COSMIC STRING CONSTRAINTS Pogosian etal 08 semi-analytic models (cf. numerical string models Bevis 07)



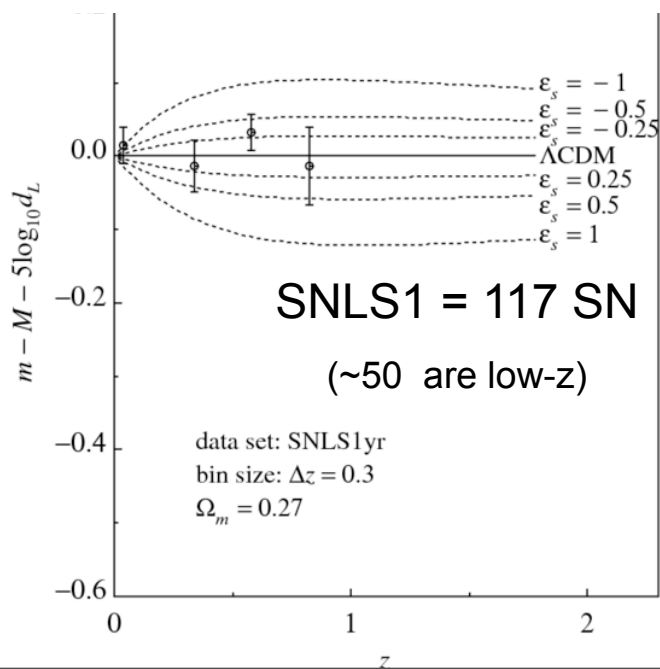
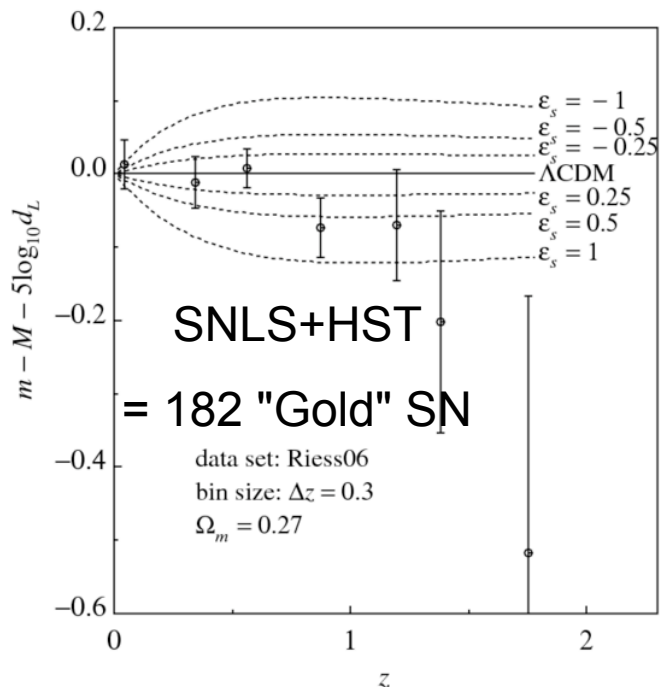
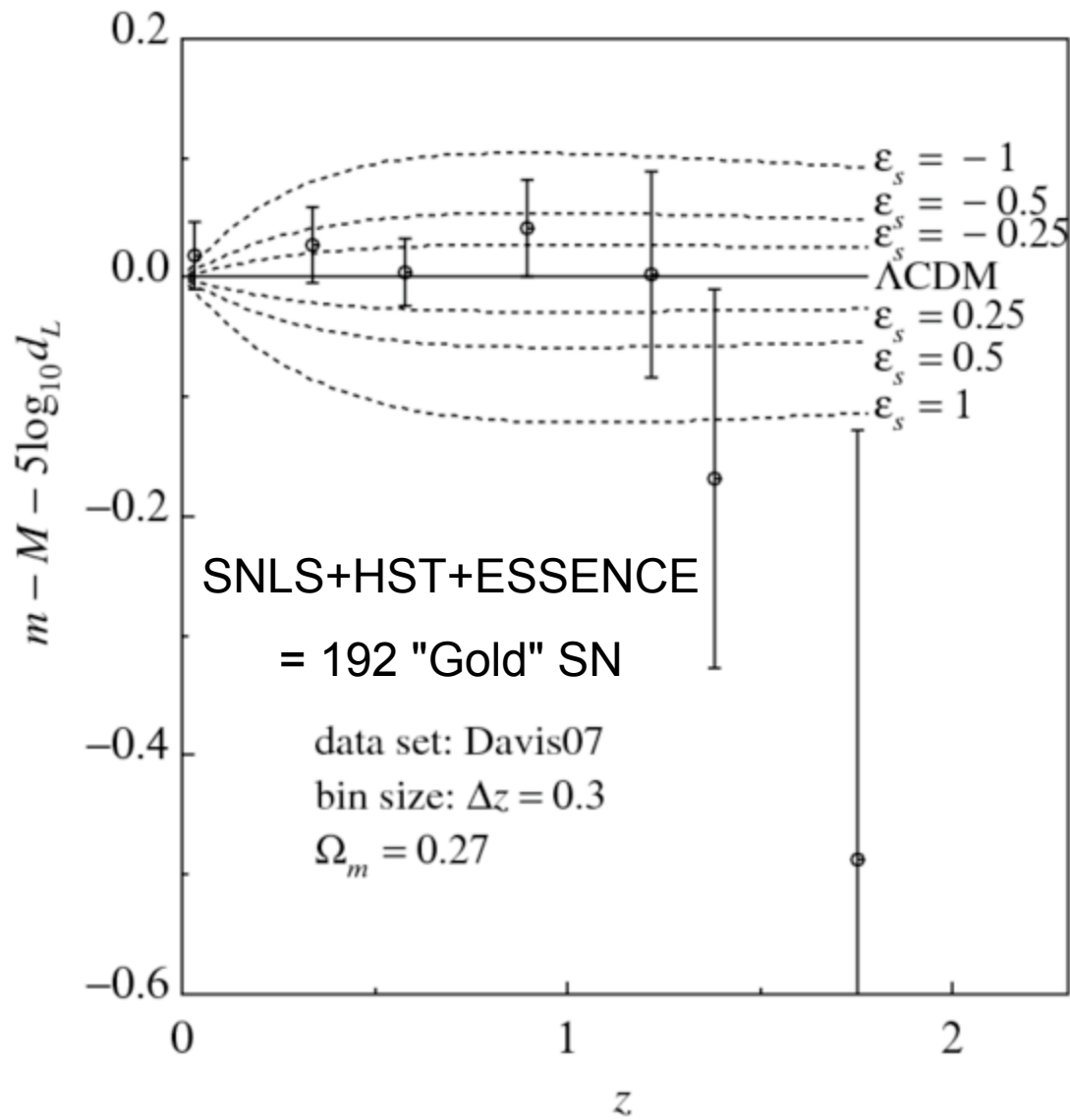
$w(a)=w_0+w_a(1-a)$ models

