Magnetic helicity in the interstellar medium

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physique et chimie du milieu interstellaire

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1. What?
2. Why?
3. How?

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2. Why?
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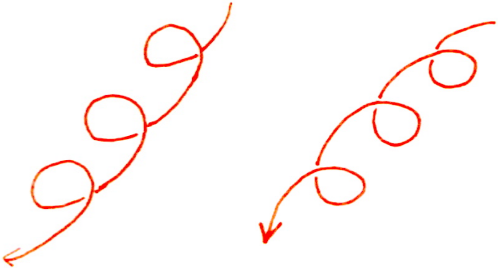
Magnetic helicity – the handedness of the field lines



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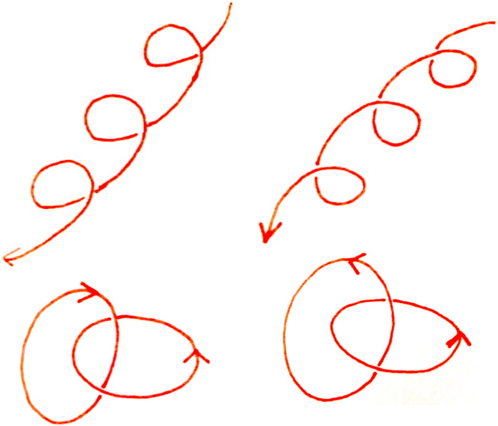

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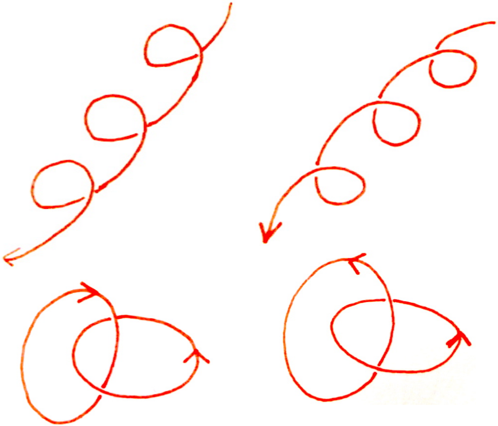

$$H = \int_V \mathrm{d}^3x\, \vec{A} \cdot \vec{B}$$

conserved if:

- conductivity high
- nothing happens at the surface of V

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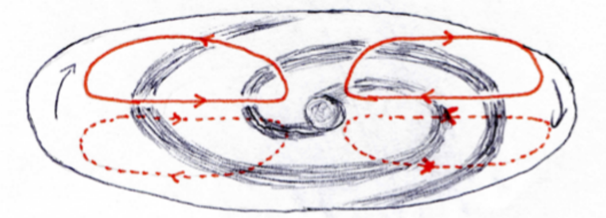
The α-Ω-dynamo



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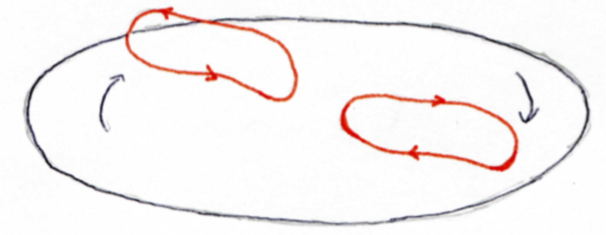

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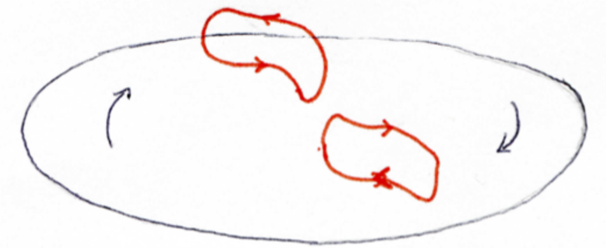

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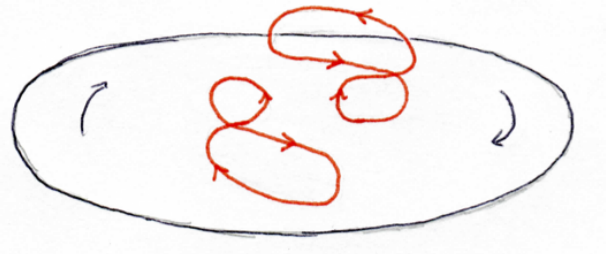

