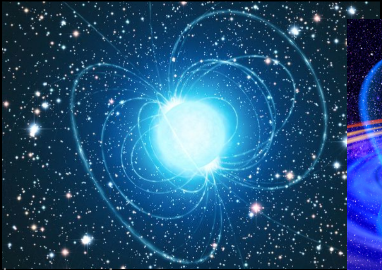
Magnetic fields seen through Faraday rotation – from the Milky Way to cosmic scales

Niels Oppermann

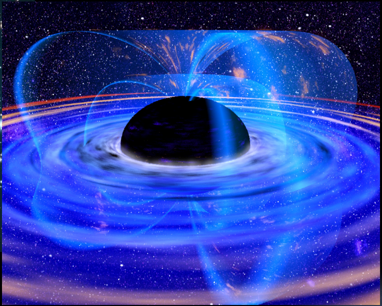
Canadian Institute for Theoretical Astrophysics
L'institut Canadien d'astrophysique théorique

with: Torsten Enßlin, Valentina Vacca, Henrik Junklewitz,
Bryan Gaensler, Dominic Schnitzeler, Jeroen Stil, Jo-Anne Brown,
. . .

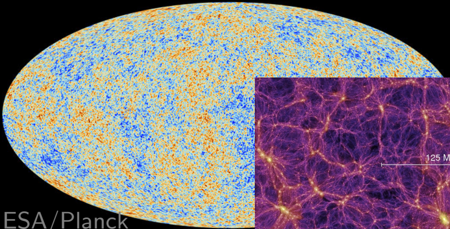
Astronomy Seminar, University of Calgary, 2015-04-07



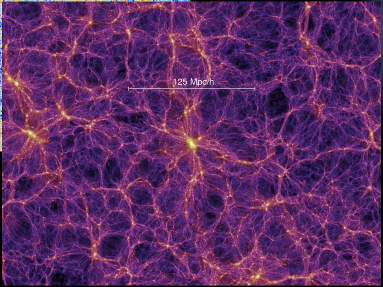
h+ magazine



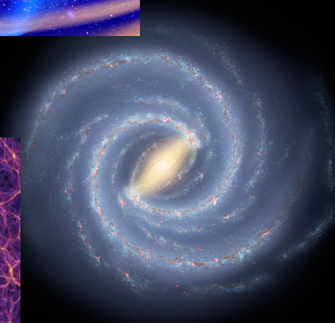
Dana Berry, NASA



ESA/Planck

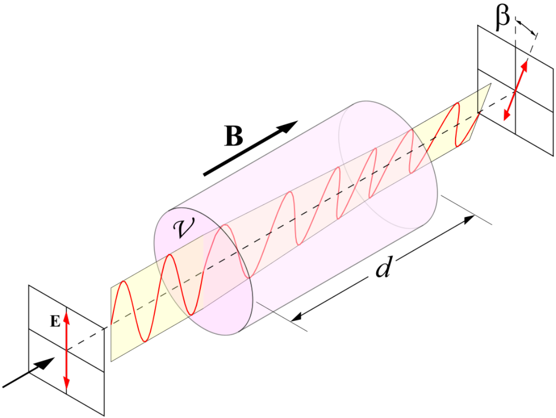


MPA



NASA/JPL-Caltech

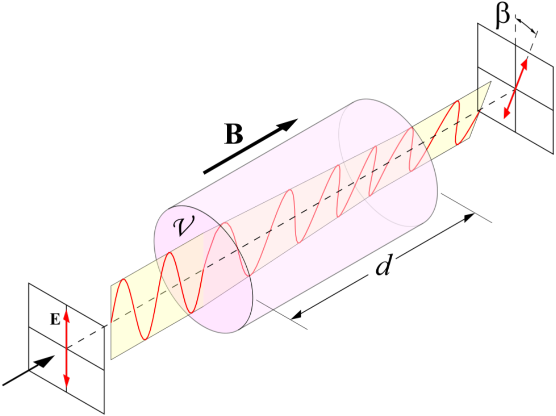
Faraday rotation



$$\mathrm{d}\beta \propto \lambda^2 n_\mathrm{e}\, B_r\, \mathrm{d}r$$

$$\Rightarrow \quad \beta \propto \lambda^2 \int_{r_\mathrm{source}}^{0} (1+z)^{-2}\; n_\mathrm{e}\, B_r\, \mathrm{d}r$$