cf. SNLS+HST+ESSENCE = 192 "Gold" SN

illustrates the near-degeneracies of the contour plot



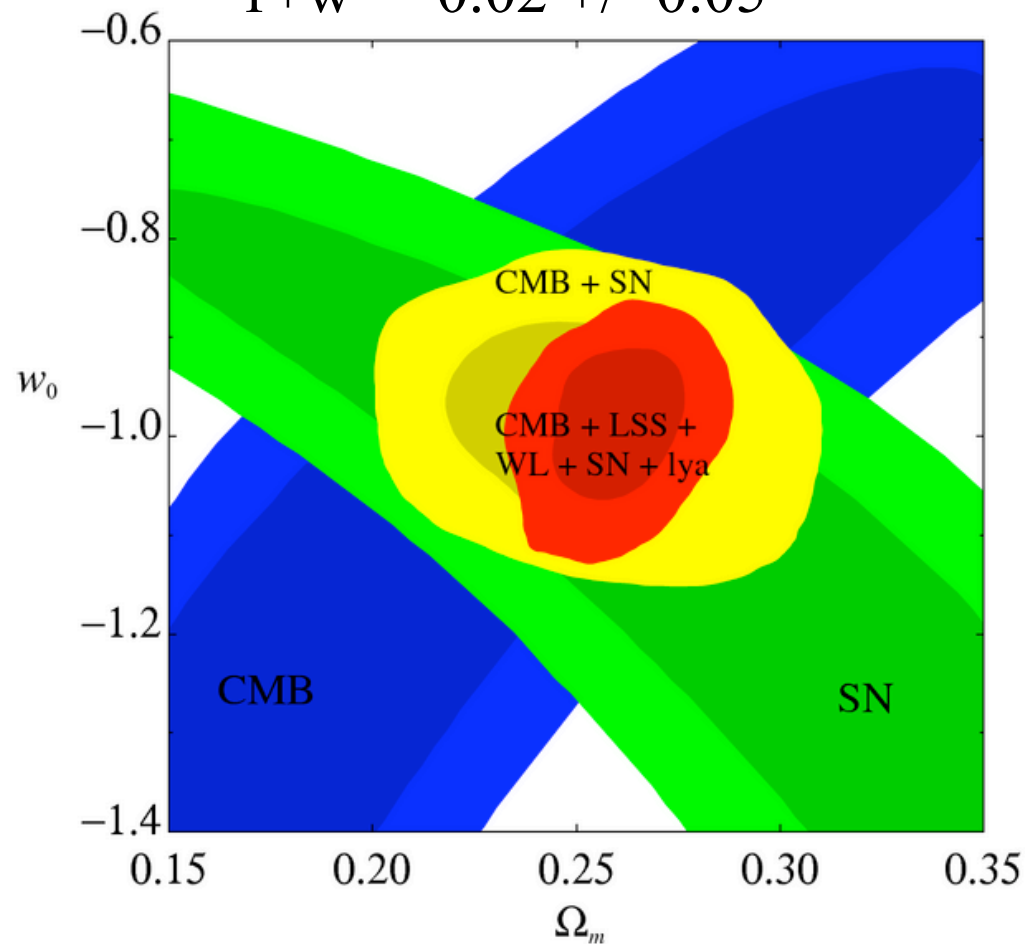
45 low-z SN + ESSENCE SN + SNLS 1st year SN
+ Riess high-z SN, all fit with MLCS



Measuring w (Apr07 SNe+CMB+WL+LSS)

$$w(a) \equiv \frac{p(a)}{\rho(a)}$$

$$1+w = 0.02 \pm 0.05$$



Measuring w (Apr08 SNe+CMB+WL+LSS)

$$w(a) \equiv \frac{p(a)}{\rho(a)}$$

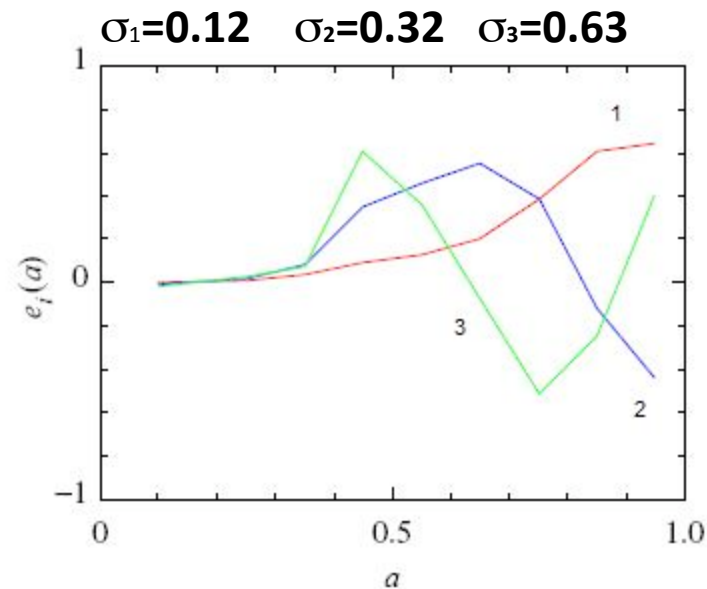
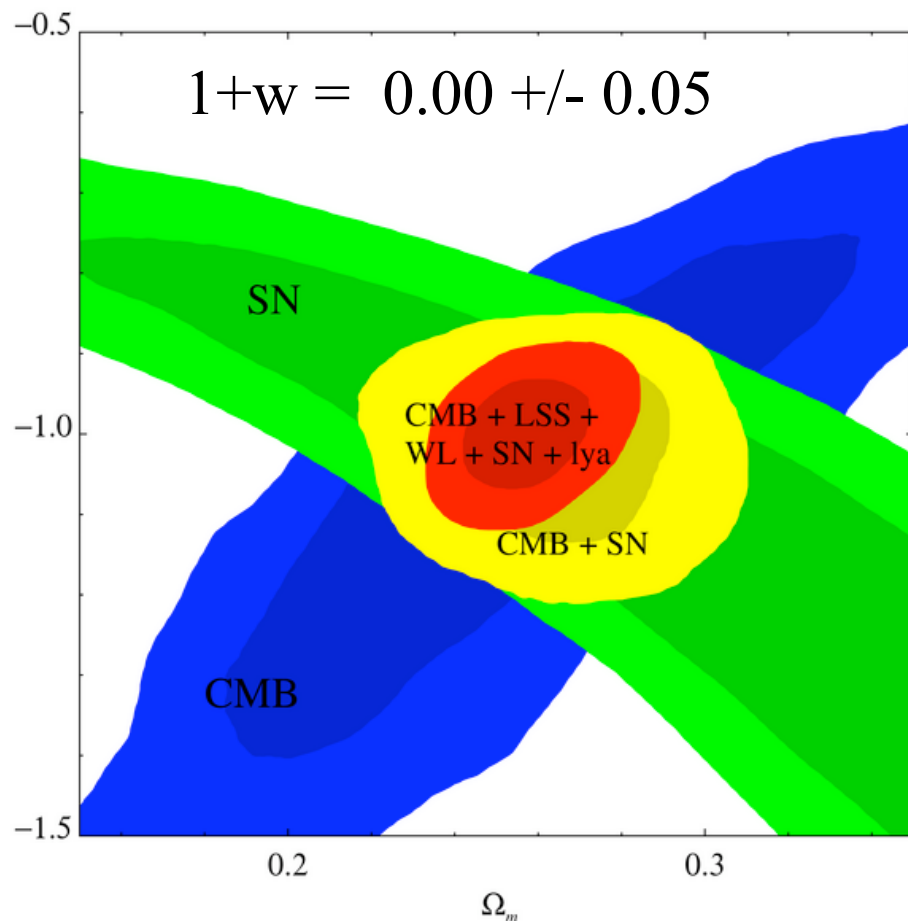
$$w(a) = w_0 + w_a(1-a)$$