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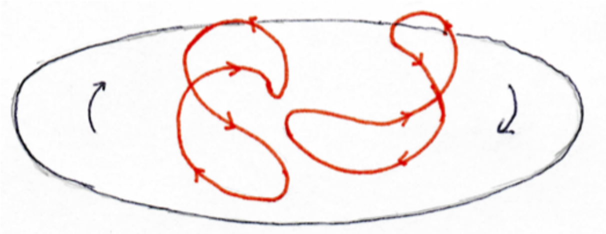

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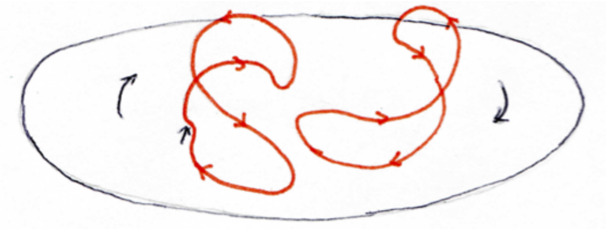

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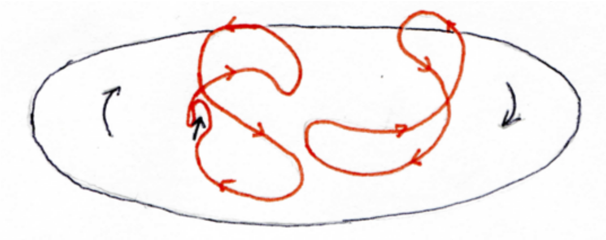

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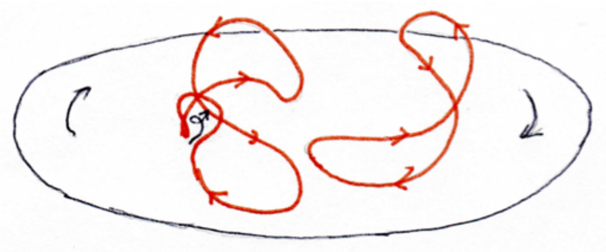

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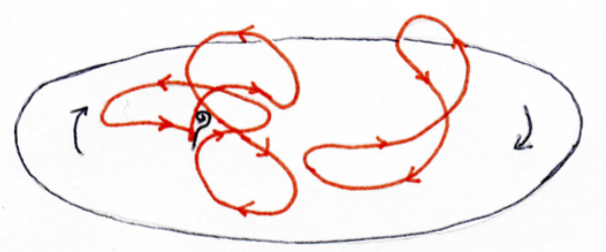

Ingredients: mean-field theory

induction equation:

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{v} \times \vec{B} \right) + \eta \, \Delta \vec{B}$$

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with $\vec{B} = \langle \vec{B} \rangle + \delta\vec{B}$ and $\vec{v} = \langle \vec{v} \rangle + \delta\vec{v}$:

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$$H = \int_V \mathrm{d}^3x\,\vec{A}\cdot\vec{B} \qquad\qquad \frac{\partial H}{\partial t} = -2\eta\int_V \mathrm{d}^3x\,\vec{j}\cdot\vec{B} \approx 0$$

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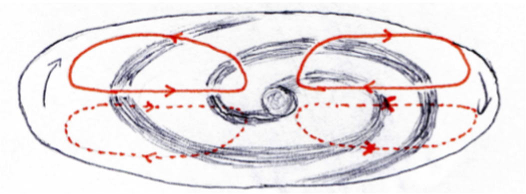

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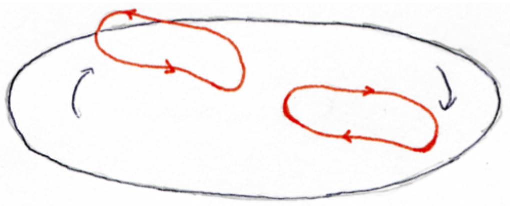

