The Kinematics of Inflation, Preheating and Heating: a Playground for Kolmogorov-Sinai and Shannon Entropies

Dick Bond @ IAS18_5

what are the degrees of freedom / parameters of the ultra early Universe? TBD

Quantum Inflation - if quantum energy then quantum gravity (entangled) then gravitons

Phonons ***density fluctuations = Trace strain = spatial 3-volume fluctuations***

=> *combined* **entropy-like measure** ζ =inflaton

$$\zeta(x,t) = \int_{\text{field-path}} (dE+pdV)/3(E+pV) = \text{Trace } \delta\alpha^{i}_{j} + \int_{\text{field-path}} \delta \ln \rho_c / 3(1+w)$$

Gravitons ***tensor perturbations transverse traceless strain $P_{GW} = r\, P_\zeta$ grail $r < .07$ now, to $< .001$***

Isocons ***when multiple particle-species - orthogonal scalar degrees of freedom to inflaton/phonon***

Dilatons ***4-volume fluctuations - Higgs inflation $L_G(R)$ gravity - conformally-flatten potentials***

moduli, axions ***connection to particle physics models "fundamental scalars" .. string theory***

fermions, vector gauge fields, Higgs ***Standard model of particle physics*** **. vector perturbations**

begin-inflate => inflate => end-inflate => preheat => non-equilibrium heat+entropy
=> ***Standard Model particle physics*** **QG plasma radiation dominated**
=> dark matter dominated ***structure via gravitational instability*** **=> dark energy now**

fit into a UV-complete theory (ultra-high energy to the Planck scale) strings, landscape, ..
& IR-complete theory (post-inflation heating -> quark/gluon plasma)??? TBD

TBD *inflate => end-inflate => preheat => non-equilibrium heat+entropy via a* ***|cg <=> fg> condensate/fluctuation framework****, cf. quantum cold atom system? using coherent states (very overcomplete basis, but quantum optics, classical-like approach with* ***hbar****). includes* ***Bogoliubov*** *transformations for fluctuations as condensate evolves =>* ***particle creation*** *in heating and inflating regime.*

Dick Bond @IAS18_05

ζ *all cosmic structure from* **entropy!**

linear (*bst1983*) =>**nonlinear** $\zeta(x,t) = \int_{\text{field-path}} (dE+pdV) / 3(E+pV) = \alpha_{|H}$

coarse-grained *horizon scale cf. fine-grained fluctuations*

volume deformation = **isotropic strain**

$\ln V / \langle V\rangle|_\rho = \ln \det A^i_j(x,t) / \langle a\rangle|_\rho$ = Trace $\delta\alpha_c{}^i_j(x,t)|_\rho$

$\ln \rho(x,t)/\rho_c(x,t)$ **phonon flucs** $\ln \rho_c(x,t)/\langle\rho\rangle|_V /3(1+w)$ **condensate**

along coarse-grain trajectories $d\zeta = [\bar{d}\zeta](fg\text{->}cg)$ $(-\bar{d}\zeta](cg\text{->}fg))$

regimes: 1. stochastic inflation non-adiabatic $[\bar{d}\zeta](fg\text{->}cg)$

gradient flow ***+stochastic jitter,*** **simple Hamilton principle function S~H(ϕ_{cg})**

2. ballistic phase adiabatic thru EoI, but caustics & Kolmogorov-Sinai entropy

3. shock-in-time, *cg <=> fg, origin of almost all entropy* $S_{U,m+r} \sim 10^{88.6}$

cf. $S_G \sim 10^{121.9}$ *asymptotic DE*

further S generation in early Universe: phase transitions, out-of-equilibrium decays?

further ***d****bar S: reionization epoch & beyond via nuclear/accretion, gravitational collapse* ***CIB***

CMB/LSS observations give access to limited partial operator-paths in field-space aka **kinematics** => limited glimpse of a **UV/IR complete theory** aka **dynamics**. so far just through the **ζ-fluctuation spectrum** encoding the quantum diffusion of (**longitudinal/inflaton**) paths. **Transverse** field-motion aka **isocons** may influence $\mathcal{P}_{\zeta\zeta}(k)$ and **ζ-nonG**
stochastic dynamical systems theory for kinematics of cg variables

coarse-grained *stochastic inflation k~Ha resolution/dynamics relation*

$$d\zeta(x,Ha) = [\mathcal{P}_{\zeta\zeta}(k)]^{1/2} \, [d\ln Ha]^{1/2} \eta_{GRD}(x,Ha)$$ ***quantum fluctuations & no drift***

order parameters from the ultra early U? so far ζ ≈GRF, $\mathcal{P}_{\zeta\zeta}(k)$ 2 ζ-params

ζ = **an adiabatic invariant**, conserved even with large field-strains
to get nonG need to break adiabaticity => nonlinear fluctuations

Kinematics of Preheating:
BBFH are developing a **trajectory bundle evolution framework** for nonG in **post-inflation preheating -> heating**. field-space path deviations aka strains and their shear can characterize smoothed-bundle "ballistic" chaotic evolutions. arrested by nonlinear fluctuation-mode generation. **metric entropy (KS) in the ballistic phase -> Shannon entropy in the fluctuation-mode phase**. nonlinear multi-field classical coupled kinematics/dynamics using lattice simulations. via **symplectic defrost++ code + spectral code** => high accuracy to unveil small effects

condensate evolves chaotically? => phase-transition-like melting into phonon+ modes,
fast scramble of fundamental fields, evolution of statistically-simple energy-density phonons
can one use coherent states to address this quantum mechanically? TBD

$\zeta_{fi}(x,t) = \mathrm{Trace}\,(\alpha_f - \alpha_i) + \int_i^f d\ln \rho_c / \rho_i / 3(1+w)$

cf. $ds_{fi} = ds_{KSfi} - d\mathrm{Tr}\,\varepsilon_{fi} = \delta s_{fi}$ *(fg-> cg) FokkerPlanck for Prob(*δs_{fi}*| control)*

$\zeta_{NL}(x)$ from *"isocon" degrees of freedom cf.* $\zeta_{NL}(x)$ from ***inflaton***

modulated heating, ***ballistic chaos, caustics, shock-in-time,***

modulators isocon $\chi(x)$, ***axionic-isocon***(x) ***couplings*** $g(x)$ super-horizon accessible

complex field configs *Oscillons, Stringy configurations, curvatons, .. Bubble Collisions*

aside on *LSS:* late-time **CDM-ish web ~strain web** $\zeta_{fi}(x,t) = \ln \rho \det A / 3$

if cold DM $p/\rho \sim 0 \Rightarrow \zeta(x,t\,|cdm)$ *is conserved (mass-energy an adiabatic flow)*

before shell crossing (analogue of preheating)
KS entropy in cool 2LPT dynamics
then shock of shell crossing = Shannon entropy development = heat of CDM
long adiabatic KS phase cf. should we call it chaotic - tho fits the definition?

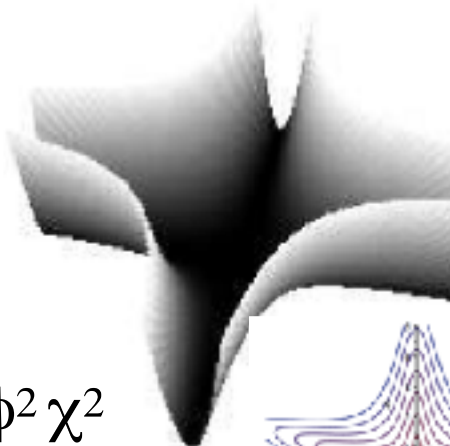
what is the **inflaton+isocons potential**?

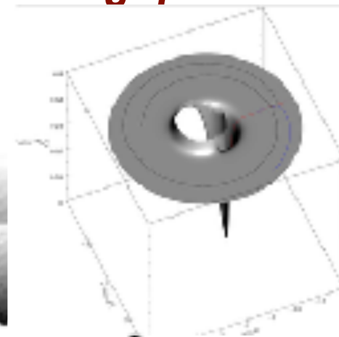
around a minimum** is the **HOT /heating question

how was *matter & entropy generated at the end of acceleration = inflation*?

Relate to Higgs & standard model?

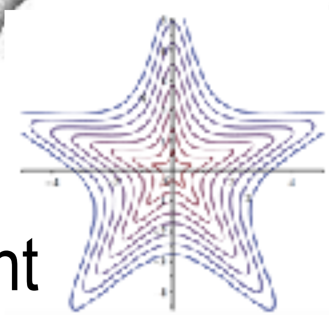
2 filament?





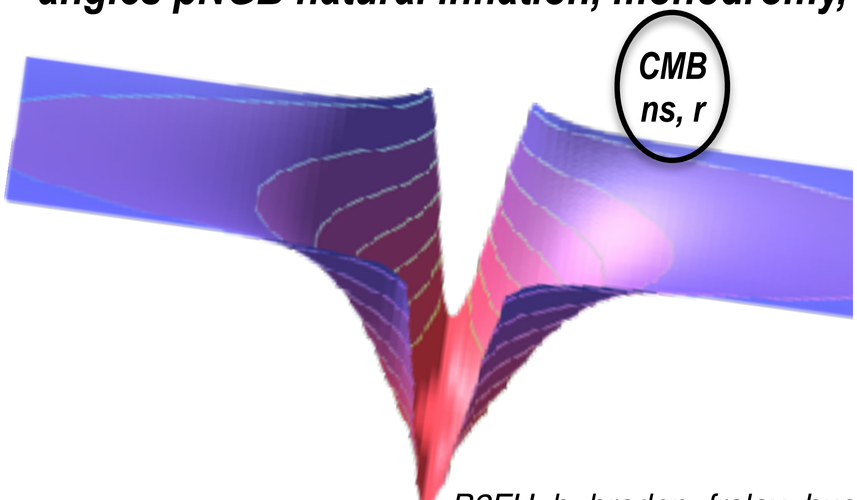
4 filament

$1/4\lambda\phi^4 + 1/2 g^2 \phi^2 \chi^2$



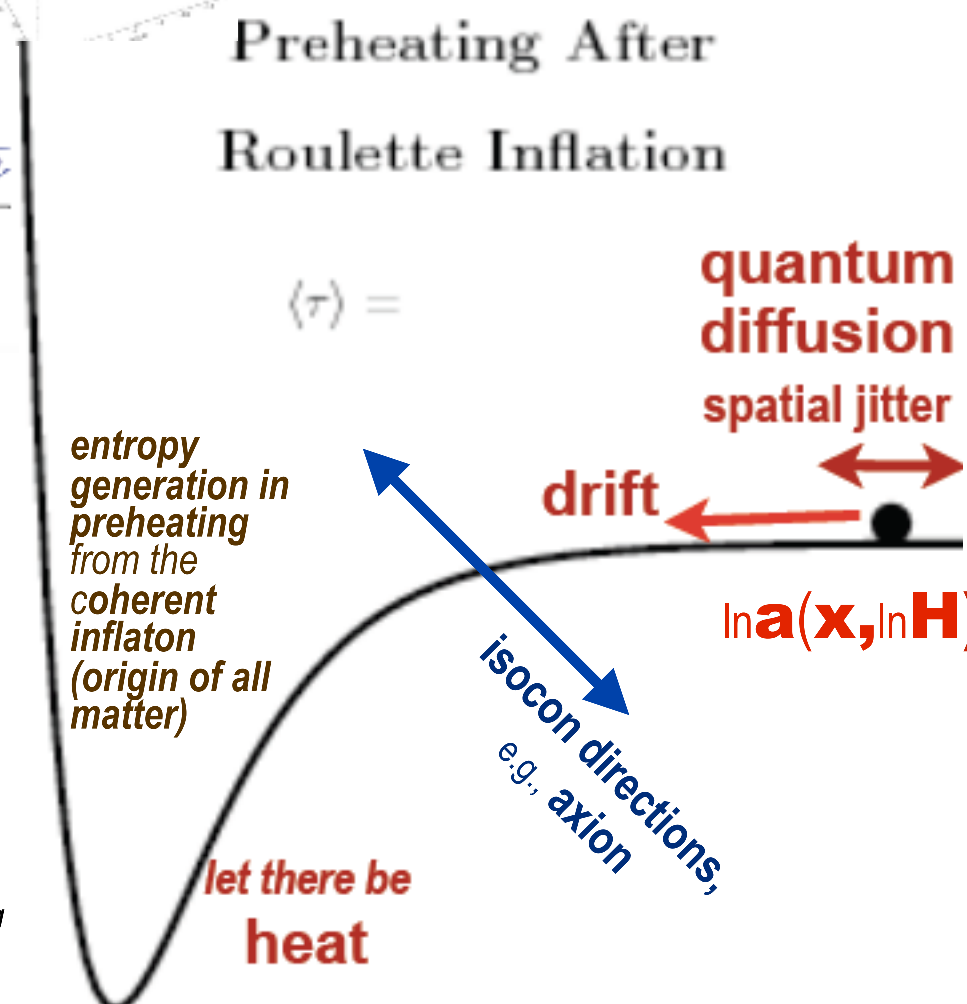
3-filament 5-filament

angles pNGB natural inflation, monodromy, ..