$$1+w_0 = -0.11 \pm 0.14, w_a = 0.4 \pm 0.4$$

piecewise parameterization
4,9,40 modes in redshift

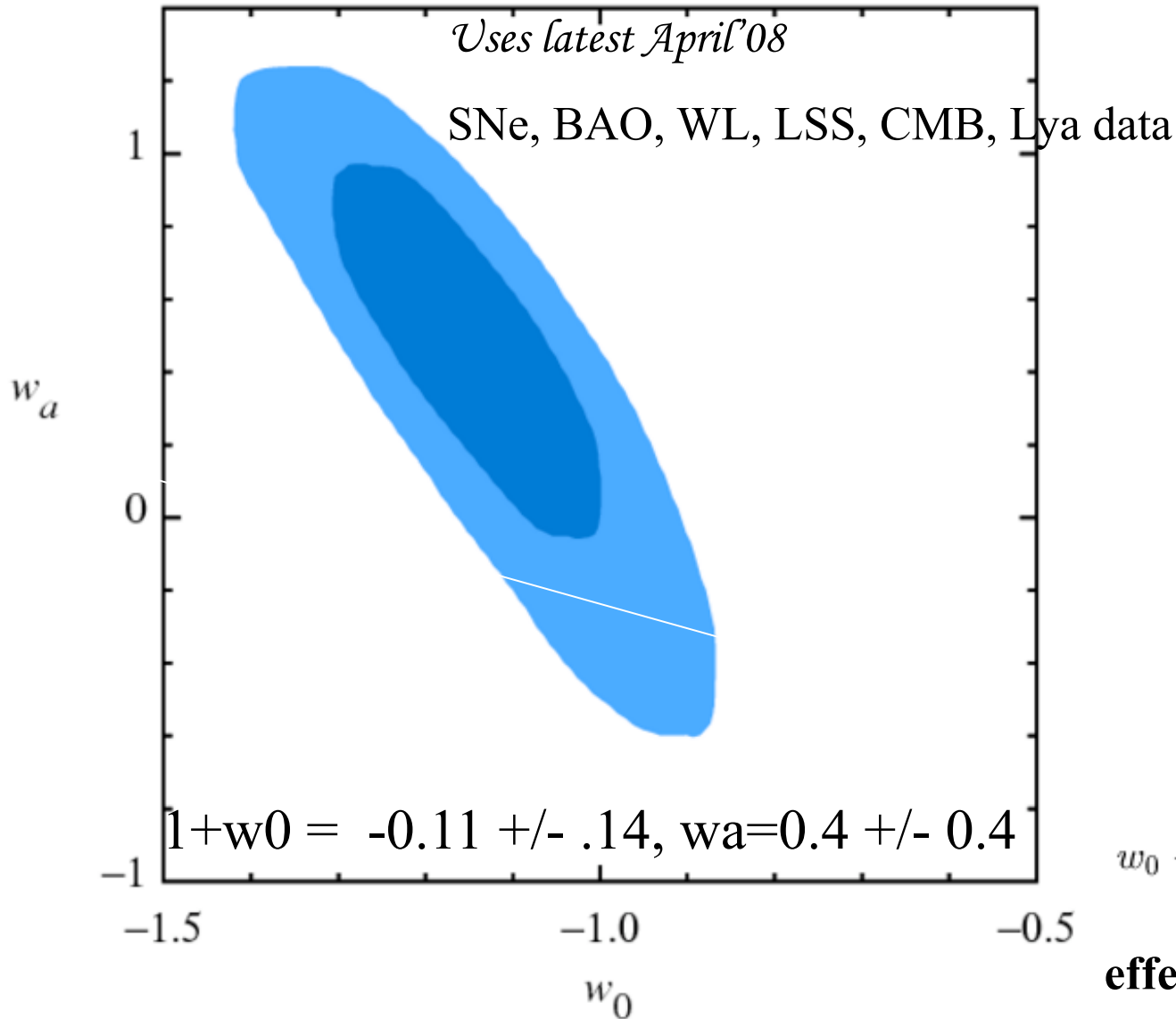
9 & 40 into Parameter eigenmodes

data cannot determine >2 EOS parameters
DETF Albrecht etal06, Crittenden etal06, hbk07



$$w(a) \equiv \frac{p(a)}{\rho(a)}$$

$$w(a) = w_0 + w_a(1-a)$$

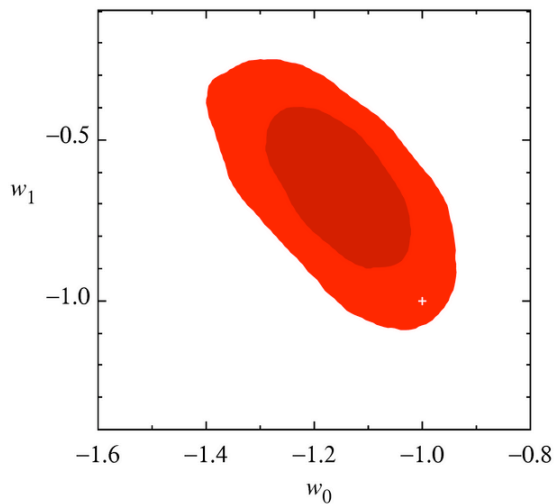
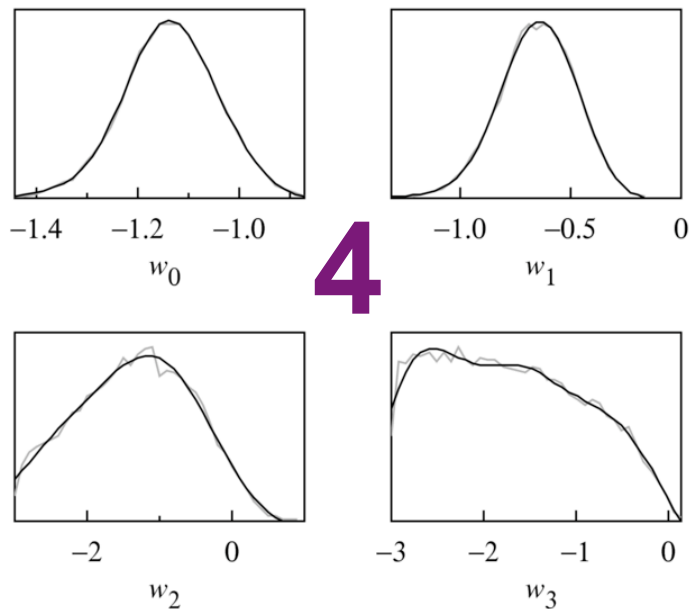


$$w_0 + w_a \left(0.264 + \frac{0.132}{\Omega_{m0}} \right) = -1$$

effective constraint eq.