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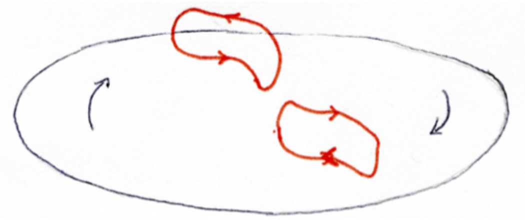

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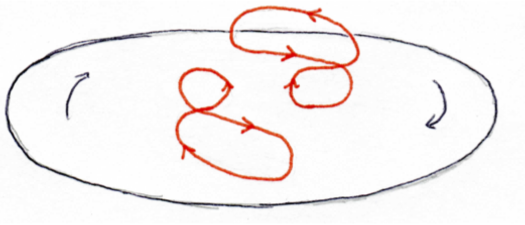

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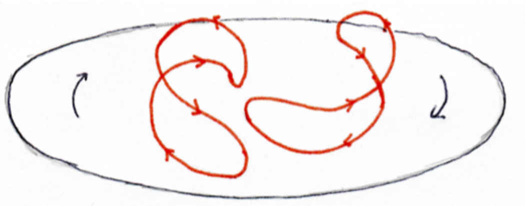

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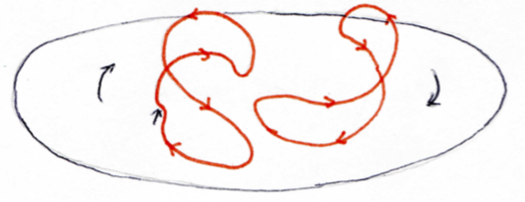

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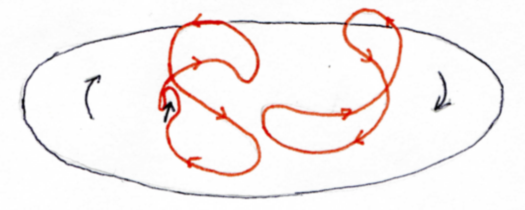

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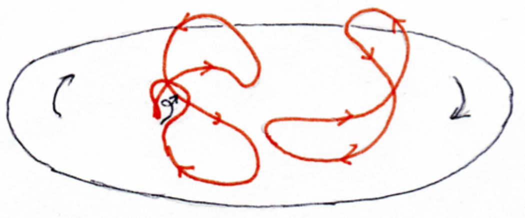

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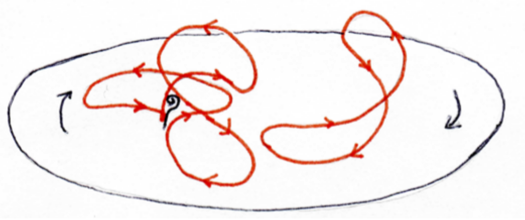

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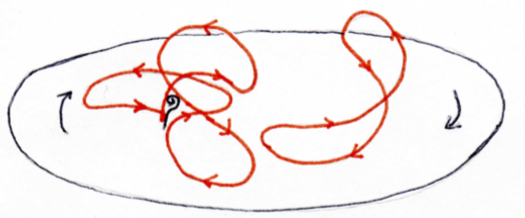
$$\frac{\partial\langle\delta H\rangle}{\partial t} \approx -2\int_V \mathrm{d}^3x\,\vec{\mathcal{E}}\cdot\langle\vec{B}\rangle$$

Ingredients: mean-field theory + helicity conservation

$$\frac{\partial\langle\vec{B}\rangle}{\partial t} = \overbrace{\underbrace{-\left(\langle\vec{v}\rangle\cdot\vec{\nabla}\right)\langle\vec{B}\rangle}_{\text{advection}} + \underbrace{\left(\langle\vec{B}\rangle\cdot\vec{\nabla}\right)\langle\vec{v}\rangle}_{\text{stretching}}}^{\text{large-scale motions}} + \vec{\nabla}\times\underbrace{\left\langle\delta\vec{v}\times\delta\vec{B}\right\rangle}_{\vec{\mathcal{E}}} + \eta\,\Delta\langle\vec{B}\rangle$$