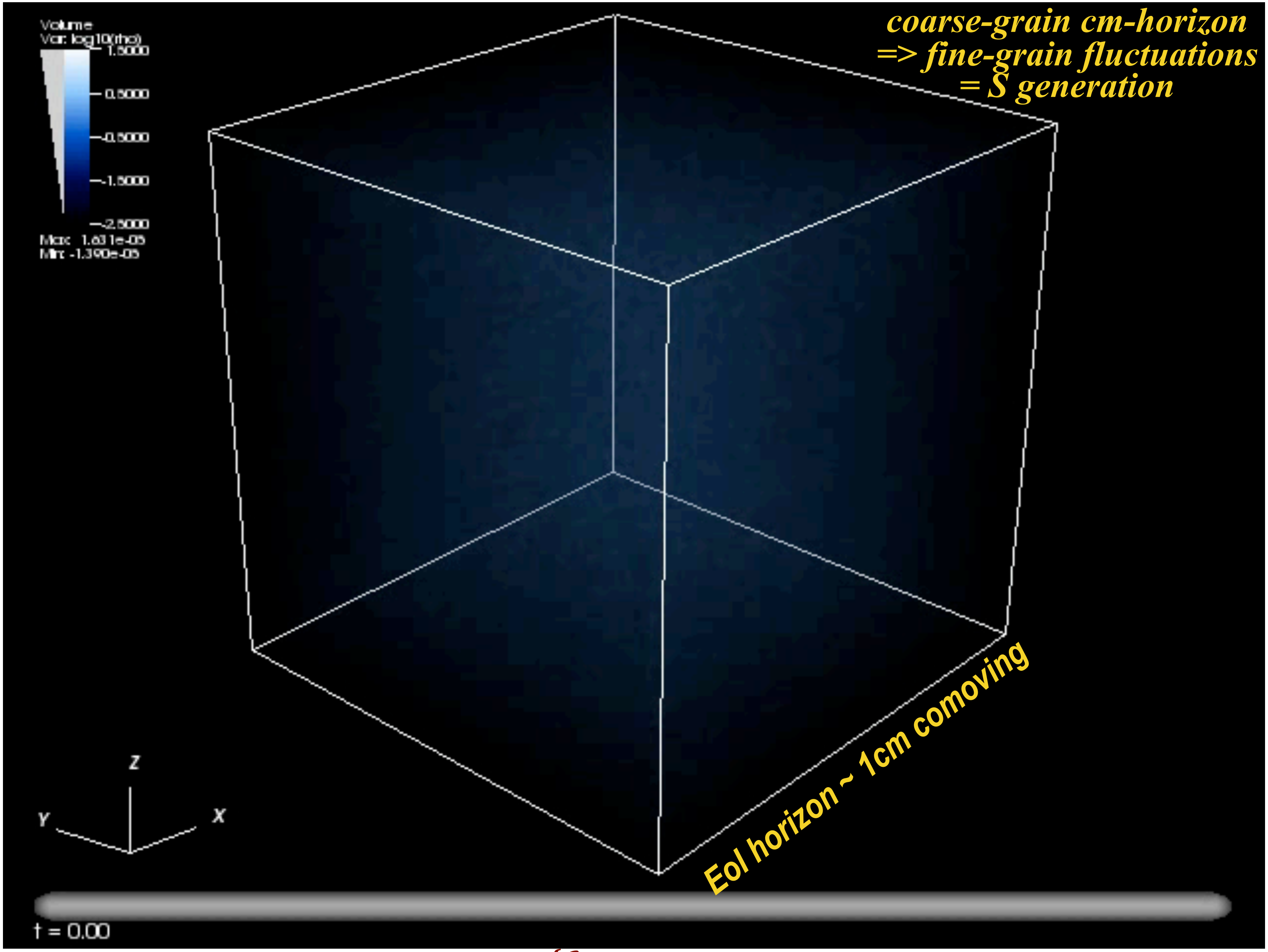
B2FH, b+braden+frolov+huang

***conformal potential-flattening eg Higgs inflation** SBB89 etc*



semi ETERNAL INFLATION

quartic inflaton $V(\phi,\chi) = 1/4\ \lambda\phi^4 + 1/2\ g^2\phi^2\chi^2$



***log-normal pdf (density** aka ζ), in k-bands too; normal pdf (velocity)*

$$T^a{}_b = \rho_{(c)} U^a U_b + U^a J_{(e)b} + U_b J_{(e)}{}^a + p_{(c)}(\delta^a{}_b + U^a U_b) + \Pi^a{}_b$$

spatial complexity** but ∃ **statistical simplicity (1-pt, 2-pt)

post-shock ⇒ total stress-energy $T^a{}_b$

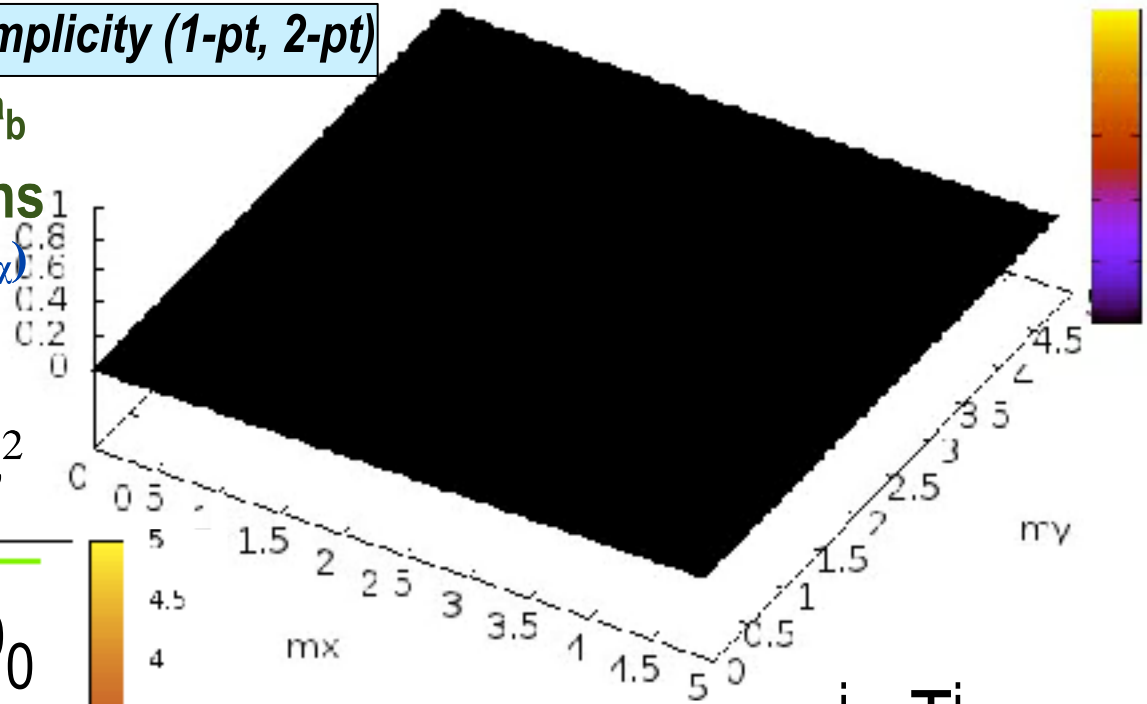
hydrodynamics description phonons

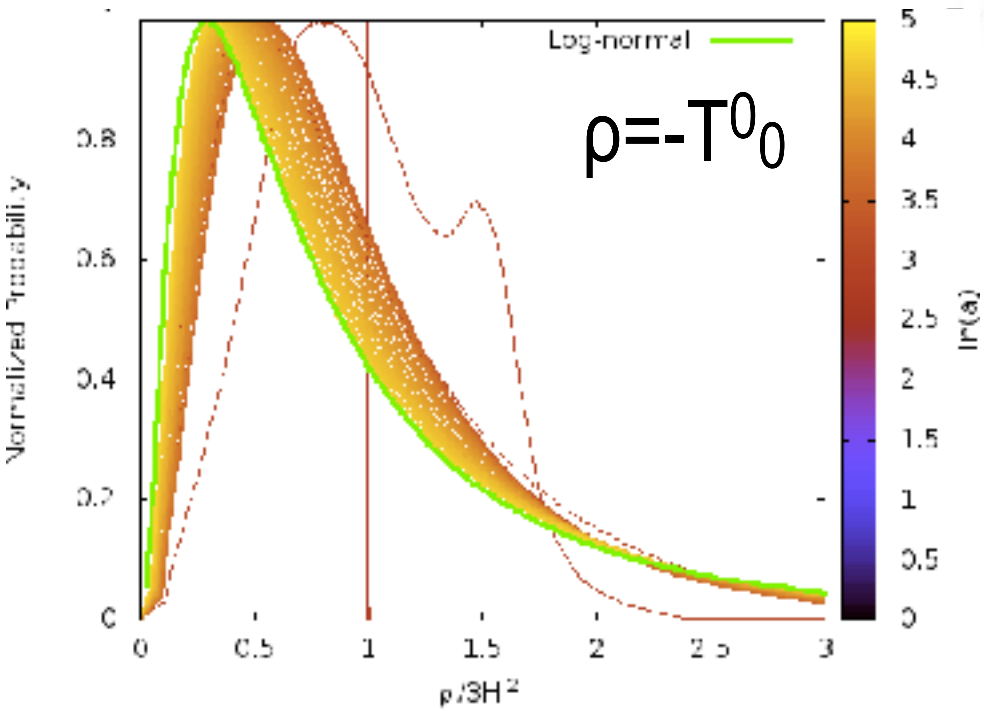
***entangled primary fields** ($\varphi, \Pi_\varphi, \chi, \Pi_\chi$)*

=> particle creation description

e.g., 100s of phonons at a time

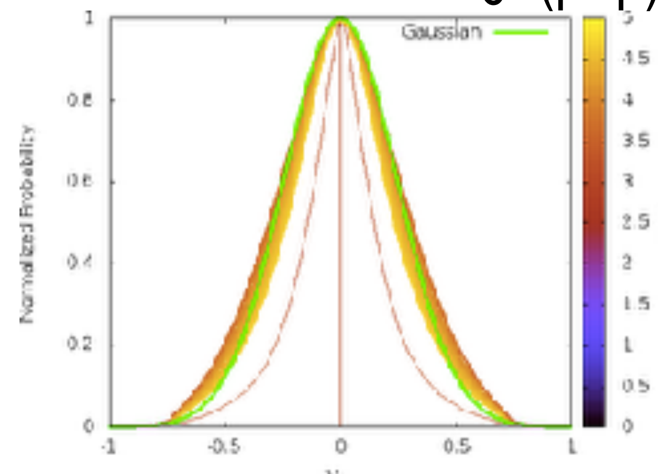
$$V(\phi,\chi) = 1/4\,\lambda\,\phi^4 + 1/2\,g^2\,\phi^2\,\chi^2$$