z-modes of $w(z)$

piecewise parameterization 4,9,40

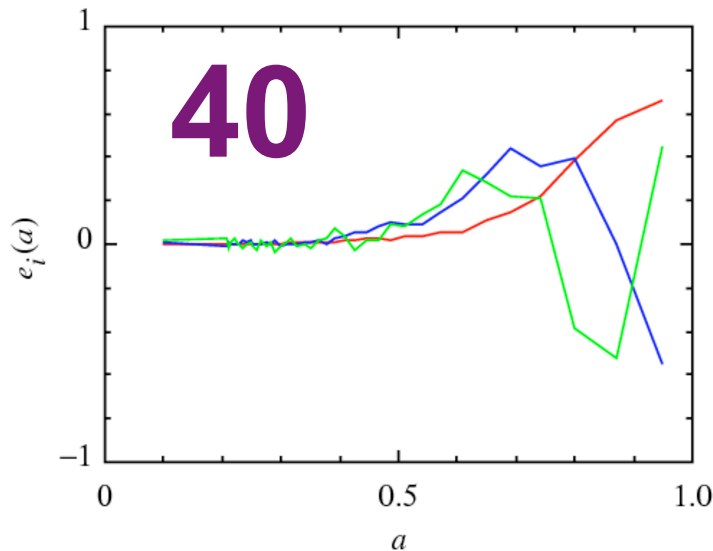
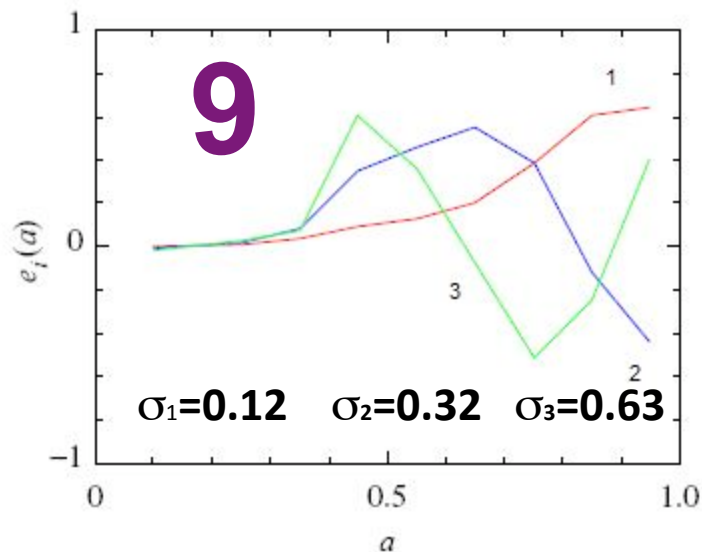


Data used 07.04:
CMB+SN+WL
+LSS+Lya

Higher Chebyshev expansion is not useful:
data cannot determine >2 EOS parameters

9 & 40 into Parameter eigenmodes

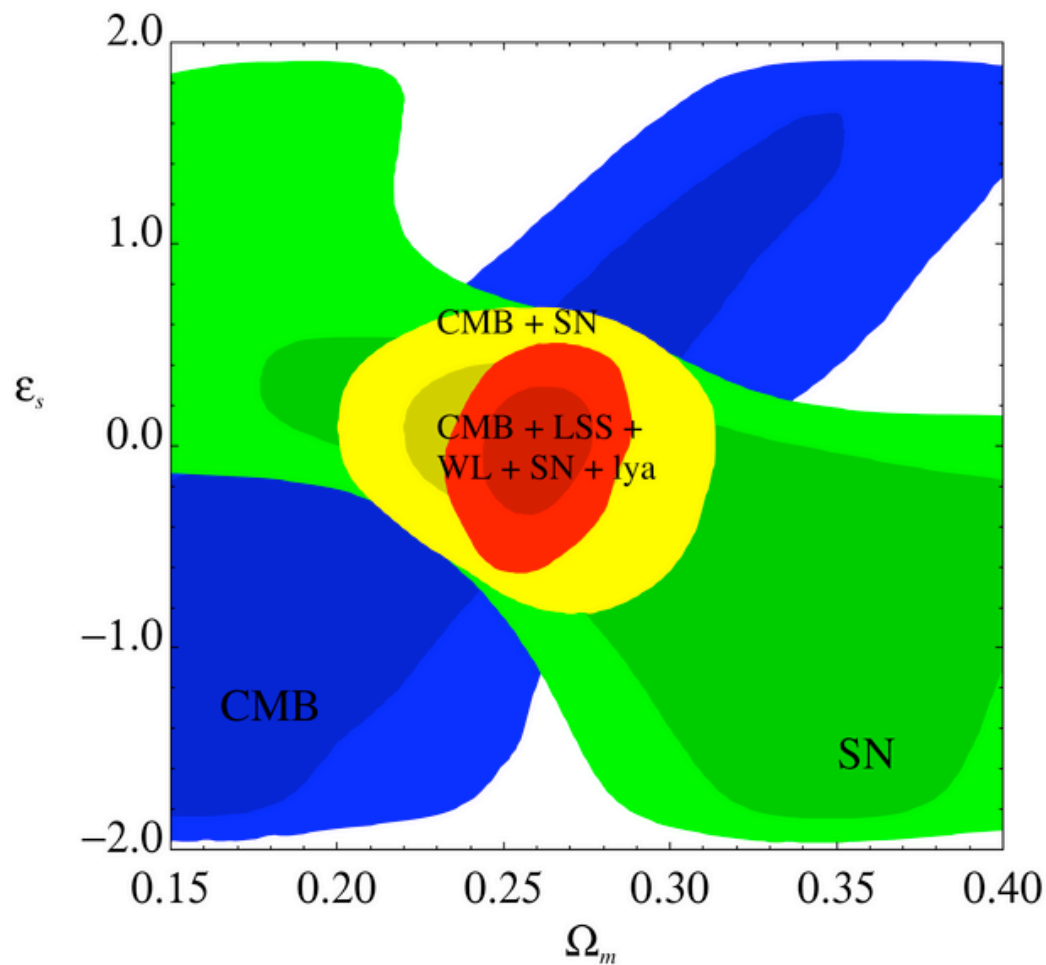
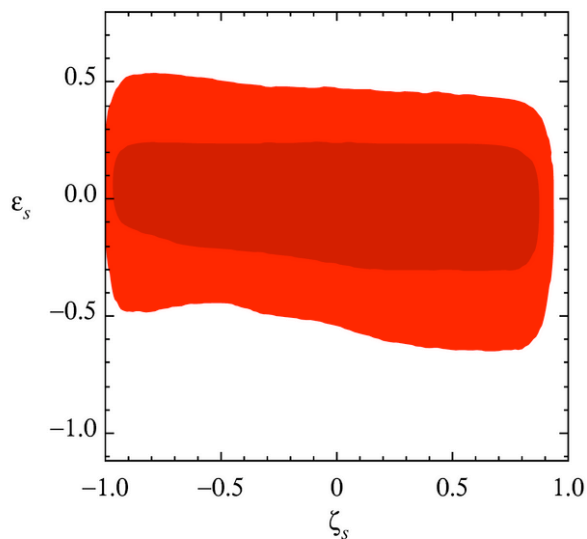
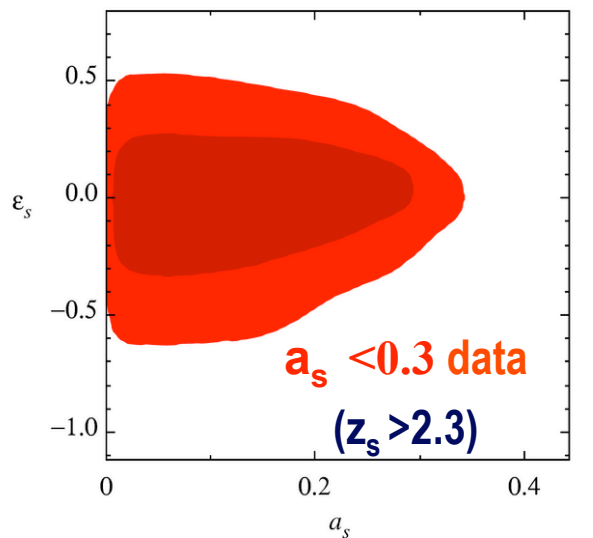
DETF Albrecht etal06, Crittenden etal06, hbk07