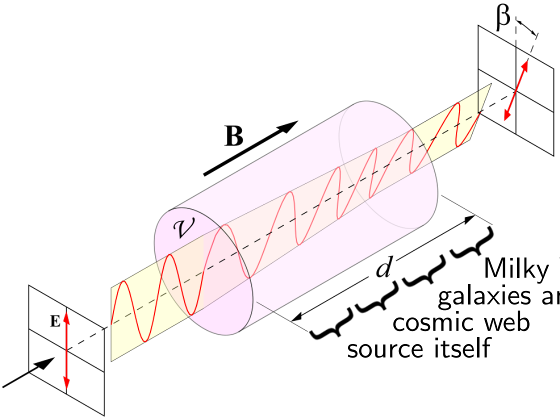
Faraday rotation

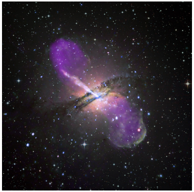


Faraday depth: $\phi \propto \int_{r_{\rm source}}^{0} (1+z)^{-2}\; n_{\rm e}\, B_r\, {\rm d}r$

$$\beta = \phi\lambda^2$$

Faraday rotation

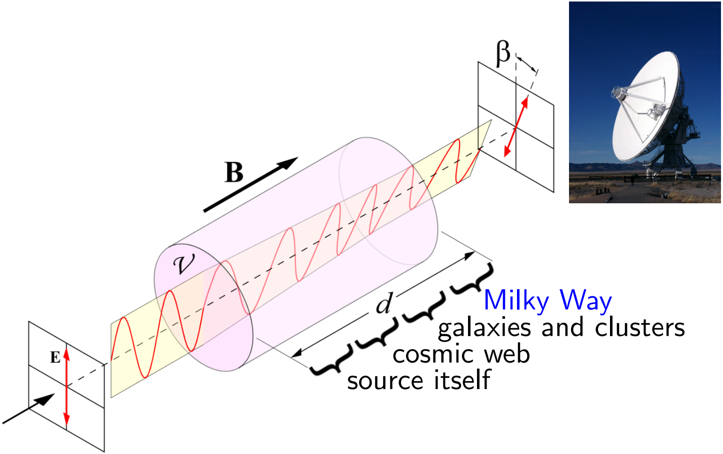


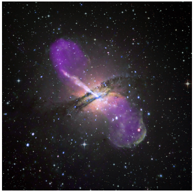


Faraday depth: $\phi \propto \int_{r_{\mathrm{source}}}^{0} (1+z)^{-2}\; n_{\mathrm{e}}\, B_r \,\mathrm{d}r$

$$\beta = \phi \lambda^2$$

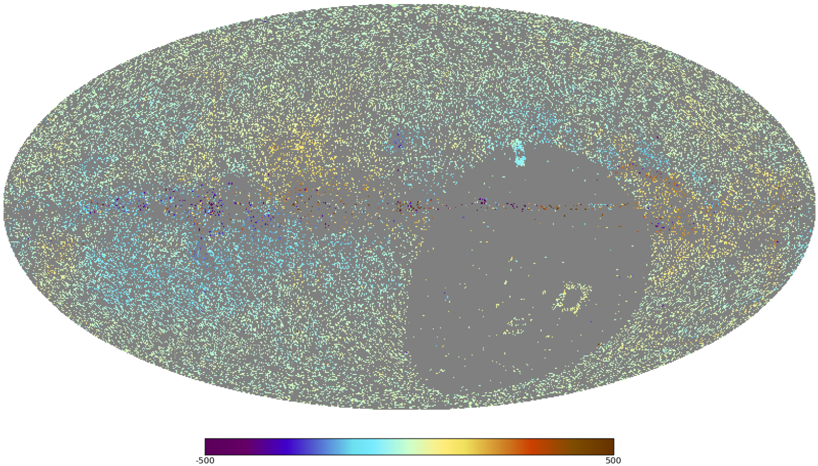
Extracting the Galactic contribution



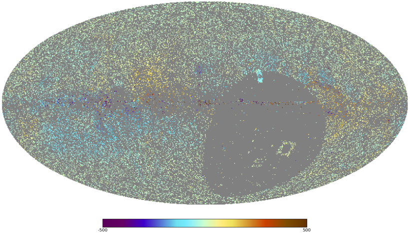


Galactic Faraday depth:

$$\phi_{\mathrm{g}} \propto \int_{r_{\mathrm{MilkyWay}}}^{0} (1+z)^{-2}\; n_{\mathrm{e}}\, B_r \,\mathrm{d}r$$

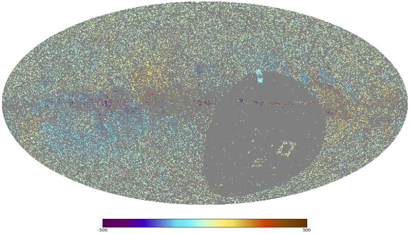


$\gtrsim 40\,000$ data points



Challenges

- Regions without data
- Extragalactic contributions unknown
- Uncertain error bars



Challenges

- Regions without data
- Extragalactic contributions unknown
- Uncertain error bars
  - $n\pi$
  - multiple components along a LOS
  - ionosphere
  - ...