$\rho = -T^0{}_0$

$v^i = a T^i{}_0 / (\rho + p)$



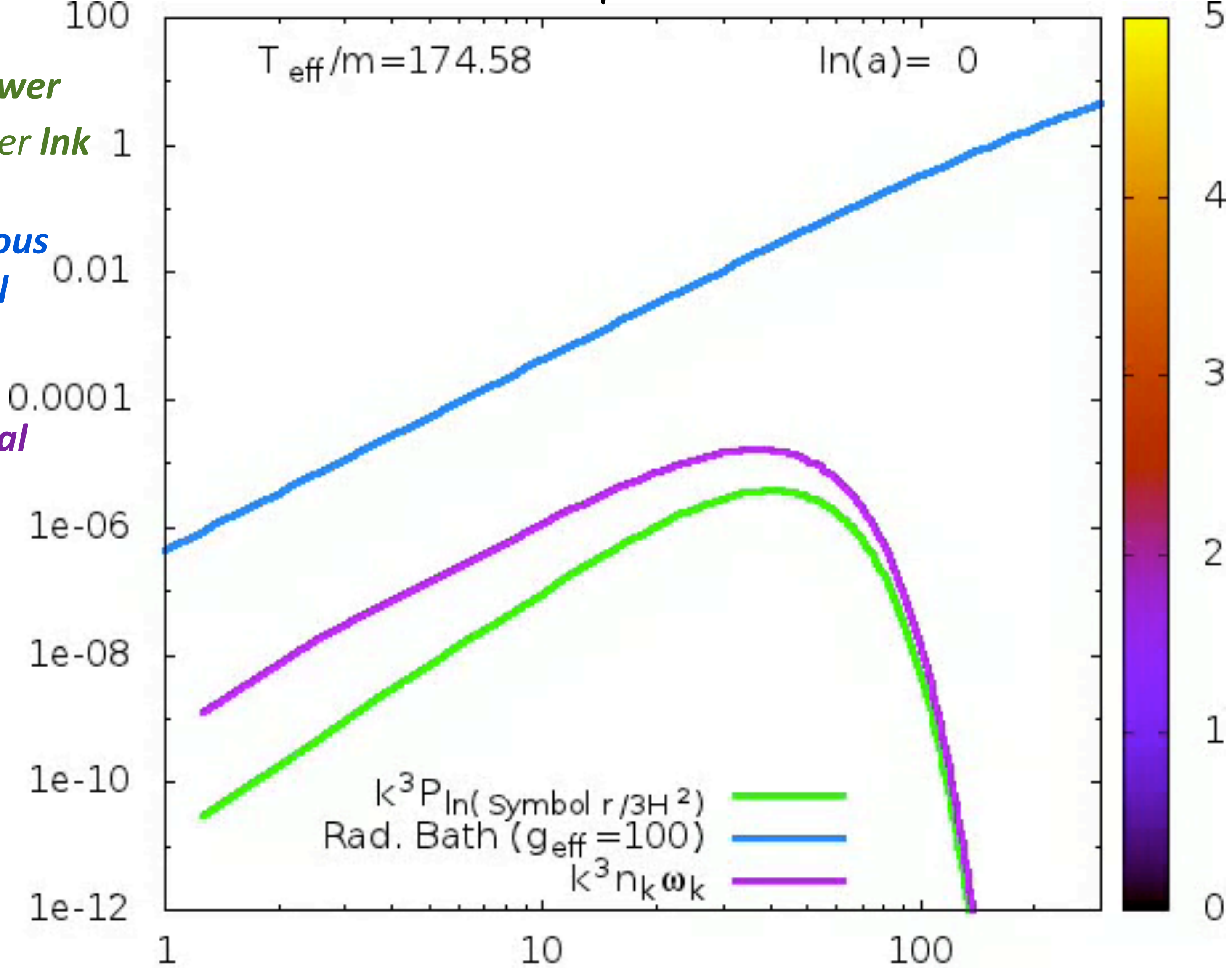
nearly Gaussian in ln ρ/<ρ> & in k-bands! ***B+Braden+Frolov*** ***nearly Gaussian in v***

coherent inflaton => incoherent mode cascade of fields thru a shock-in-time to thermal equilibrium

s_{Ui}~*0;* $s_{Utot,m+r}/n_b$ **~1.66x10^10 bits/b;** s_γ/n_γ *=5.2 bits/ϒ =2130/411*; s_ν *=21/22*

s_γ

ln ρ/<ρ> power spectrum per lnk
cf. instantaneous full thermal spectrum
cf. conventional energy spectrum using a pseudo particle occupation number



momentum

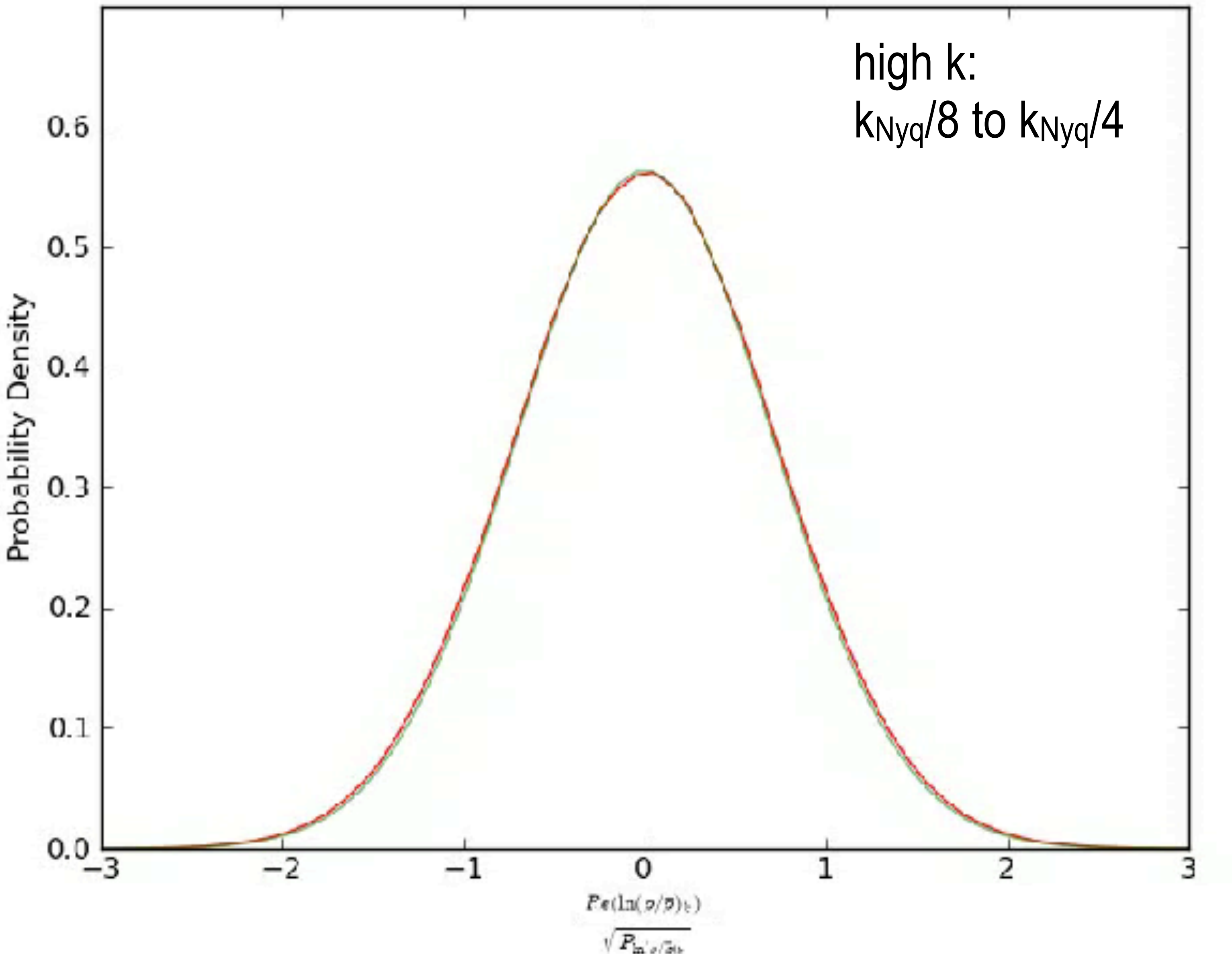
$V(\phi,\chi) = 1/2\, m^2 \phi^2 + 1/2\, g^2 \phi^2$

B+Braden+Frolov

box L=10m and N=1024^3

but Statistical Simplicity

FourierTransform(*ln* density) PDF ~ log-normal after initial transient



B+Braden+Frolov

coherent inflaton => incoherent mode cascade of fields thru a shock-in-time to thermal equilibrium

s_{Ui}~*0*; $s_{Utot,m+r}/n_b$ ~***1.66x10^10*** **bits/b**; s_γ / n_γ = ***5.2 bits/γ*** = *2130/411*; s_ν =*21/22* s_γ

ln ρ/<ρ> power spectrum

cf. instantaneous full thermal spectrum

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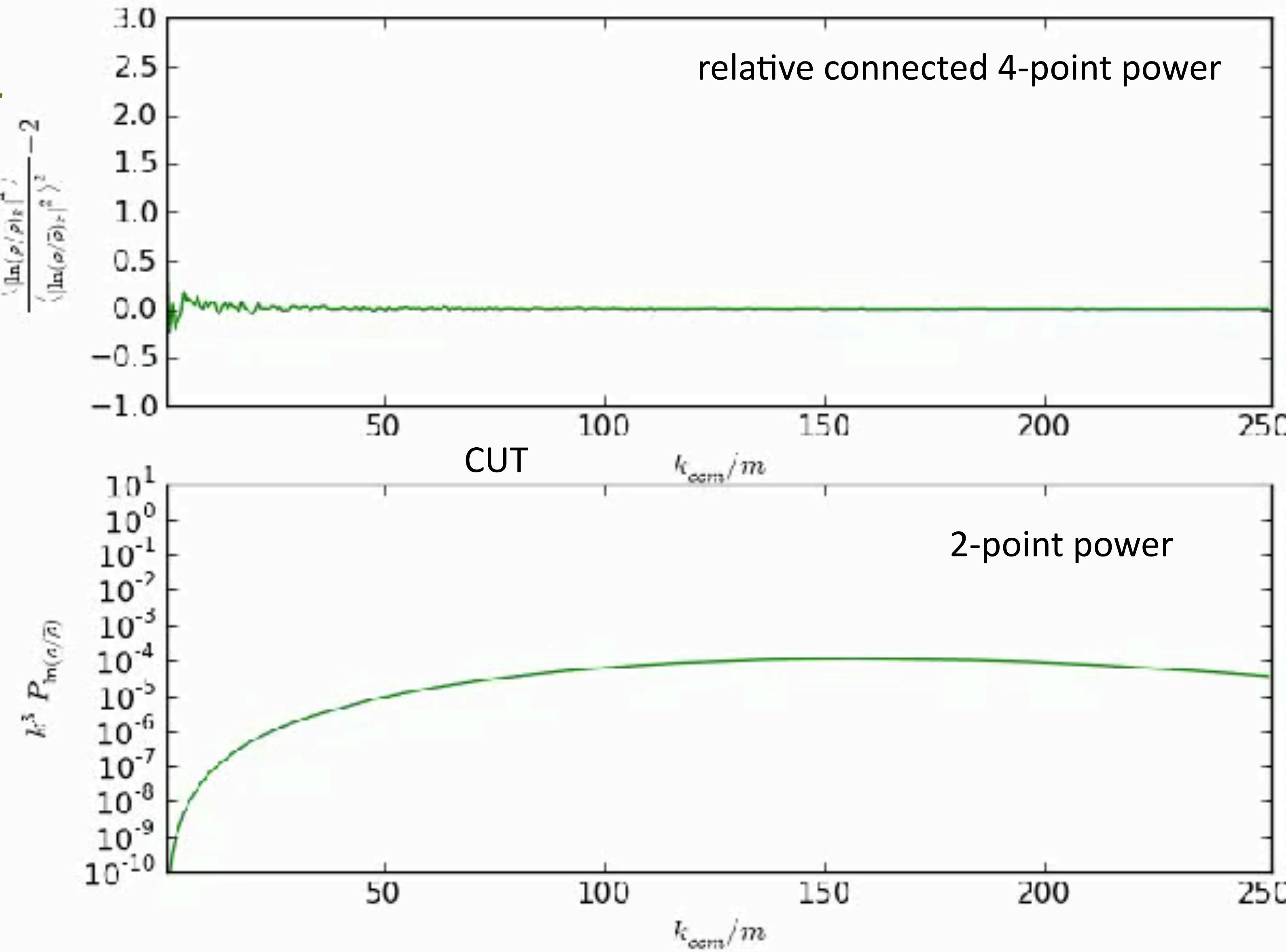
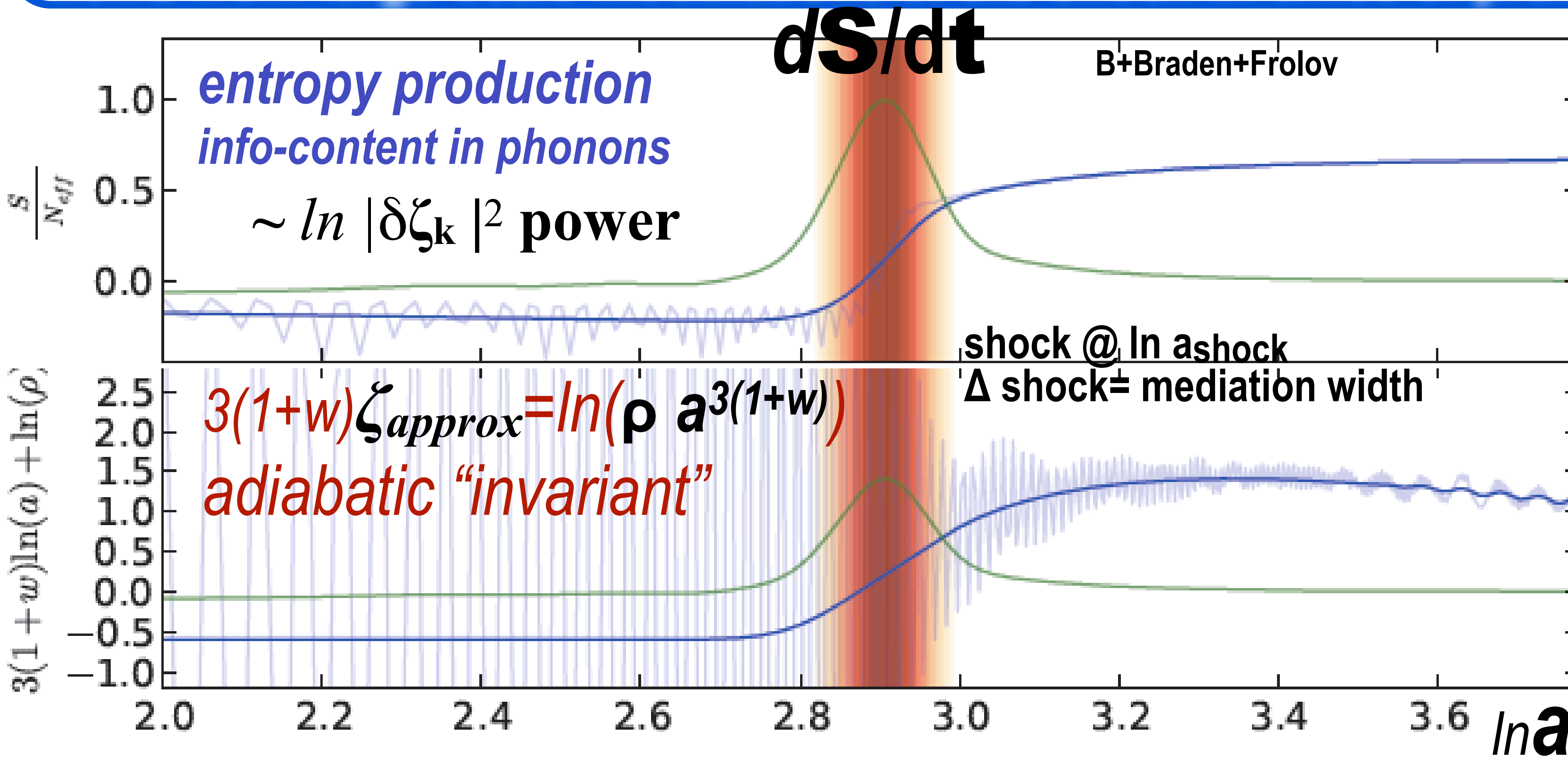