Measuring the 3 parameters with Apr07 data

- Use 3-parameter formula over $0 < z < 4$ $w(z > 4) = w_h$
 $\epsilon_s = -0.01 \quad +0.23 \quad -0.24 \quad 1 \text{params}$

$$\epsilon_s = -0.00 \quad +0.20 \quad -0.24 \quad 1 \text{params}$$

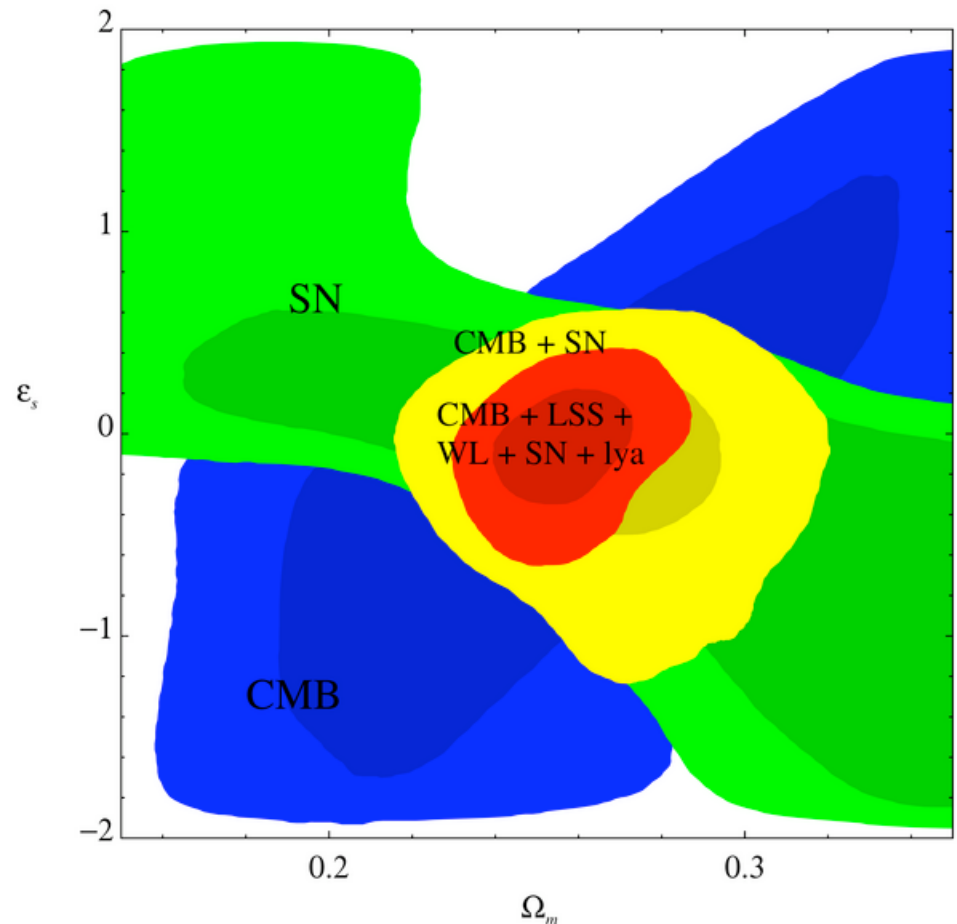
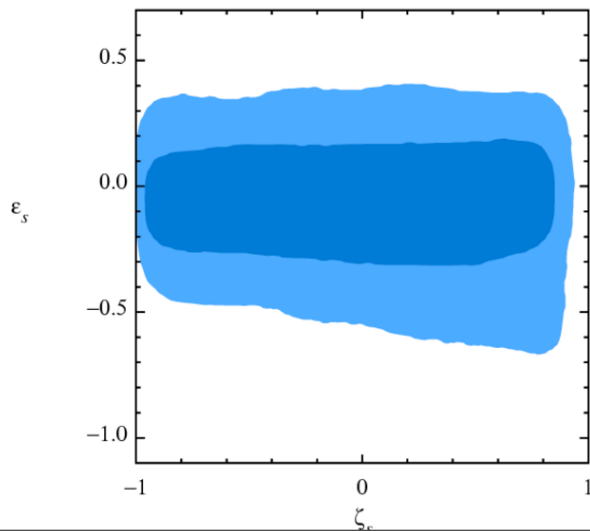
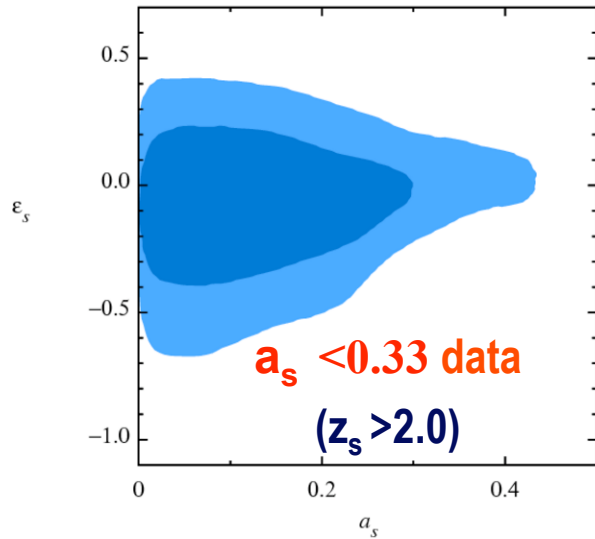


Measuring the 3 parameters with Apr08 data

- Use 3-parameter formula over $0 < z < 4$ & $w(z > 4) = w_h$

$$\epsilon_s = -0.00 \quad +0.20 \quad -0.20 \quad \text{1params}$$

$$\epsilon_s = -0.06 \quad +0.19 \quad -0.21 \quad \text{3params}$$

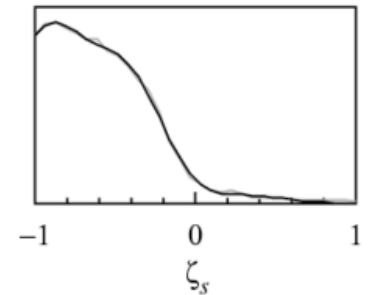
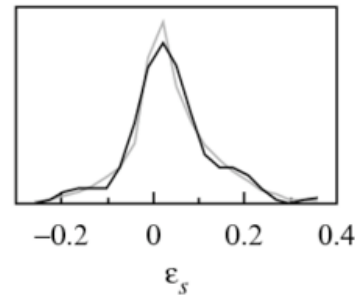
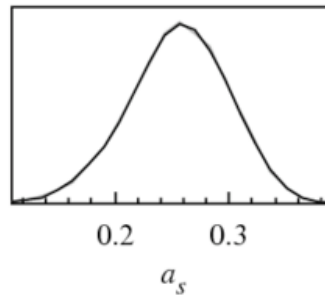


Thawing, freezing or non-monotonic?