"electromotive force": $\vec{\mathcal{E}} \approx \overbrace{\left[\,\underbrace{\left\langle\delta\vec{j}\cdot\delta\vec{B}\right\rangle}_{\text{current helicity}} - \underbrace{\left\langle\delta\vec{v}\cdot\left(\vec{\nabla}\times\delta\vec{v}\right)\right\rangle}_{\text{kinetic helicity}} + \ldots\right]}^{\alpha}\langle\vec{B}\rangle + \ldots$

$$\overbrace{\text{current helicity}} \approx \left\langle\delta\vec{A}\cdot\delta\vec{B}\right\rangle$$



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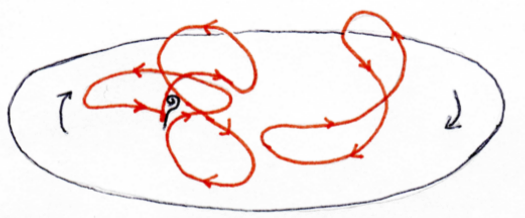

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problem: Helicity quenches the dynamo.

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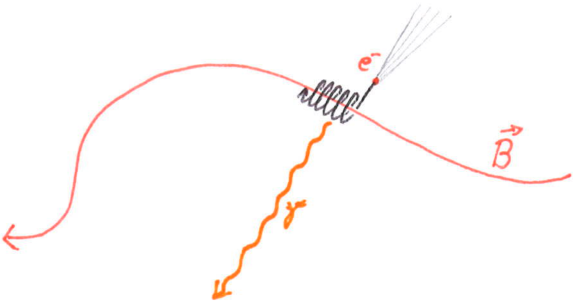
solution: Move helicity around.

Predictions

- If kinetic helicity due to cyclonic turbulence:
  $\Rightarrow$ opposite signs above and below the plane
  e.g.: Ferrière (1998), Brandenburg et al. (2014)
- Due to helicity conservation:
  $\Rightarrow$ opposite signs on small and large scales
  e.g.: Subramanian (2002)
- If helicity flux dominates kinetic helicity:
  $\Rightarrow$ same sign on all scales, but opposite signs above and below the plane
  e.g.: Vishniac et al. (in prep.)

1. What?
2. Why?
3. How?

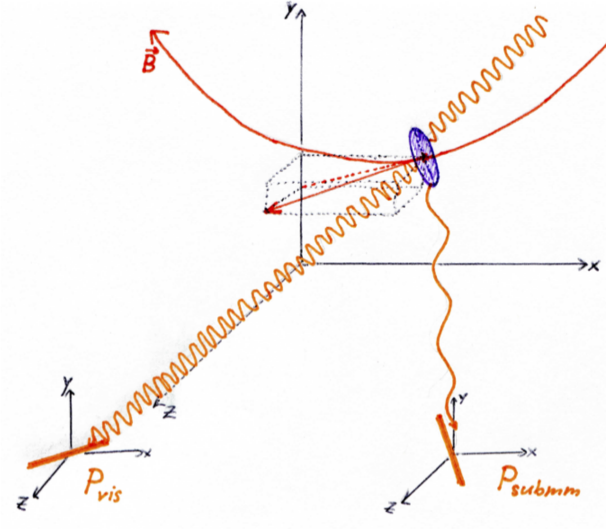
Synchrotron



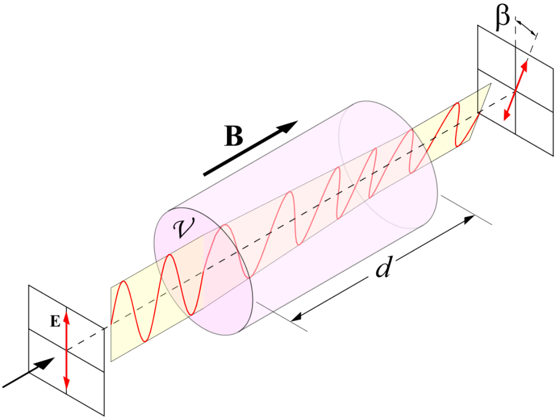
for $n_{\mathrm{CRE}}(E) \propto E^{-\gamma}$:

$$P(\lambda) = Q(\lambda) + iU(\lambda) \propto \lambda^{\frac{\gamma-1}{2}} \int \mathrm{d}z\, n_{\mathrm{CRE}}\, B_\perp^{\frac{\gamma+1}{2}}\, \mathrm{e}^{2i\chi}$$