momentum

$V(\phi,\chi) = 1/2\, m^2 \phi^2 + 1/2\, g^2 \phi^2$

B+Braden+Frolov

nonG from large-scale modulations of the shock-in-times of preheating



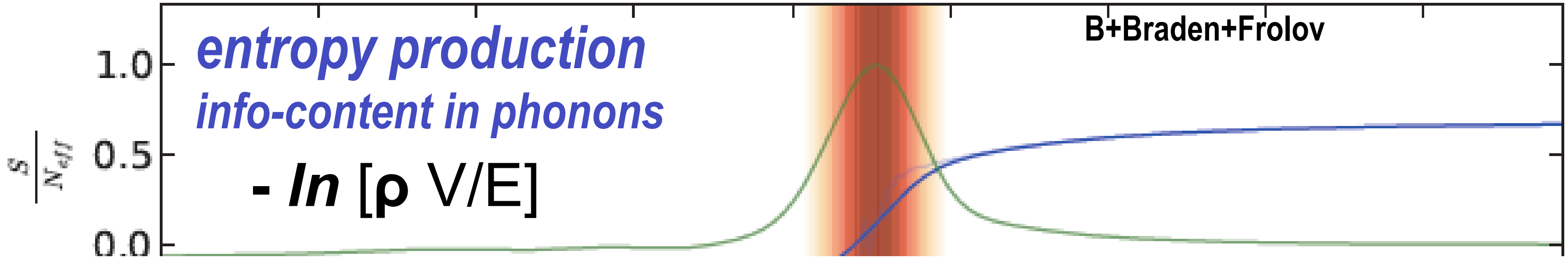
true thermal equilibrium far off ➤ & on to coupling to standard model degrees of freedom

coarse-grain <ζ> <=> fine-grain $\delta\zeta_k$ gradients, δV

$$\zeta_{final}(\mathbf{x}, t_f\ |\chi_{cg,eoi}(\mathbf{x}), g^2/\lambda) \sim \zeta_{shock}$$

$$V(\phi,\chi) = 1/4\ \lambda\phi^4 + 1/2\ g^2\phi^2\chi^2$$

nonG from large-scale modulations of the shock-in-times of preheating

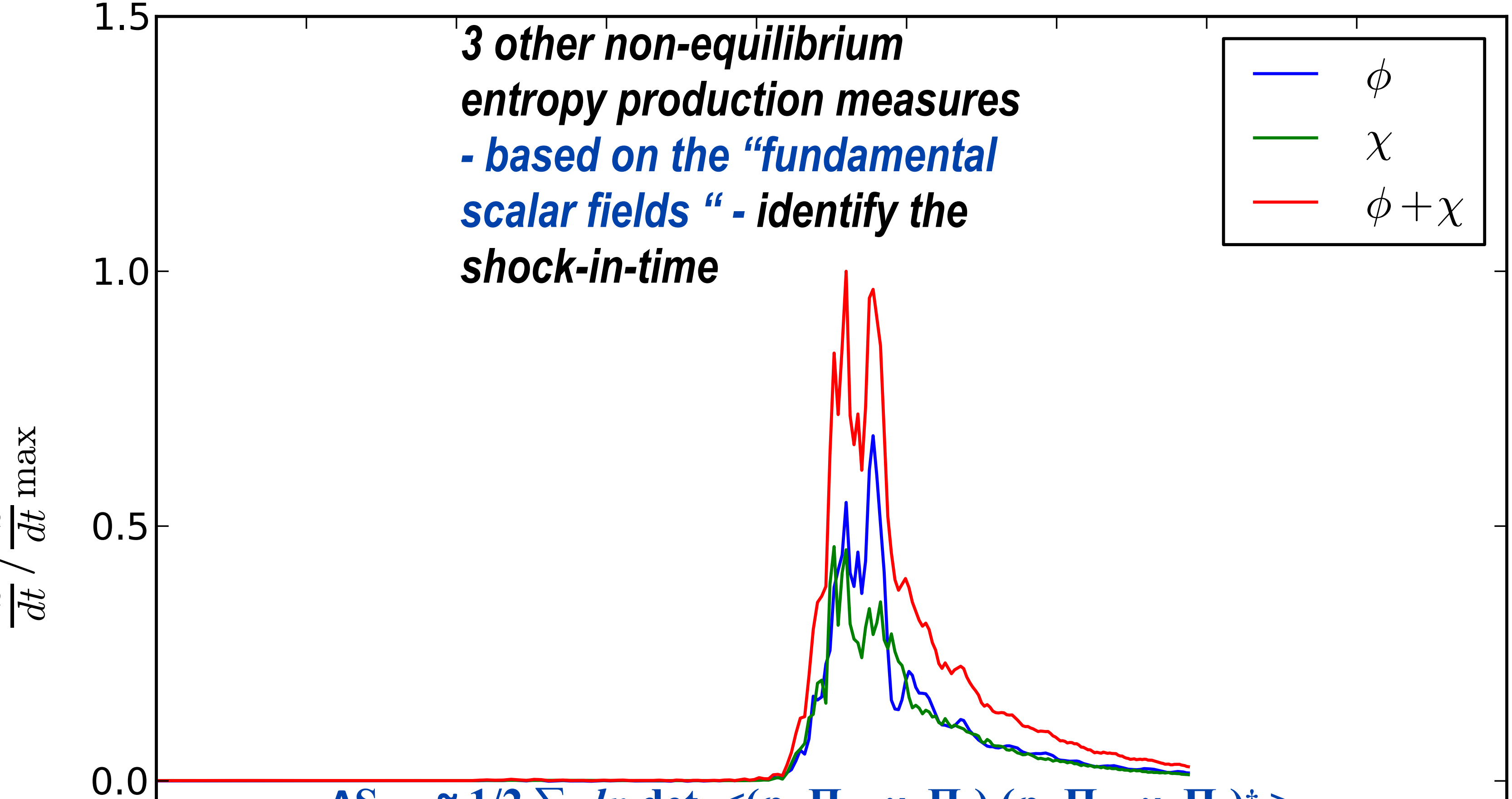


true thermal equilibrium far off ➤

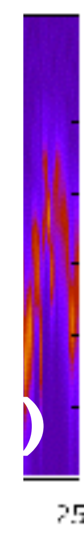
entangled primary fields $(\varphi, \Pi_\varphi, \chi, \Pi_\chi)$

to

decay rates (Feynman diagrams) and transport theory difficult to make accurate through preheating



of



$\Delta S_{\varphi+\chi} \sim 1/2 \sum_k ln \det \langle(\varphi, \Pi_\varphi, \chi, \Pi_\chi)(\varphi, \Pi_\varphi, \chi, \Pi_\chi)^\dagger\rangle$

A Shocking End to Post Inflation Mean Field Dynamics

Shock-in-space t = const

$v_{bulk}^2 > c_s^2 \Rightarrow v_{bulk}^2 < c_s^2$

supersonic ⇒ subsonic

Characteristic spatial scale

Jump Conditions: $\Delta T^{\mu\nu}$

Randomizing Shock Front: ΔS

Mediation: width via viscosity or collisionless dynamics

post-shock evolution, slow, of temperature, etc.

coherent condensate breakup?