- *Thawing*: $1+w$ monotonic up as z decreases
- *Freezing*: $1+w$ monotonic down to 0 as z decreases
- $\sim 15\%$ thaw, 8% freeze, most non-monotonic with flat priors

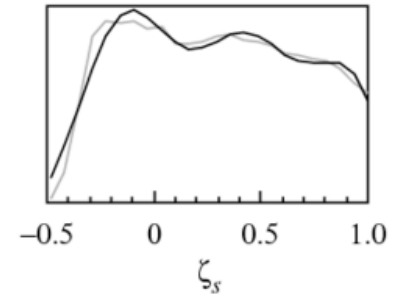
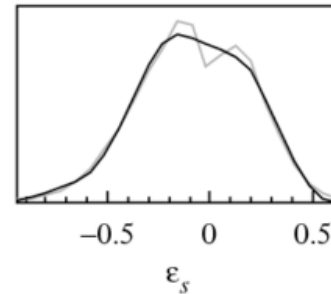
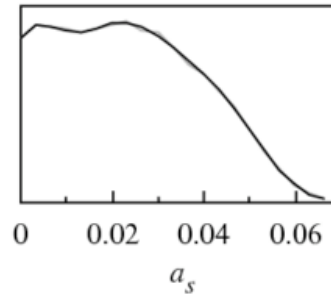
With freezing prior:

$$\epsilon_s = -0.02^{+0.10}_{-0.06}$$

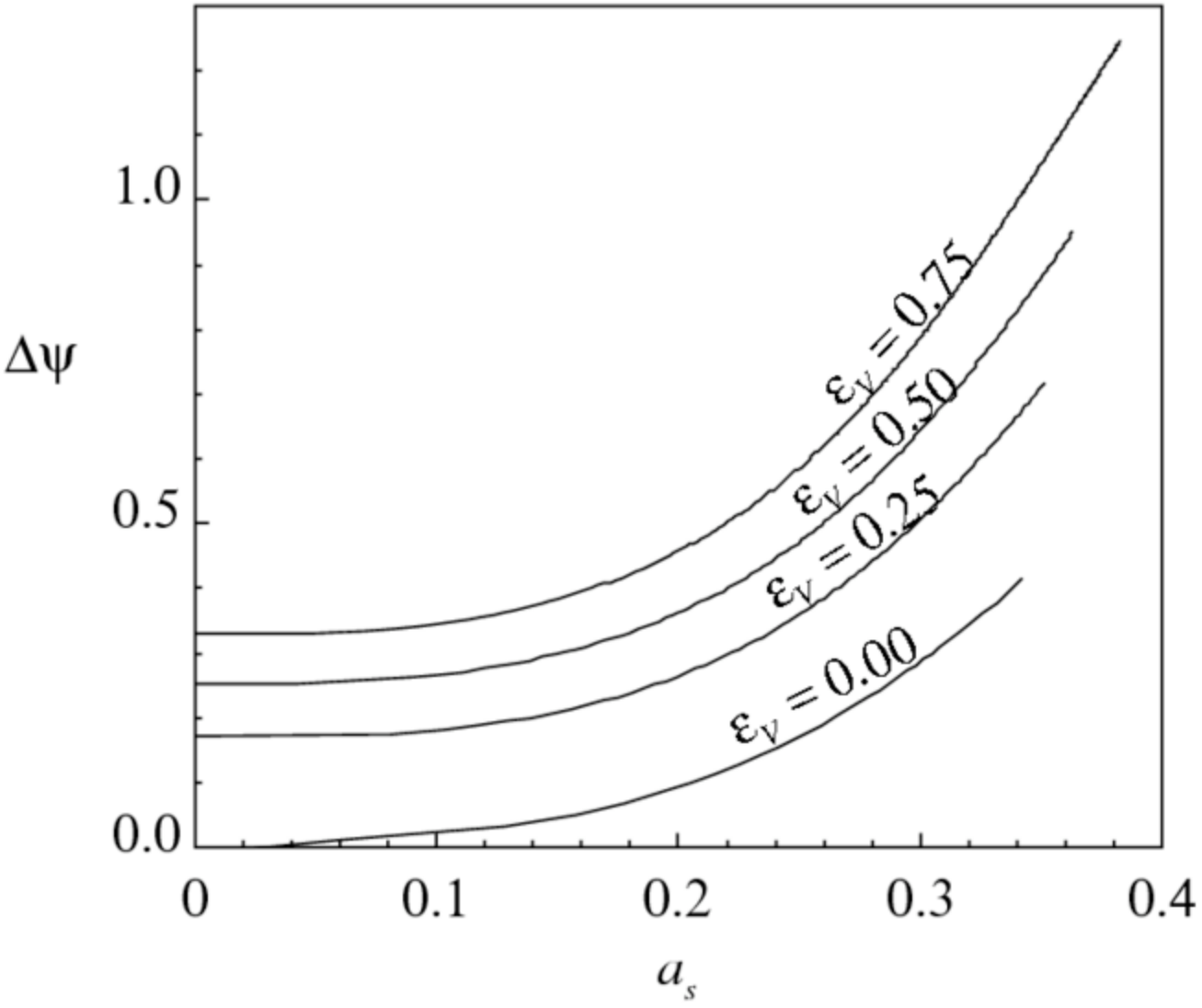


With thawing prior:

$$\epsilon_s = -0.08^{+0.29}_{-0.27}$$



the quintessence field is below the reduced Planck mass



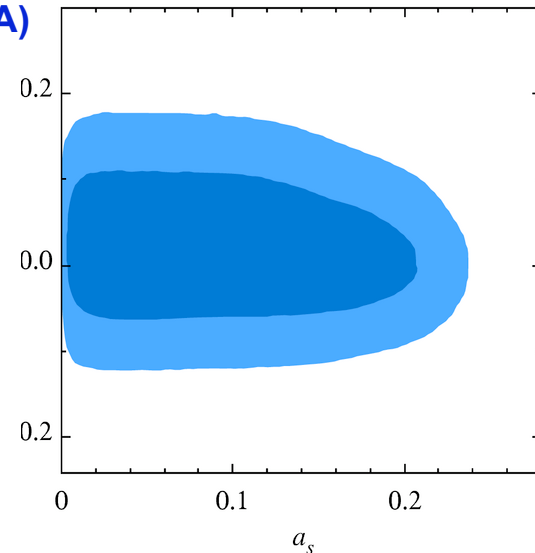
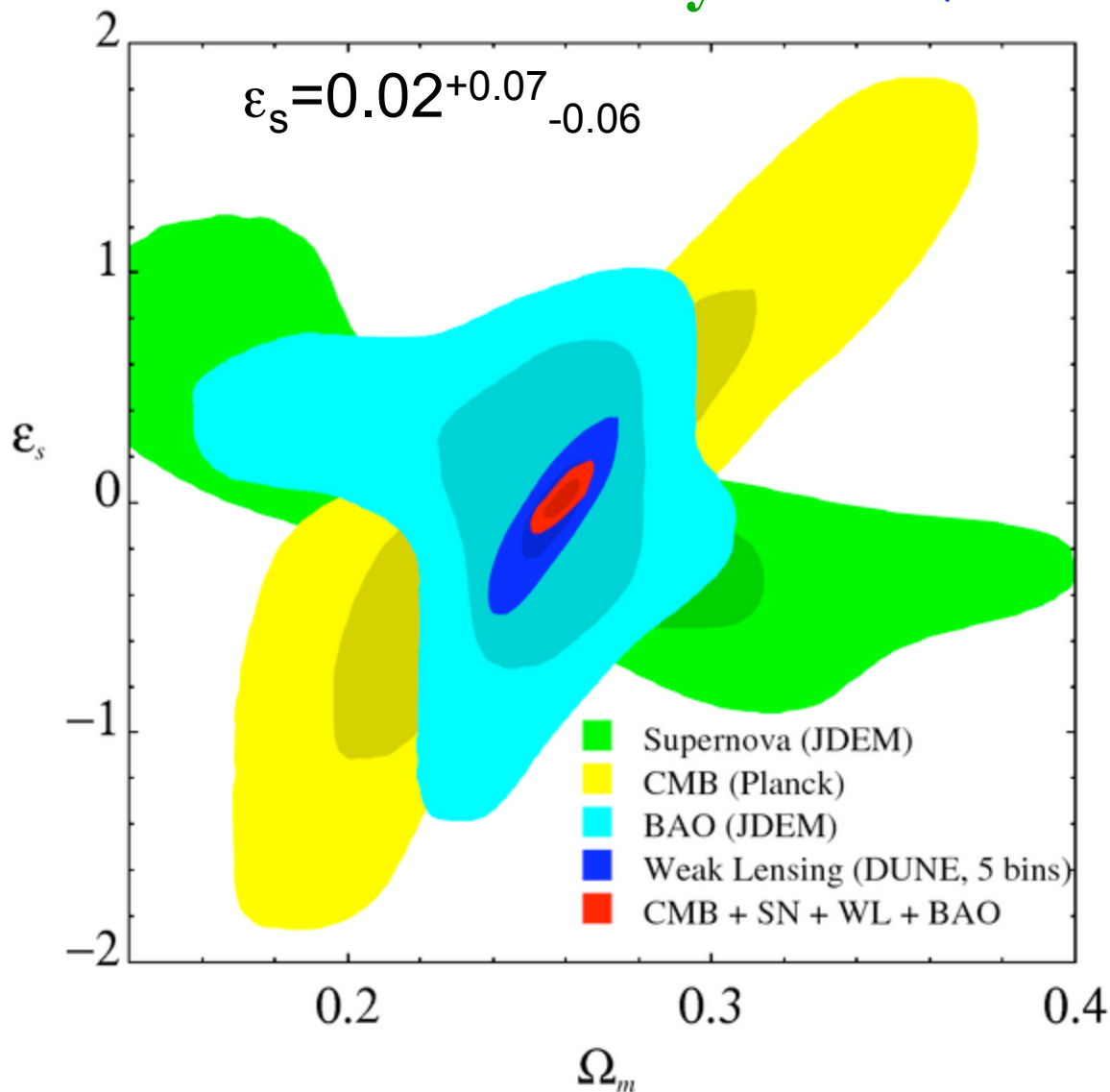
**INFLATION
NOW**

**PROBES
THEN**

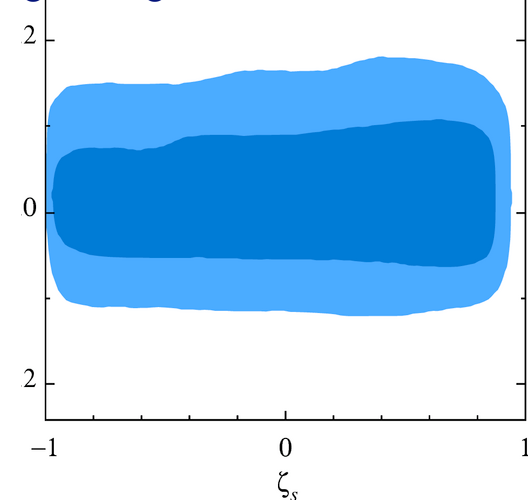
Forecast: **JDEM-SN** (2500 hi-z + 500 low-z)
 + **DUNE-WL** (50% sky, gals @z = 0.1-1.1, 35/min²) +
Planck1yr **ESA (+NASA/CSA)**

$a_s < 0.21$ (95%CL)

($z_s > 3.7$)



ζ_s ($\sim d\epsilon_s / d\ln a$) ill-determined



Inflation now summary

- the data cannot determine more than 2 w -parameters (+ c_{sound} ?). general higher order Chebyshev or spline expansion in $1+w$ as for “inflation-then” $\varepsilon=(1+q)$ is not that useful. **Parameter eigenmodes** show what is probed
- Any $w(a)$ leads to a viable DE model. The $w(a)=w_0+w_a(1-a)$ phenomenology requires baroque potentials
- Philosophy of HBK07: **backtrack from now ($z=0$) all w -trajectories arising from quintessence ($\varepsilon_s > 0$) and the phantom equivalent ($\varepsilon_s < 0$); use a 3-parameter model to well-approximate even rather baroque w -trajectories, as well as thawing & freezing trajectories.**
- **We ignore constraints on Q -density from photon-decoupling and BBN because further trajectory extrapolation is needed. Can include via a prior on $\Omega_Q(a)$ at z_{dec} and z_{bbn}**
- For general slow-to-moderate rolling one needs 2 “dynamical parameters” (a_s, ε_s) & Ω_Q to describe w to a few % for the not-too-baroque w -trajectories. A 3rd param $\zeta_s, (\sim d\varepsilon_s/d\ln a)$ is ill-determined now & in a Planck1yr-CMB+JDEM-SN+DUNE-WL future.