One slide on statistics

One slide on statistics

$$d = \phi_{\rm g} + \phi_{\rm e} + n$$

Covariance matrices:

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$
$$E_{ij} = \delta_{ij}\,\sigma_{\rm e}^2$$
$$N_{ij} = \delta_{ij}\,\sigma_i^2$$

Wiener filter:

$$\hat{\phi}_{\rm g} = G\,(G+E+N)^{-1}\,d$$

One slide on statistics

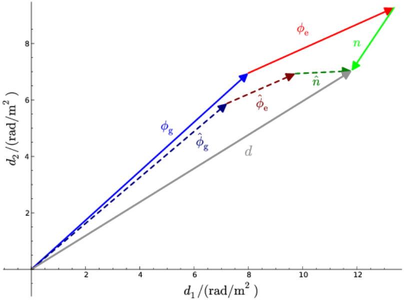
$$d = \phi_\mathrm{g} + \phi_\mathrm{e} + n$$

Covariance matrices:

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$
$$E_{ij} = \delta_{ij}\,\sigma_\mathrm{e}^2$$
$$N_{ij} = \delta_{ij}\,\sigma_i^2$$

Wiener filter:

$$\hat{\phi}_\mathrm{g} = G\,(G+E+N)^{-1}\,d$$

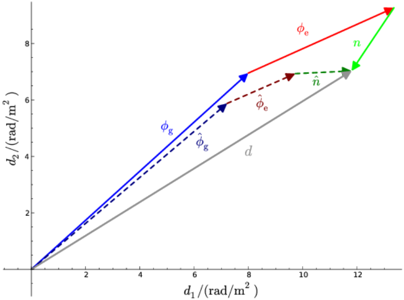


One slide on statistics

$$d = \phi_{\rm g} + \phi_{\rm e} + n$$

Wiener filter:

$$\hat{\phi}_{\rm g} = G\,(G + E + N)^{-1}\,d$$



Covariance matrices:

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$
$$E_{ij} = \delta_{ij}\,\sigma_{\rm e}^2$$
$$N_{ij} = \delta_{ij}\,\sigma_i^2$$

Posterior uncertainty:

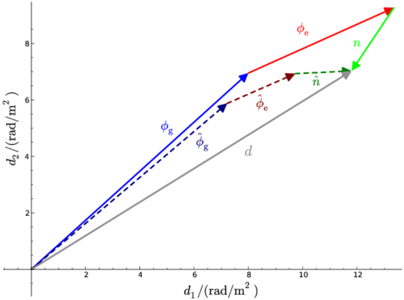
$$D_{\rm g} = \left(G^{-1} + (E+N)^{-1}\right)^{-1}$$
$$D_{\rm e} = \left(E^{-1} + (G+N)^{-1}\right)^{-1}$$
$$D_n = \left(N^{-1} + (G+E)^{-1}\right)^{-1}$$

One slide on statistics

$$d = \phi_\mathrm{g} + \phi_\mathrm{e} + n$$

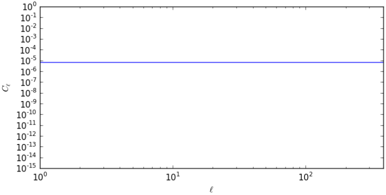
Wiener filter:

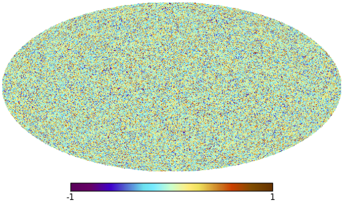
$$\hat{\phi}_\mathrm{g} = G\,(G + E + N)^{-1}\,d$$



Covariance matrices:

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$
$$E_{ij} = \delta_{ij}\,\sigma_\mathrm{e}^2$$
$$N_{ij} = \delta_{ij}\,\sigma_i^2$$