Shock-in-time x = const (deviations for nonG)

$\langle\rho\rangle \gg \delta\rho \Rightarrow \langle\rho\rangle \ll \delta\rho$

Homogeneous ⇒ Fluctuations

Characteristic temporal scale

Jump Conditions: $\Delta T^{\mu 0}$

Randomizing mode cascade & Particle Production: ΔS

Mediation: width via gradients and nonlinearities

post-shock evolution, slow, of fluctuations

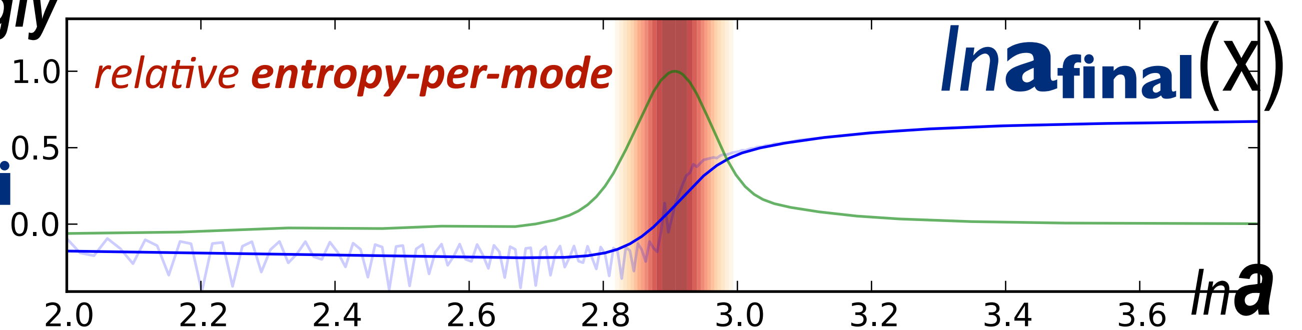
coherent condensate breakup E-phonon soup

- a difference: chaotic lead in to shock

Preheating is a shockingly efficient entropy source

$\ln a_{shock}(x)/a_{eoi}$

& the mediation width

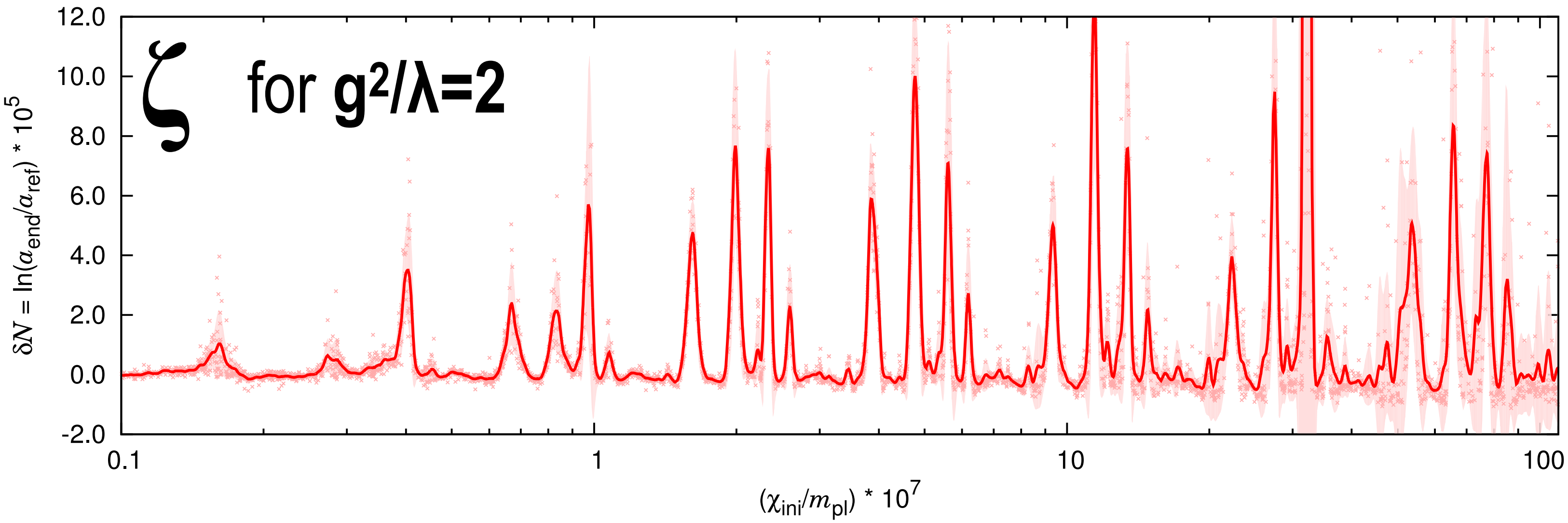


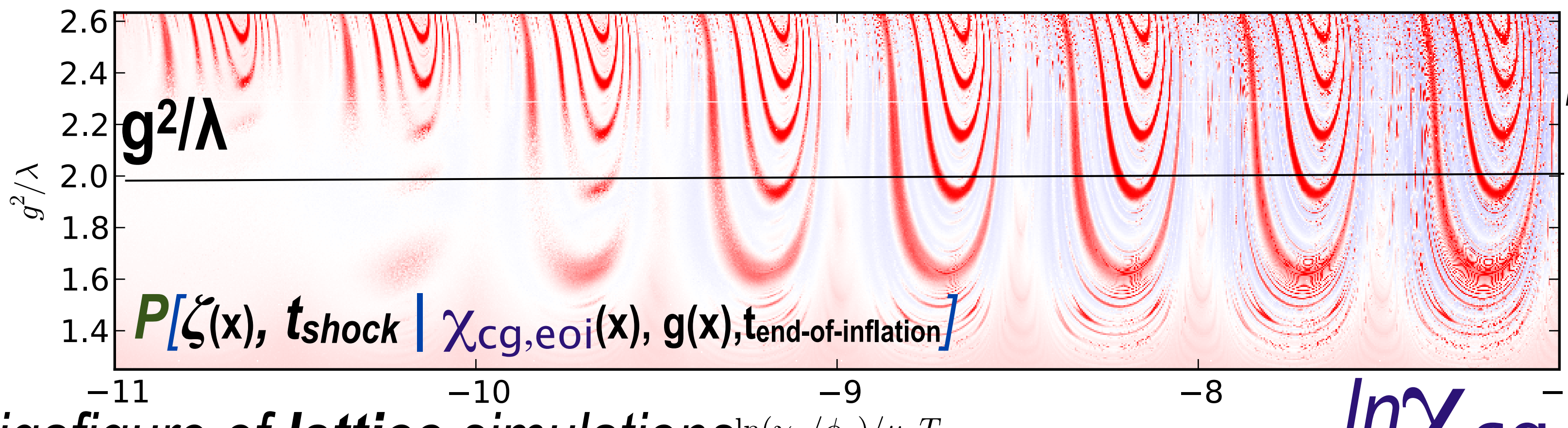
$dS/dt(t,g)$ => *the Shock-in-time: entropy production rate*

$\zeta_{shock}(\chi_{cg,eoi}(x)\ |g^2/\lambda))$ => ***Chaotic Billiards: NonG from Parametric Resonance in Preheating***

B+Frolov, Huang, Kofman 09
B+Braden, Frolov, Huang 18

$V(\phi,\chi)=1/4\ \lambda\phi^4 + 1/2\ g^2\phi^2\chi^2$





computational tour de force

huge number of 64^3 sims to show the wondrous complexity of $\zeta(\chi_i, g^2/\lambda)$

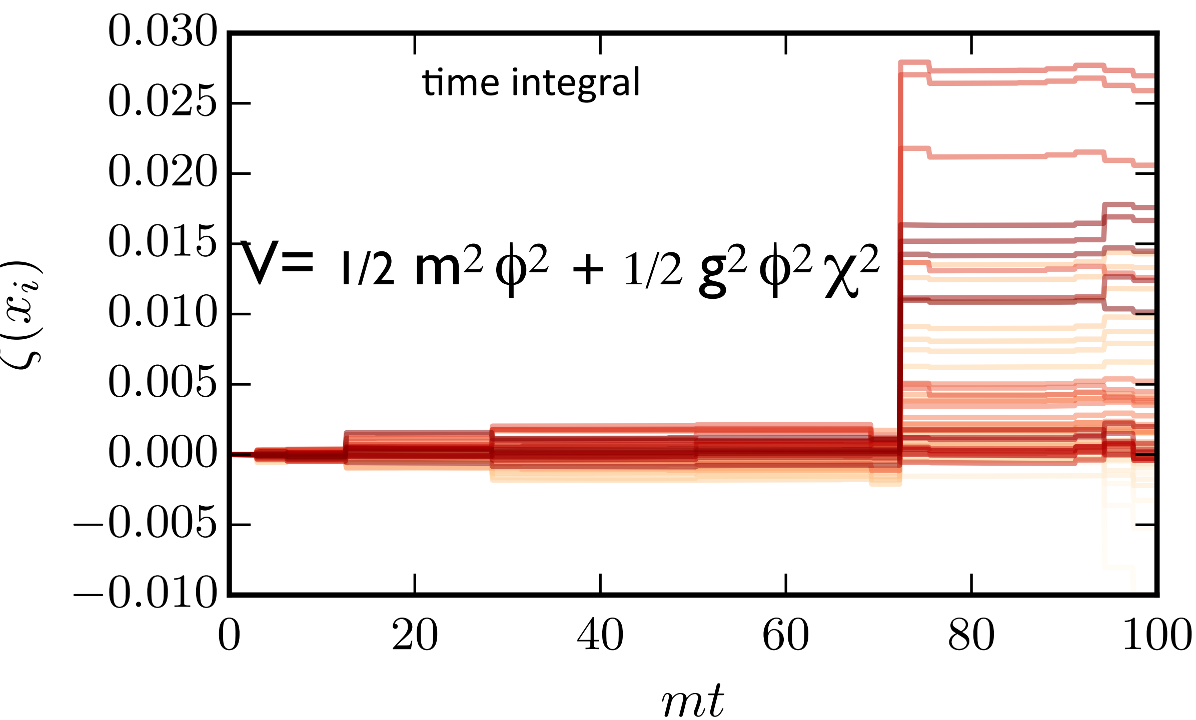
*gigafigure of **lattice** simulations*

$\ln\chi_{cg,eoi}$

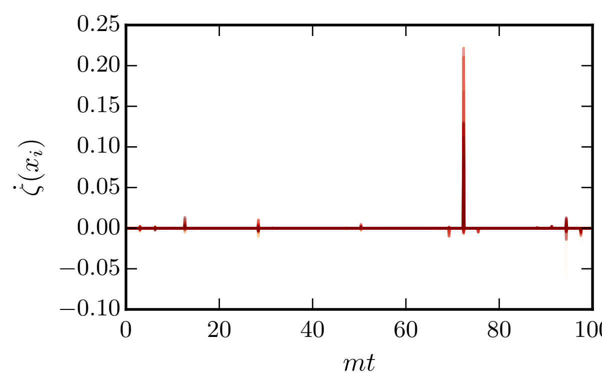
ζ conserved along trajectories until the "shock-in-time" when high **k** fluctuations (fine-grain) develop from coarse-grain, measure is $s_{Shannon} = -\ln \mathcal{P}\sqrt{g}$

but $-D\ln \mathcal{P}/dt = \mathrm{Trace}\ d\mathcal{E}/dt$ does change ~ KS entropy (rate)

stretching of phase strings. begin with anisotropic Gaussian at EoI and watch it stretch, $\mathcal{E}$ grows, rotates, locally OK as distorted ellipsoid, but strain depends upon the central value => phase tubes



evolution of the **ζ** source term



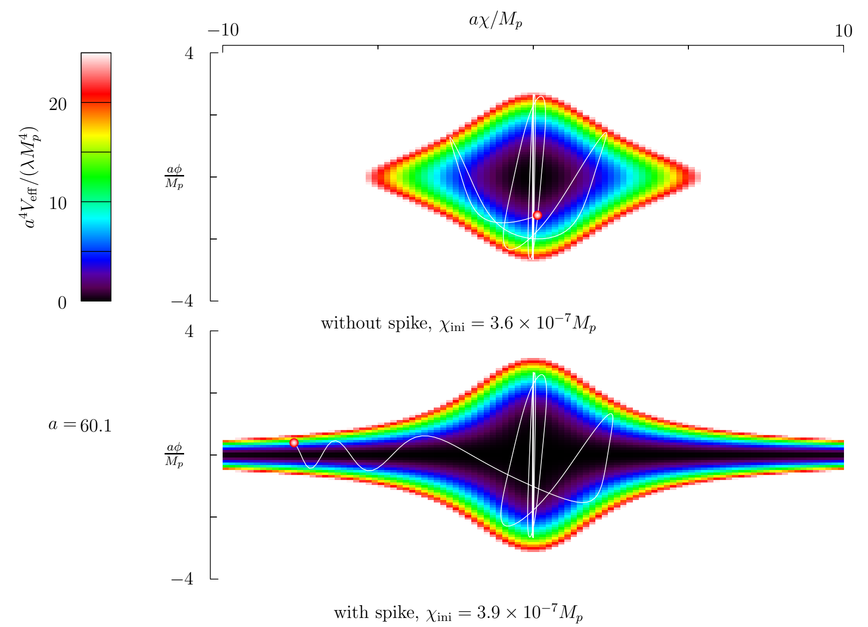
understanding the ζ -spike structure, B²FH

qualitatively YES quantitatively OK

arresting chaotic orbits via a shock-in-time, incoherent cf. coherent (caustic) trajectory bundles