Dust

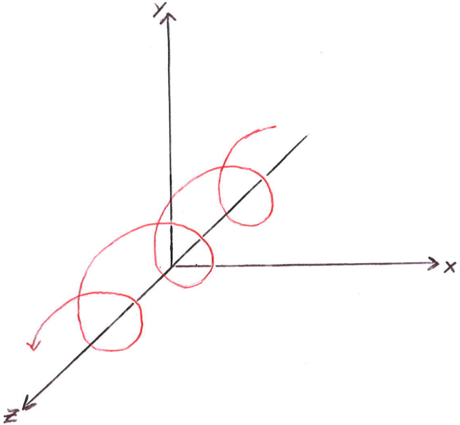


Faraday rotation



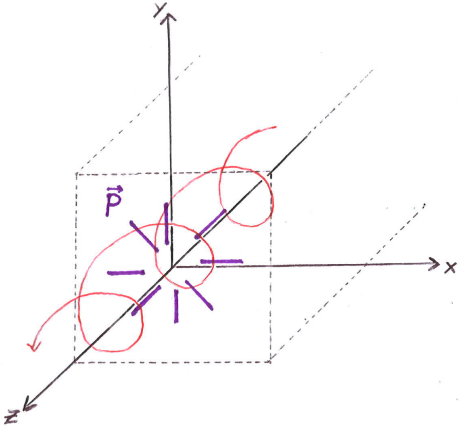
$$\beta \propto \lambda^2 \, \phi(r_{\rm source})$$

$$\phi(r_{\rm source}) \propto \int_{r_{\rm source}}^{0} n_{\rm e}(\vec{x}) \, B_r(\vec{x}) \, {\rm d}r$$



`LITMUS` procedure

Junklewitz et al. (2011)
Oppermann et al. (2011)



LITMUS procedure

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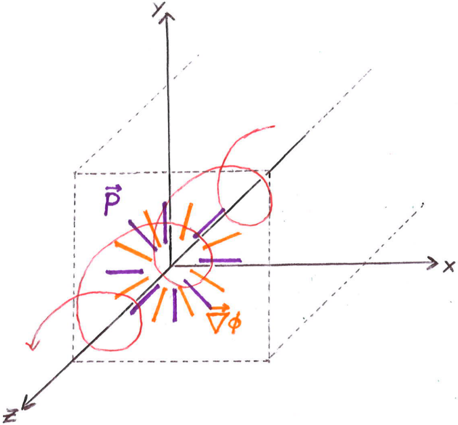

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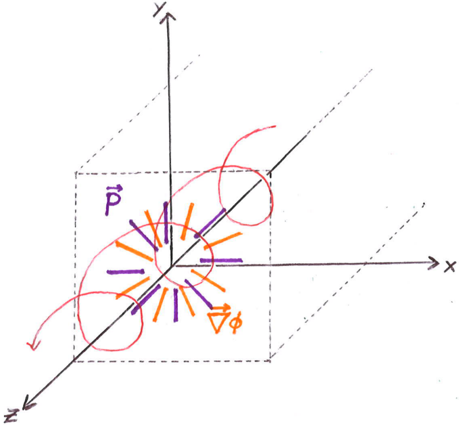
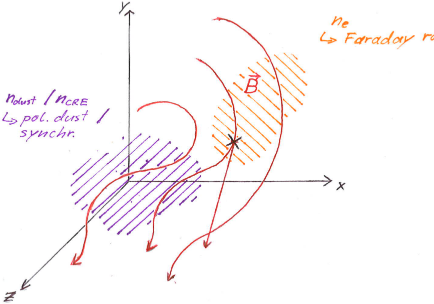


but:

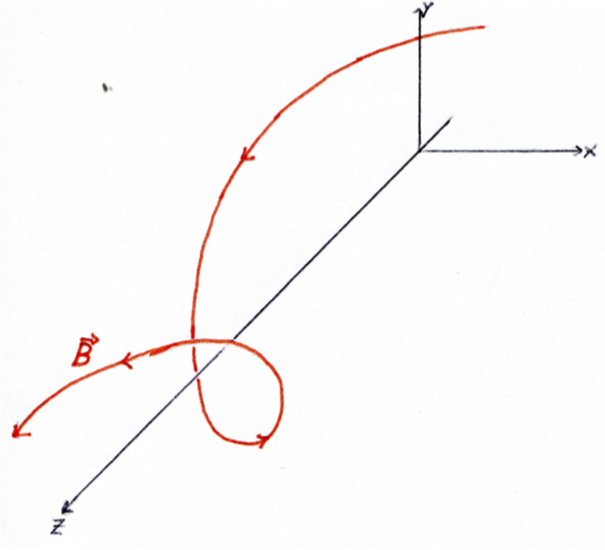


Junklewitz et al. (2011)
Oppermann et al. (2011)

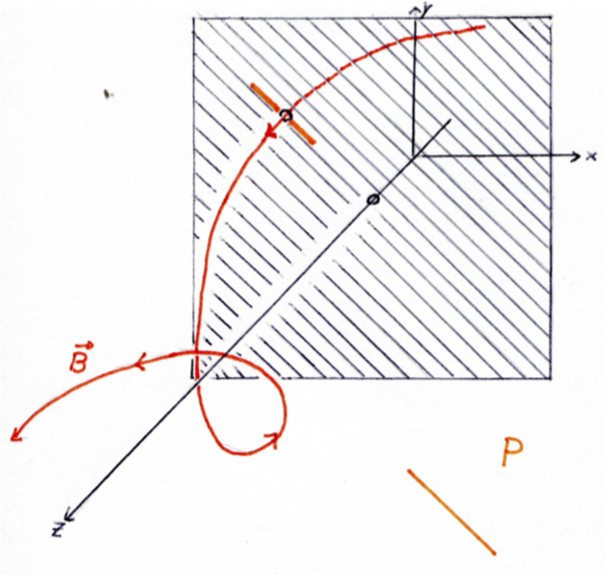
Faraday rotated synchrotron radiation

$$P(\lambda) \propto \int_0^\infty \mathrm{d}z \, p(z) \, \mathrm{e}^{2i \, \lambda^2 \, \phi(z)}$$

in general: Faraday depolarization



Brandenburg et al. (2014)



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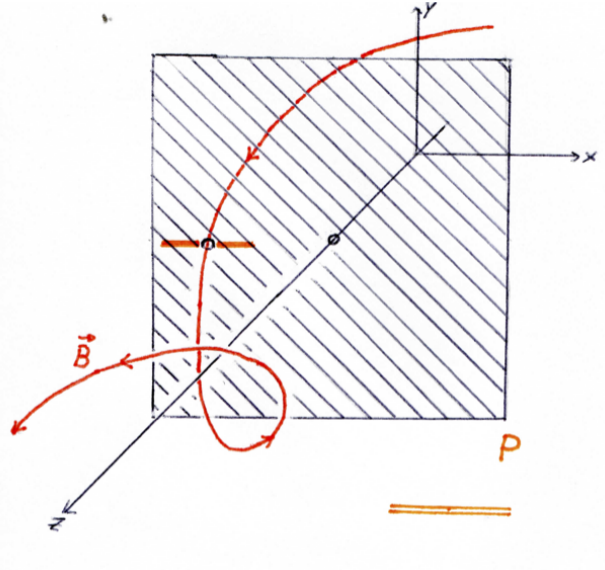