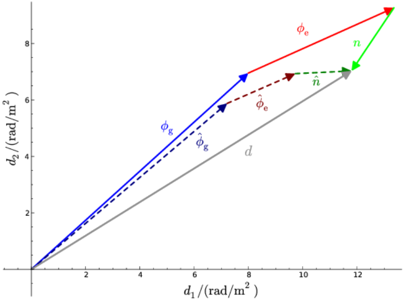


One slide on statistics

$$d = \phi_{\rm g} + \phi_{\rm e} + n$$

Wiener filter:

$$\hat{\phi}_{\rm g} = G\,(G + E + N)^{-1}\,d$$



Covariance matrices:

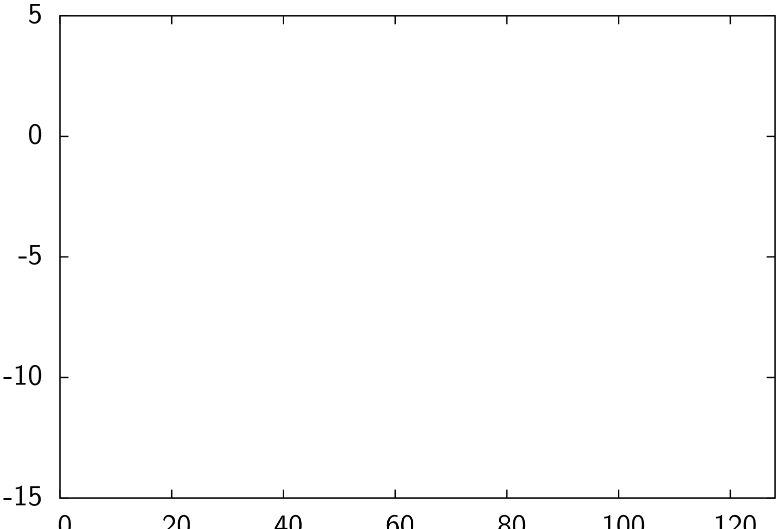
$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$
$$E_{ij} = \delta_{ij}\,\sigma_{\rm e}^2$$
$$N_{ij} = \delta_{ij}\,\sigma_i^2$$

$$(E + N)_{ij} = \delta_{ij}\,\left(\sigma_{\rm e}^2 + \sigma_i^2\right)\,\eta_i$$

1D example

Assumptions:

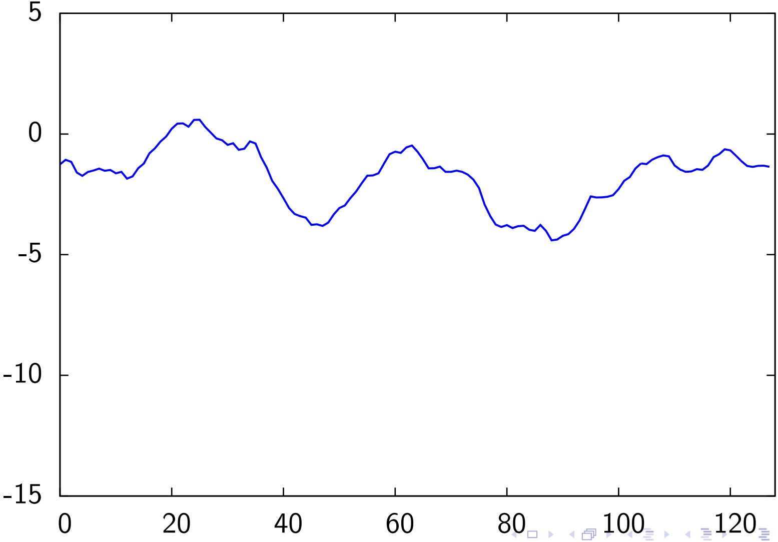
-
-



1D example

Assumptions:

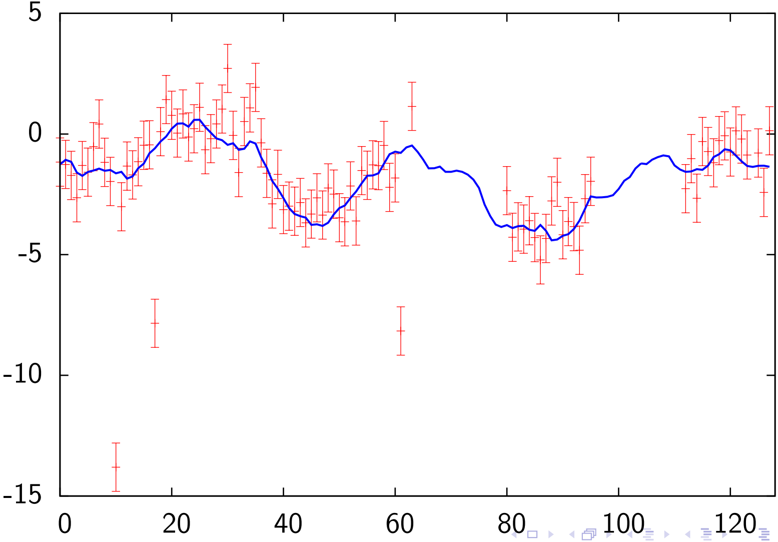
- signal field statistically homogeneous Gaussian random field
-



1D example

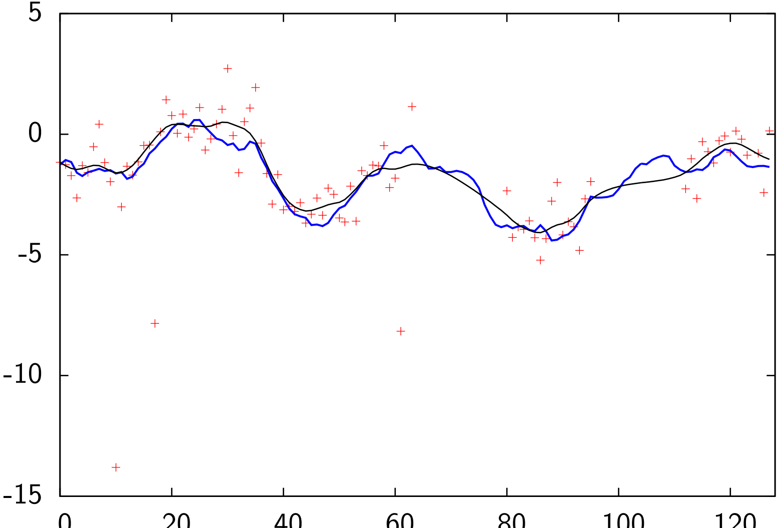
Assumptions:

- signal field statistically homogeneous Gaussian random field
- noise uncorrelated, Gaussian