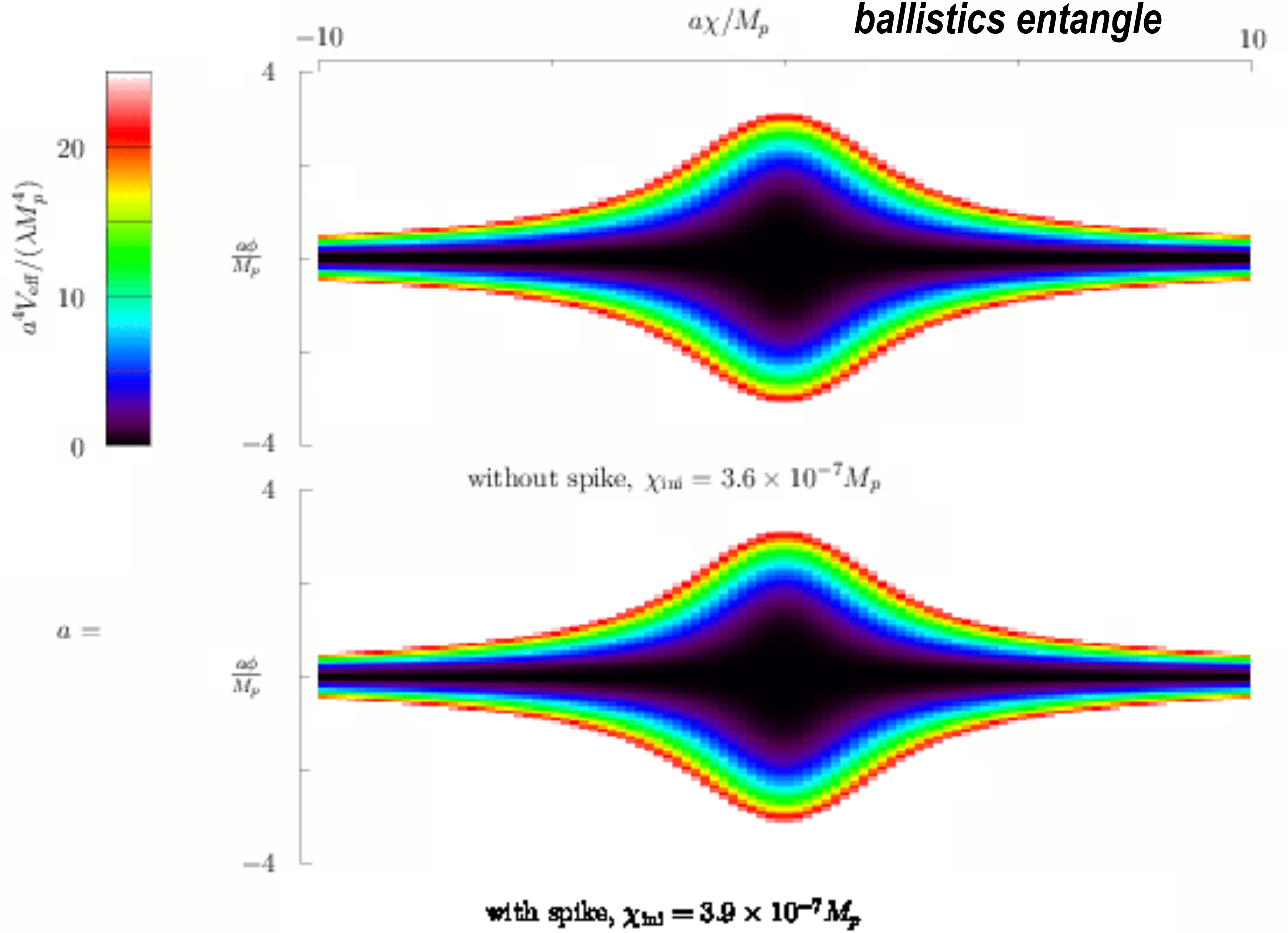
incoherent

coherent



full lattice simulations of coarse-grained k~0 trajectories ($\chi_{cg,eoi}$)

ballistics entangle



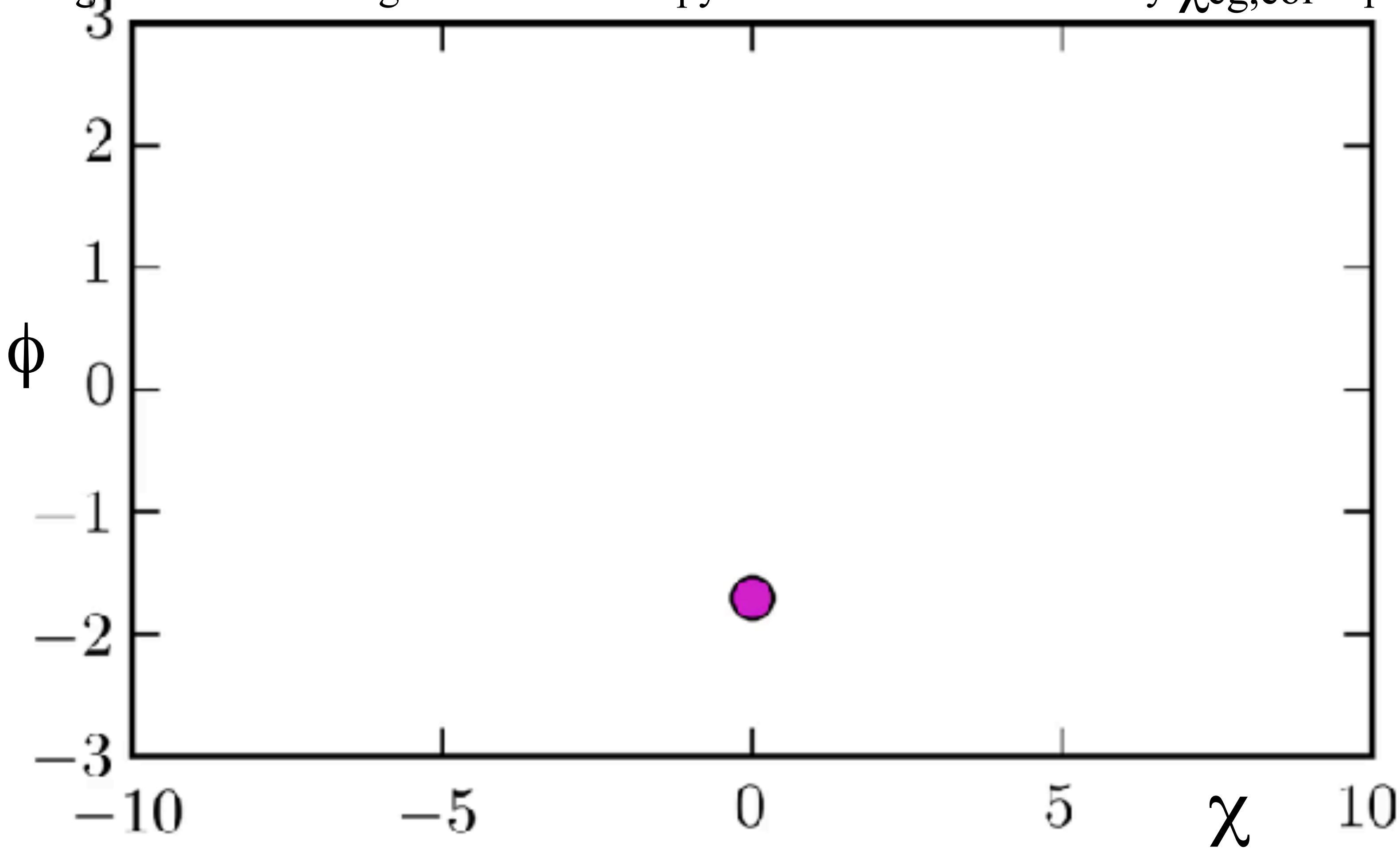
(nonlinear) **V_{eff} is trajectory-bundle dependent**

$V(\phi,\chi)=1/4\ \lambda\phi^4 + 1/2\ g^2 \phi^2 \chi^2$

ballistic billiards k=0 mode **phase space string** **evolution**

2D constrained distribution functions

stopping criterion when coarse-grained entropy of field variables rises <=> strain ε high,
ie when integral of the Kolomgorov-Sinai entropy reaches a threshold - very $\chi_{cg,eoi}$ dependent



$V = 1/4\ \lambda\phi^4 + 1/2\ g^2\phi^2\chi^2$

caustics in <q^A> *ballistic* **orbits**

$<\delta q^A\ Xt2\ |\ \delta q^B\ Xti> \sim \exp(\mathcal{E}(Xt2\ |\ Xt1))^A{}_C <\delta q^C\ Xt1\ |\ \delta q^B\ Xti>$

early U parameters: **final φ, Π_φ, χ, Π_χ, *ln* a, ln ρ, initial $\chi_{cg,eoi}$, *couplings* g, λ, ..**

parameter strain tensor in field space $\mathcal{E}^A{}_C(Xt2\ |Xt1)$ deformation $e^{\mathcal{E}}$

$d\mathcal{E}^A{}_C/dt$ strain rate ~local Lyapunov coefficients *Floquet instability charts*

instability to have nearby parameters diverge => chaotic billiards

Kolmogorov-Sinai entropy: ~ Sum of positive evalues of $d\mathcal{E}/dt$

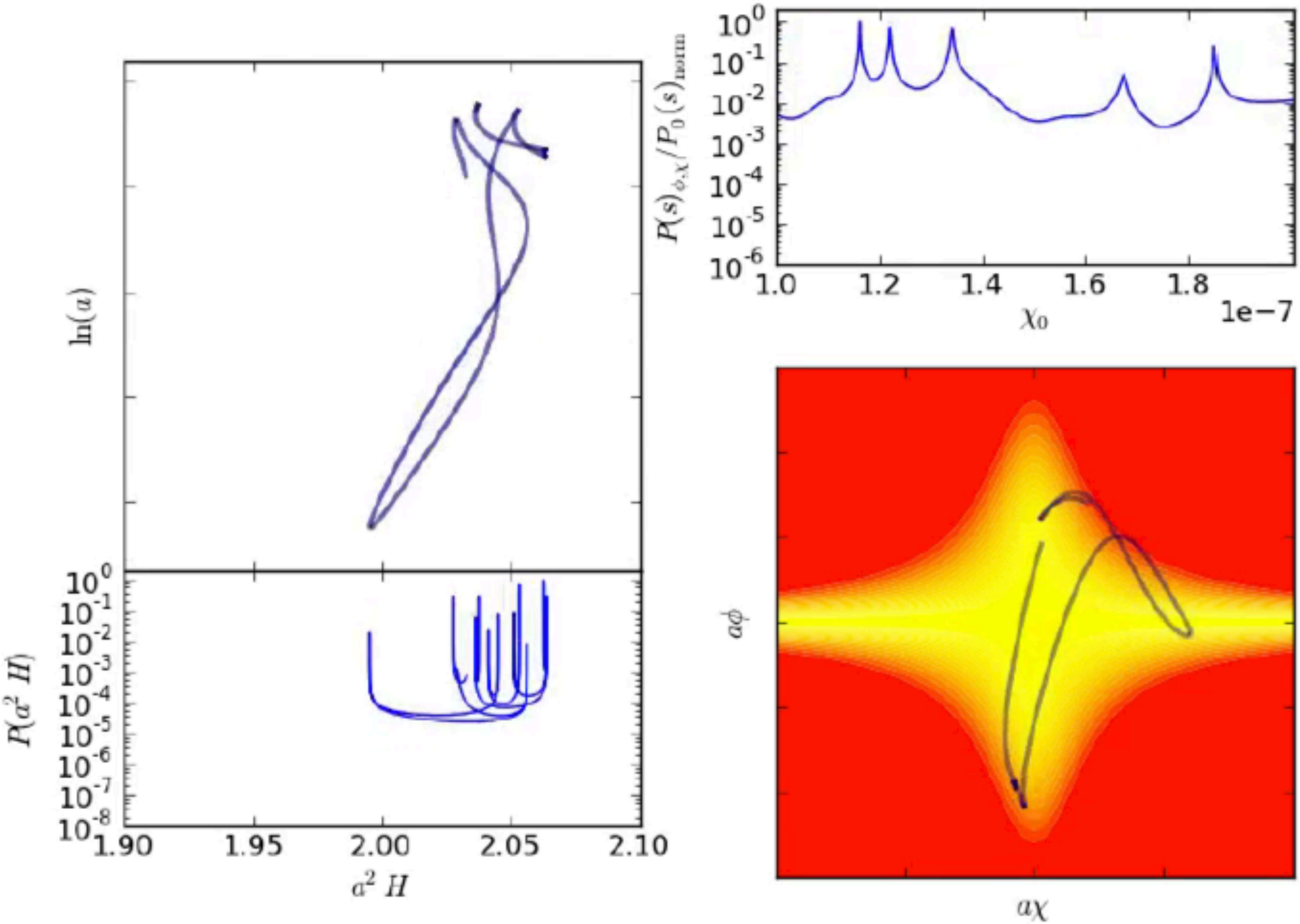
small $\mathcal{E}^A{}_A$ eigenvalues=> coherent trajectory bundles (for a time)