- $1+w(a) = \varepsilon_s f(a/a_{\Lambda\text{eq}}; a_s/a_{\Lambda\text{eq}}; \zeta_s)$

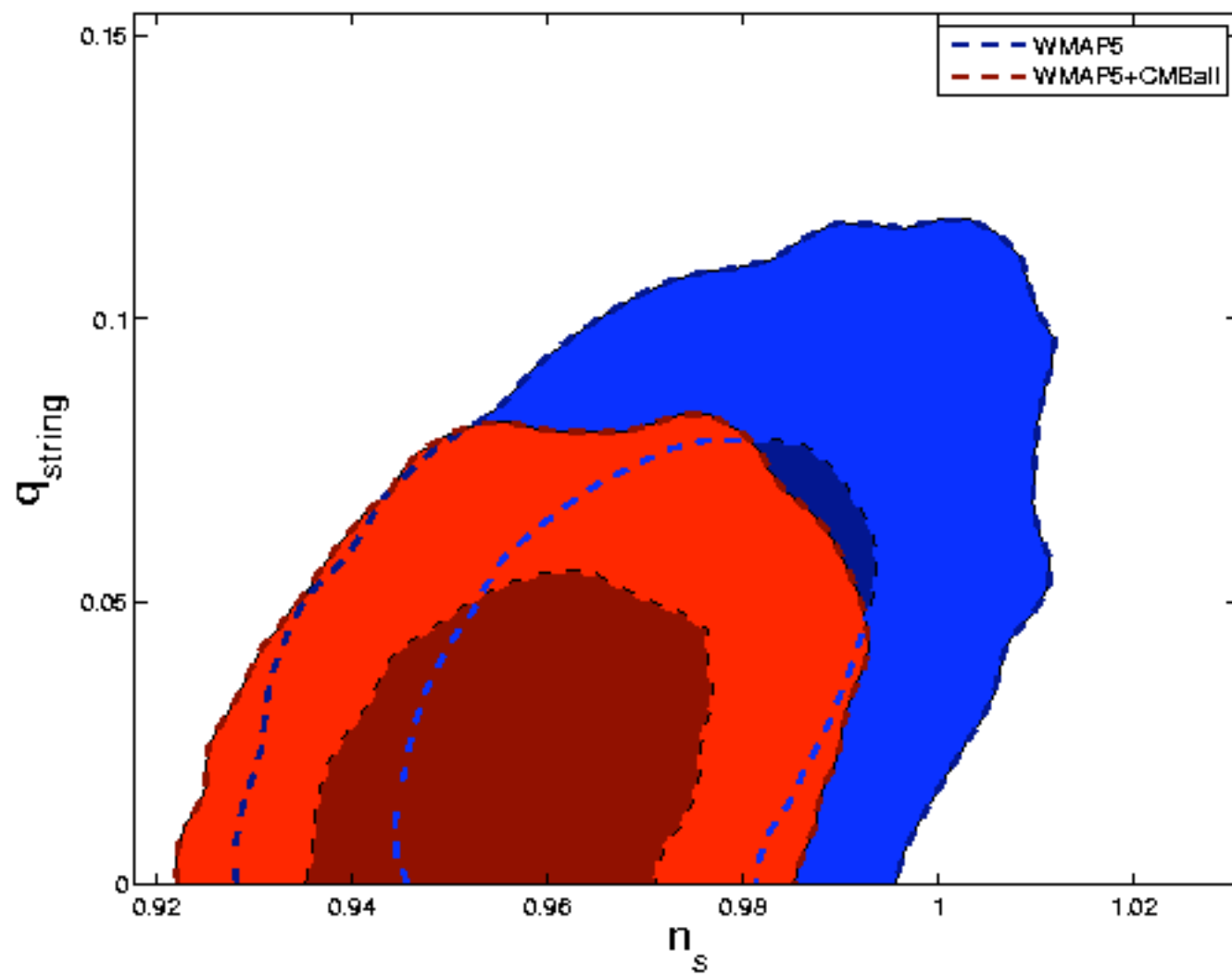
- **?? extension to $\varepsilon_s < 0$ – phantom energy, eg negative kinetic energy**

- In the early-exit scenario, the information stored in a_s is erased by Hubble friction over the observable range & w can be described by a single parameter ε_s .
- a_s is **< 0.33 current data ($z_s > 2.0$)** to **< 0.21 ($z_s > 3.7$)** in Planck1yr-CMB+JDEM-SN+DUNE-WL future
- current observations are well-centered around the cosmological constant $\varepsilon_s = -0.06 \pm 0.20$
- **in Planck1yr-CMB+JDEM-SN+DUNE-WL future ε_s to ± 0.07**
- **but one cannot reconstruct the quintessence potential, just the slope ε_s & hubble drag info**
- late-inflaton field is $<$ Planck mass, but not by a lot

END

**COSMIC
PARAMETERS
NOW & THEN**

String Amplitude vs. n_s , KS template



The Parameters of Cosmic Structure Formation

Cosmic Numerology: aph/0801.1491 – our Acbar paper on the basic 7+; **bchkv07**

WMAP3modified+B03+CBIcombined+Acbar08+LSS (SDSS+2dF) + DAS1
(incl polarization and CMB weak lensing and tSZ)

$$n_s = .962 \pm .014 \text{ (+-.005 Planck1)}$$

$$.93 \pm .03 \text{ @0.05/Mpc run\&tensor}$$

$$r = A_t / A_s < 0.47_{\text{cmb}} \text{ 95\% CL (+-.03 P1)}$$

$$< .55 \text{ CMB+LSS } \epsilon \text{ prior 5-pivot B-spline}$$

$$< .22 \text{ CMB+LSS In } \epsilon \text{ prior 5-pivot B-spline}$$

$$dn_s / d \ln k = -.04 \pm .02 \text{ (+-.005 P1)}$$

$$\text{CMB+LSS run\&tensor prior change?} < \#(1-n_s)$$

$$f_{\text{NL}} = 87 \pm 60 \text{?! (+-}$$

$$A_s = 22 \pm 2 \times 10^{-10} \text{ 5-10 P1)}$$

$$1+w = 0.02 \pm 0.05$$

'phantom DE' allowed?!

$$\epsilon_s = 0.00^{+0.20}_{-0.24} \text{ +-}.07 \text{ then}$$

$$\Omega_b h^2 = .0226 \pm .0006$$

$$\Omega_c h^2 = .116 \pm .005$$

$$\Omega_\Lambda = .72 \pm .02 - .03$$

$$h = .704 \pm .022$$

$$\Omega_m = .27 \pm .03 - .02$$

$$z_{\text{reh}} = 11.7 \pm 2.1 - 2.4$$

The Parameters of Cosmic Structure Formation

Cosmic Numerology: **wmap5+acbar08** **wmap5**

$$n_s = .964 \pm .014 \text{ (+-.005 Planck1)}$$

$$r = A_t / A_s < 0.54_{\text{cmb}} \text{ 95\% CL (+-.03 P1)}$$

<

<

$$dn_s / d \ln k = -.048 \pm .027^* \text{ (+-.005 P1)}$$

WMAP5+ACBAR08 run&tensor

$$A_s = 24 \pm 1.1 \times 10^{-10} \quad -9 < f_{\text{NL}} < 111 \text{ (+-. 5-10 P1)}$$

$$\Omega_b h^2 = .0227 \pm .0006$$

$$\Omega_c h^2 = .110 \pm .005$$

$$\Omega_\Lambda = .74 \pm .03 - .03$$

$$h = .72 \pm .027$$

$$\Omega_m = .26 \pm .03 - .03$$

$$z_{\text{reh}} = 11.0 \pm 1.5 - 1.4$$