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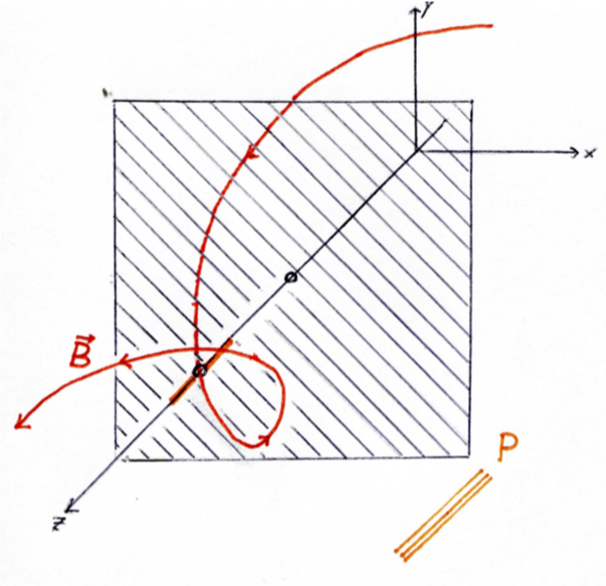
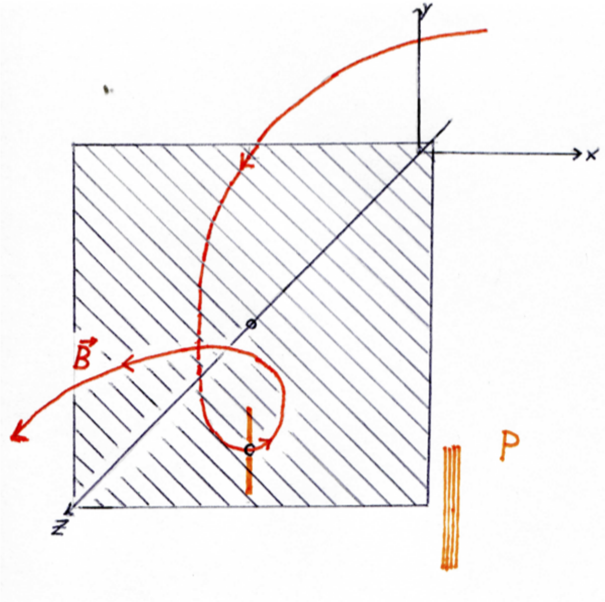


Brandenburg et al. (2014)



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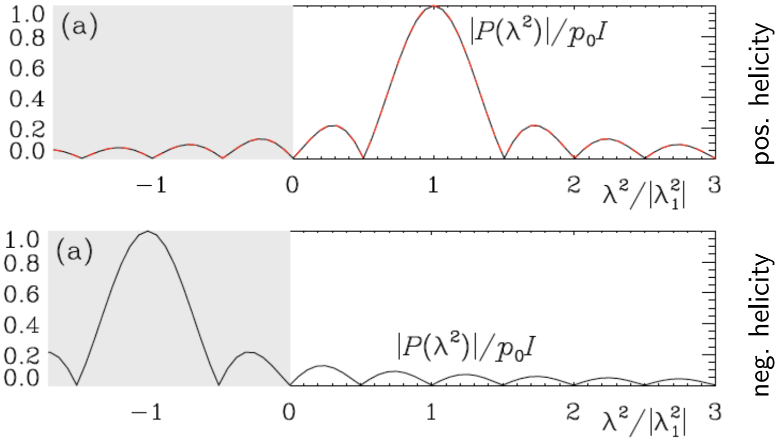
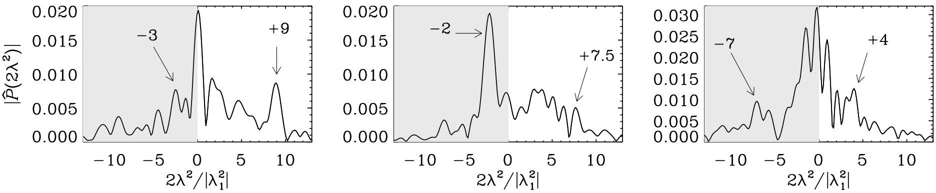


Brandenburg et al. (2014)

simulations of helical turbulence



(anti-)correlation between Faraday rotation and polarization degree

Brandenburg et al. (2014)
Volegova et al. (2010)

Magnetic helicity – summary

1. What?
   - twistiness or handedness of magnetic field
2. Why?
   - may tell us if, why, and how the Galactic dynamo works
3. How?
   - need all three B-field components in 3D
   - combine observables
   - details not entirely clear

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