= caustics (inverse -> ∞) **1/ [$\partial\, \alpha\, /\, \partial\, \chi_{cg,eoi}$]**; => peaks in $\zeta\, (\chi_{cg,eoi})$

stopping time **tstop** **(**$\chi_{cg,eoi}$**)** when $\mathcal{E}^A{}_A$ evalues get large <=> local gradients $\Uparrow$

cf. LargeScaleStructure: final Eulerian position <= initial Lagrangian position

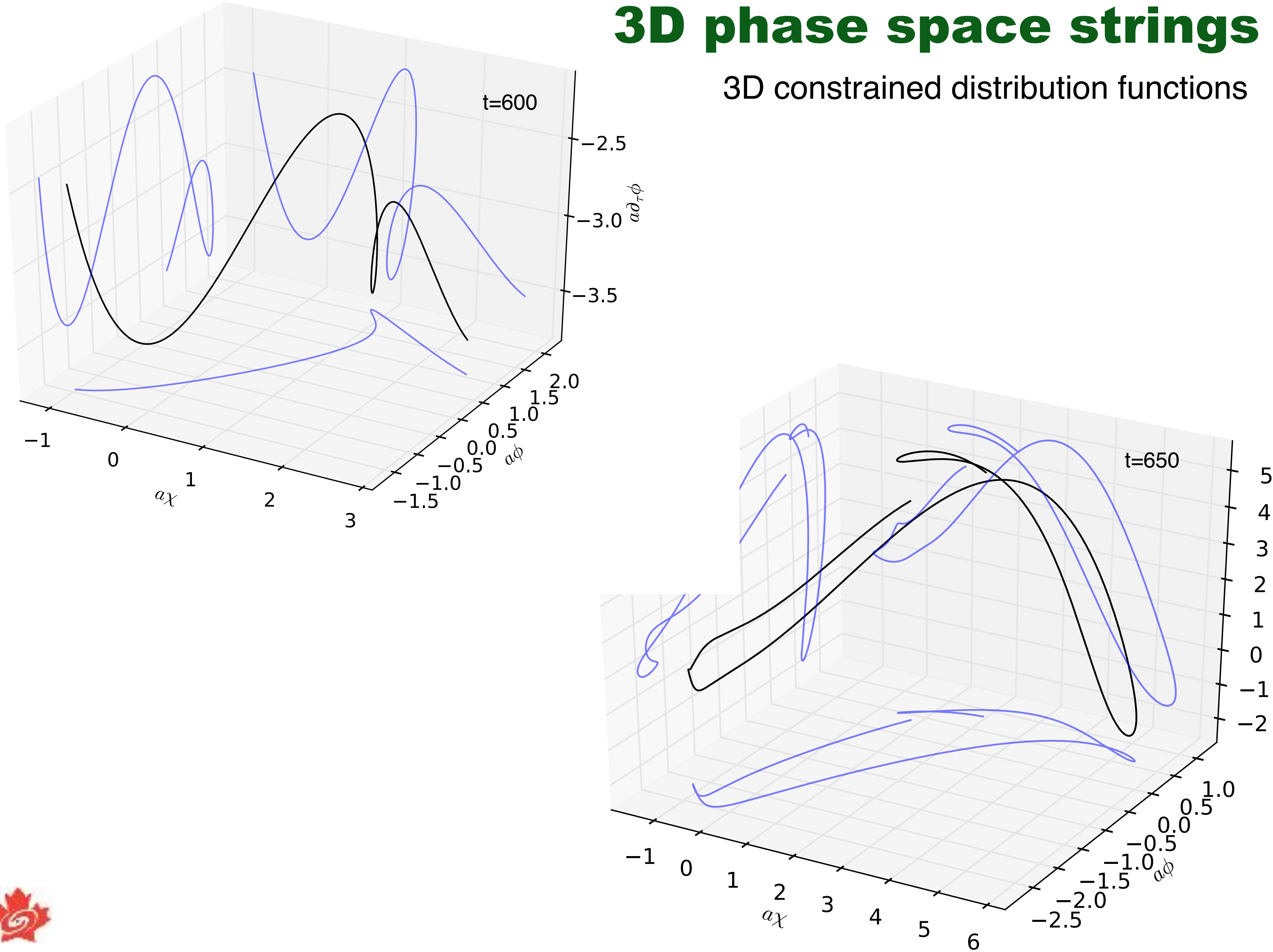
1LPT aka Zeldovich: $\partial x/\partial r = \exp(\mathcal{E})$ -> 0 density $\rho \sim \exp(-\mathrm{Tr}(\mathcal{E}))$ -> ∞



$\ln(a)$
10^{0} 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8}
$P(a^2\,H)$
1.90 1.95 2.00 2.05 2.10
$a^2\,H$
10^{0} 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6}
$P(s)_{\phi\chi}/P_0(s)_{\rm norm}$
1.0 1.2 1.4 1.6 1.8
1e−7
χ_0
$a\phi$
$a\chi$

3D phase space strings

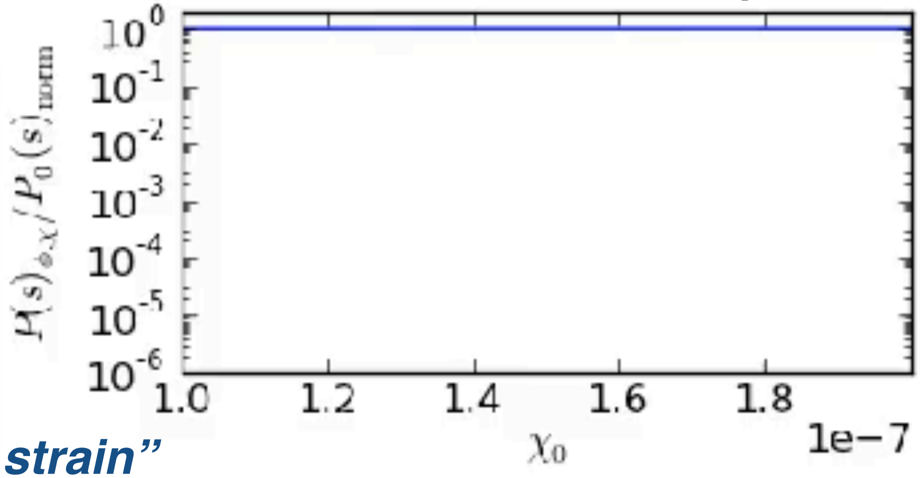
3D constrained distribution functions



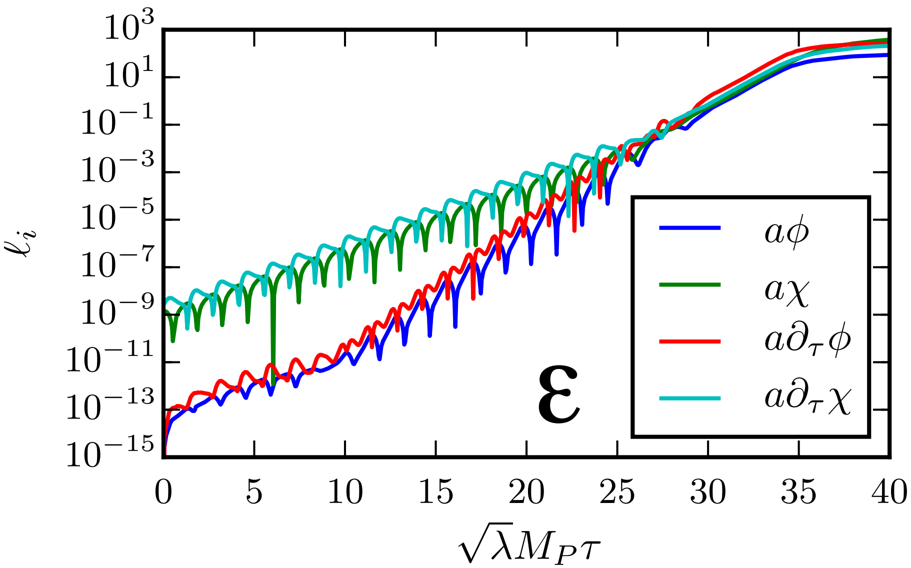
phase space strings

caustics are ubiquitous

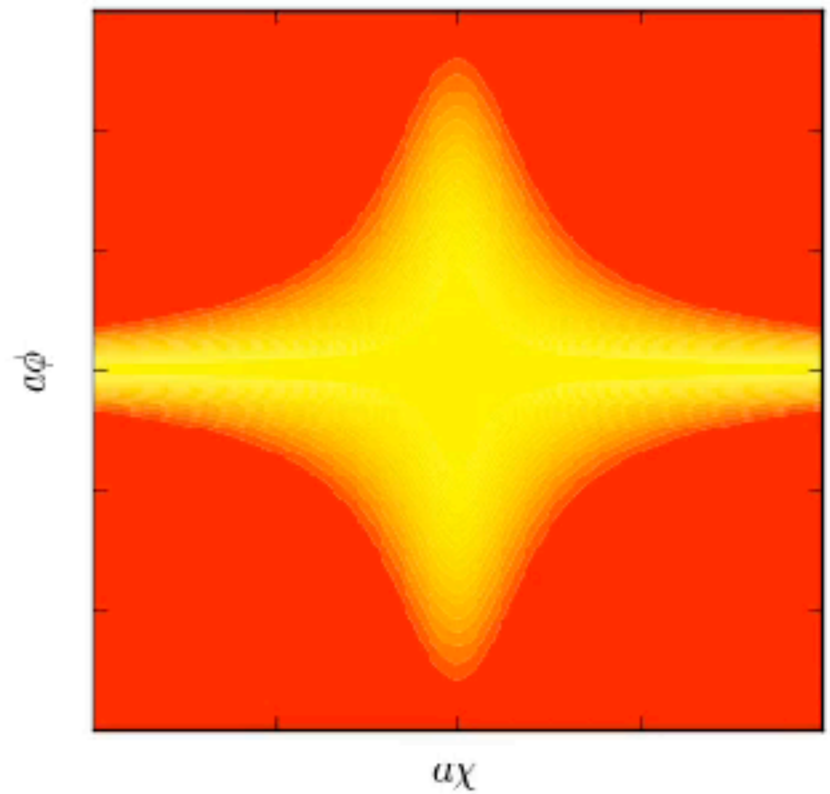
2D constrained distribution functions



phase string growth in time "parameter strain"
integral of Kolmogorov-Sinai entropy *rate*



=> 3D constrained distribution functions



B2FH, b+braden+frolov+huang

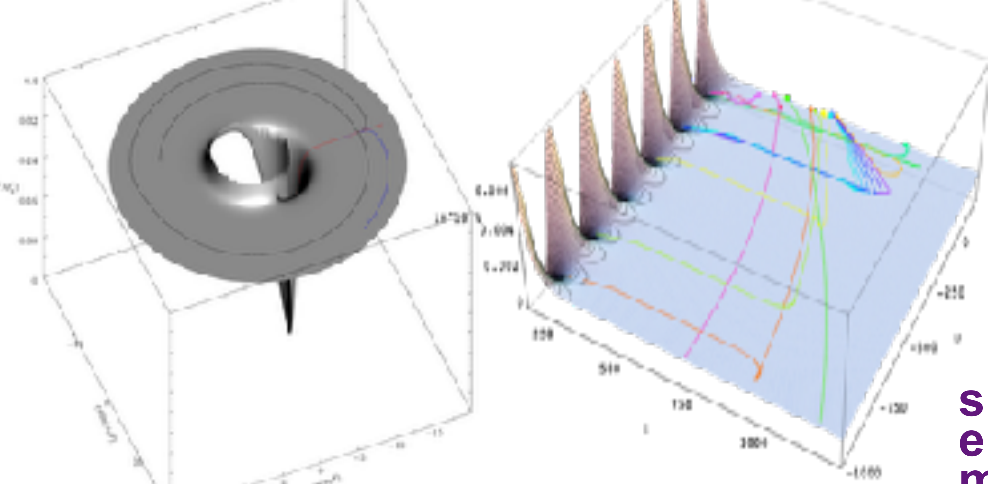
single field V heating slow, oscillating

but shaped V can give rapid heating (roulette)

coarse-grain cm-horizon =>
fine-grain fluctuations = S
generation

longitudinal instability
KS yes but no LSS modulation

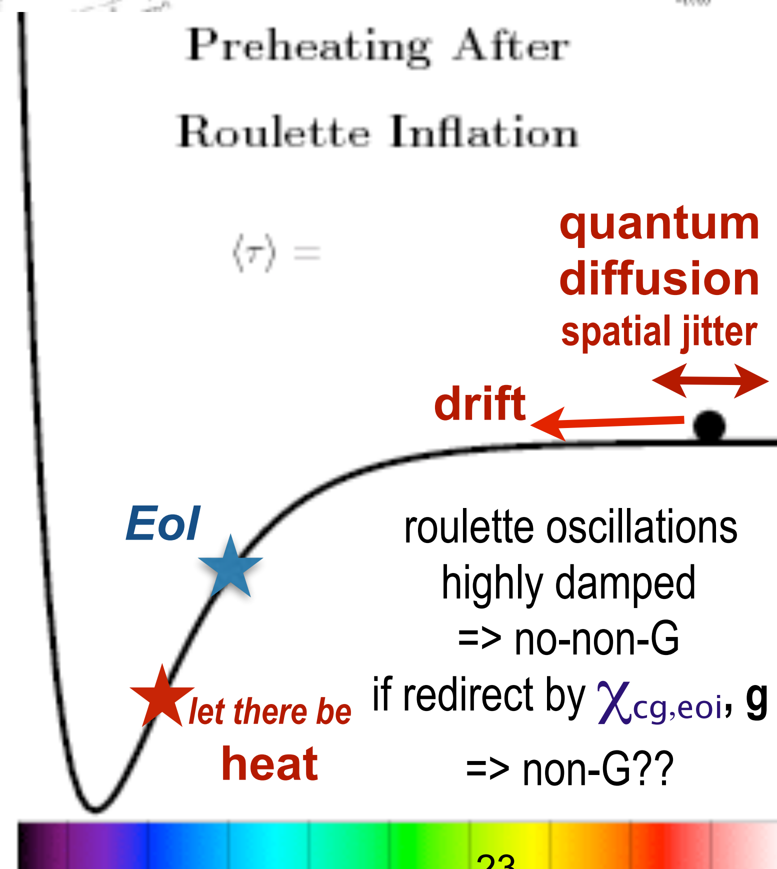
$a =\quad 1$



Preheating After

Roulette Inflation

$\langle \tau \rangle =$



semi ETERNAL INFLATION

A visualized 2D slice

in lattice simulation

EoI horizon ~ 1cm comoving

www.youtube.com/watch?v=FW__su-W-ck&NR=1

Barnaby,Bond,Huang,Kofman09

stochastic inflation *Vilenken, Starobinski* Salopek+B 90/91 & …

extend to inflation - gentle short-stretch chaos & nonG?

Stochastic inflation: insert a moving bipartite uniform k-boundary into the full field equations, cg/fg split at $k_c(t)$~lnHa => coarse-grain condensate + fine-grain quantum fluctuations

$dq_c^A = V_c^A dT + K^A_\nu \sqrt{dT}\, \eta^\nu$ (GRD) via gradient expansion
time **T=lnHa** (breaks down at eoi, but best hypersurface for wave fronts)
diffusion tensor $D^{AB} = (KK^\dagger)^{AB}/2$

+ **(linearized) fluctuation equations for q_f^A** k-modes. slow X_c possible
(constrained) Fokker-Planck equation for Shannon entropy $s(q) = -\ln \mathcal{P}(q)\sqrt{g}$

$\partial s/\partial T + (V_c + V_D)^A \partial s/\partial q^A - \partial (V_c + V_D)^A / \partial q^A = 0$ *or [ds/dT (fg->cg)]*
$\sqrt{g}$ **= parameter-volume deformation**

KS entropy *rate* **~Ds/dt ~ Trace shear** *(positive eigenvalue sum)*

diffusion velocity $V_D^A = D^{AB} \partial s/\partial q^A$ & current $J_D^A = e^{-s} V_D^A$
trajectory divergence via shear = 1/2 Trace ln g = $\partial V^A/\partial q^B$

aside: momentum kicked off the attractor is quickly damp down to the attractor => attractor approx $V_c^A \sim \partial S(q_c^A)/\partial q_c^A$ for field momentum

back to preheating:
through eoi $\mathbf{D^{AB}}$ is small, ballistic $\mathbf{dq_c^A = V_c^A dT}$ but chaotic if shear eigenvalues are positive (Kolmogorov-Sinai "entropy rate" >0)
until nonlinear couplings (shock-in-time)
often t_scramble well-separated from t_dissipation *in the MSS sense*
examples: correlated perturbative nonG cf. uncorrelated nonG subdominant to inflaton zeta fNL spike chaotic billiards

trajectory approach to nonG post-inflation:

$\mathbf{d\langle\zeta| \chi_{eoi}\rangle = Response(\chi_{eoi})\, d\chi_{eoi}}$ aka $\mathcal{E}(\zeta \mid \chi_{eoi})$ integrates to $\mathbf{\langle\zeta_{NL}| \chi_{eoi}\rangle}$

general: $\langle\zeta_{NL}| \chi_{eoi}, g, ..\rangle$ (x) via marginalization over UV (to k~1 Mpc^{-1}) and constrain in IR k <H_0 for LSS/CMB applications
complication/joy: ζ is conserved in the ballistic phase, sudden generation by fluctuation generation. but Trace shear is non-zero, the KS entropy => Shannon entropy story again

tools: fast lattice codes defrost++ and spectral code.
very fast dynamical systems theory calculations of various potentials, with conformal parameters, modified kinetic pieces in Lagrangian

condensate/fluctuation framework, classical-like approach with hbar + Bogoliubov transformations for fluctuations as condensate evolves => particle creation & fluctuation freeze-out into new condensate

stochastic inflation **| q_c : q_f>**
Langevin network evolution step: **$q_c(X,T+dT) = q_c(X,T)\, V_c dT + \delta q_f$**

condensate evolution step **$|q_c(T+dT)\rangle = \exp(V_c dT)\, \exp(\delta q_f)\, |q_c(T)\rangle$**
schematic **$\delta q_f (x,T) = \sum_{k\text{-band}} (Q_k^*(x,T) a_k^T - Q_k(x,T) a_k)$**
$V_c dT = V_c(x,T) dT (a_x^T - a_x)$
annihilation/creation operators in position and momentum a_x, a_x^T a_k, a_k^T

fluctuating part $|q_f\rangle \sim \exp(\sum \delta q_f)\, |0\rangle$ *a coherent state description?*
what is the relation to the usual **$q_{f,op} = \sum_k (Q_k^*(x,T)\, a_k^T + Q_k(x,T)\, a_k)$**
operator linear in Bunch Davies vacuum operators a_k, a_k^T (sign difference)

overcomplete basis representation, but conforms to a classical lattice simulation of inflation (no bipartite split).
still stuck with the gradient expansion for **$|q_c(T)\rangle$.**
& mixed operators $V_c dT$ and $\delta q_f(x,T)$ - can this approach work?

trajectory bundles & particle creation during inflation. **fast instabilities eg transverse + nonlinearity** or else **ζ-conservation** with no generation.
eg in state enters "chaotic unstable V-region" leaves as out state

END

how generic will caustic preheating be? structure around potential minima:

=> ‘filamentary’ potentials

=> ballistic flow channels

multi-filaments may lead to caustics

2 std inflaton, slow heating? roulette V is fast. 3-star

4 case workhorse. *the* ***5-star… ‘axionic' angles works with conformal flattening of $V(\phi_A)$ +***

cf. filaments that join at clusters in the LSS web

gentler potential structure during inflation? role for instabilities

how modulated caustics in preheating could give observable intermittency

via isocon power on large & super-horizon scales

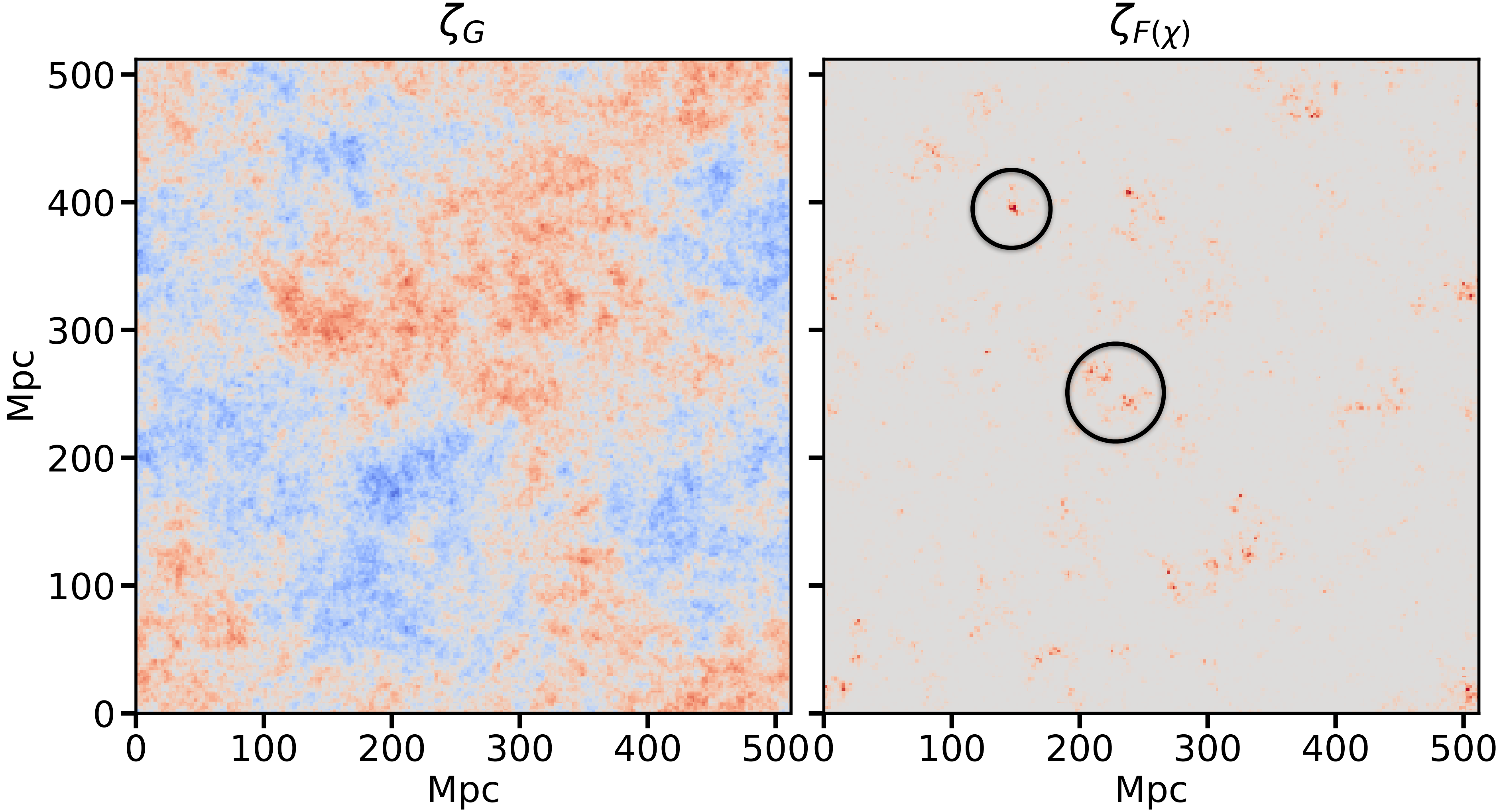
=> *light particles* ($\chi_{eoi}(x)$, couplings $g(x)$, …)

these isocons are active, NOT spectators

Primordial Non-Gaussianity in the Peak Patch method:

Intermittent Non-Gaussian case

***uncorrelated* ζ[GRF]**



Primordial Non-Gaussianity in CO

$\sigma_8 = 0.82$

In all cases