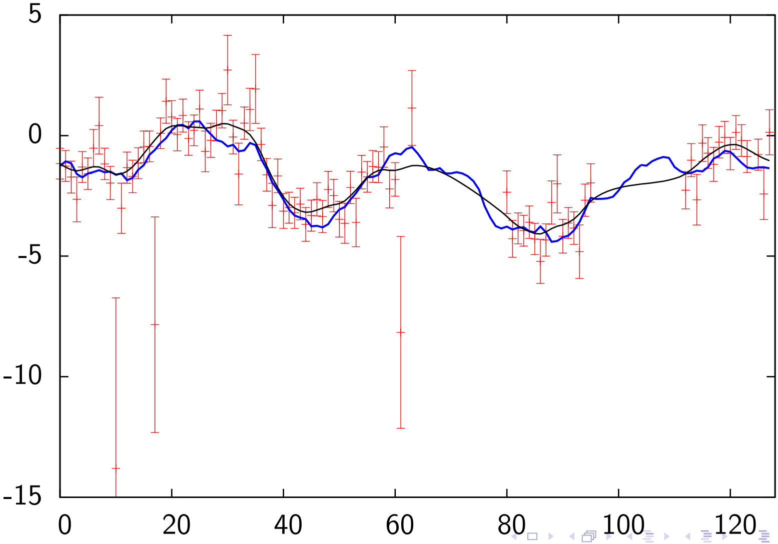
1D example

- Reconstruct (iteratively):
  signal, power spectrum, noise variance



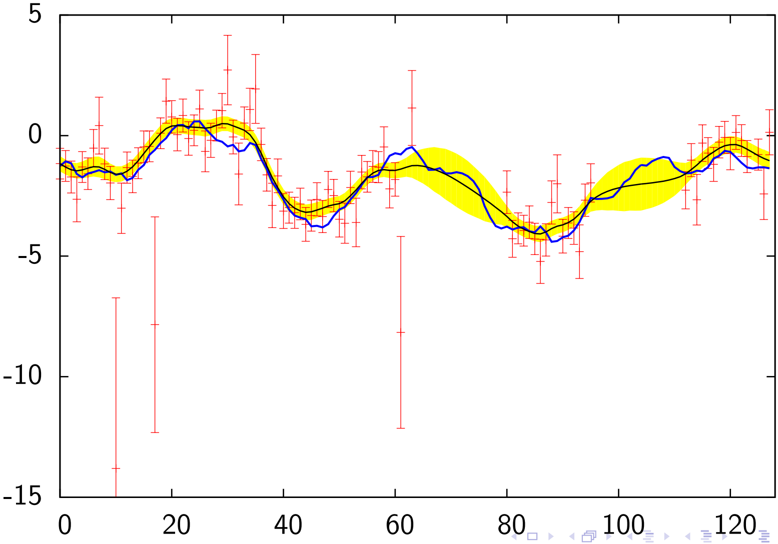
1D example

- Reconstruct (iteratively):
  signal, power spectrum, noise variance

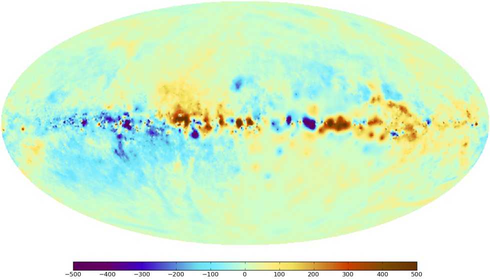


1D example

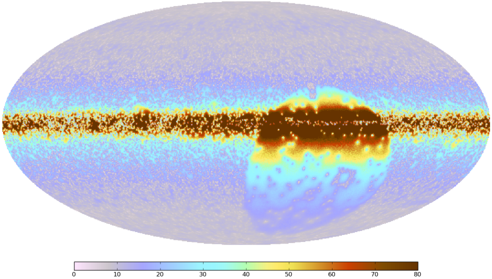
- Reconstruct (iteratively):
  signal, power spectrum, noise variance



Galactic Faraday depth

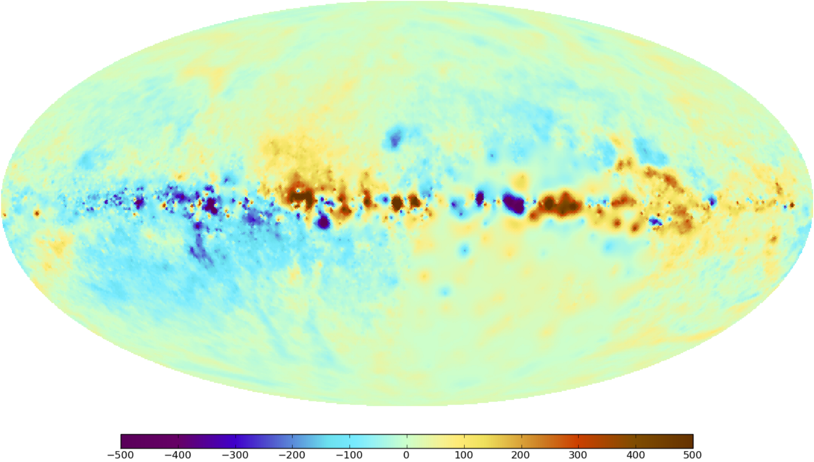


uncertainty



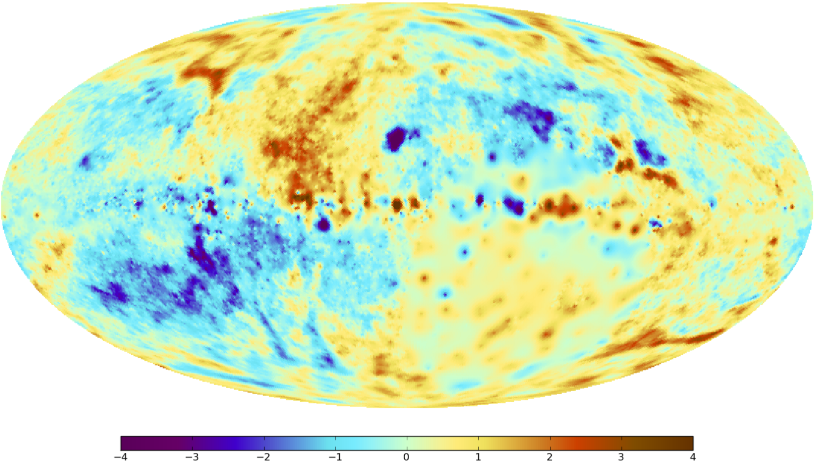
Oppermann et al. (2012/2015)

Galactic Faraday depth



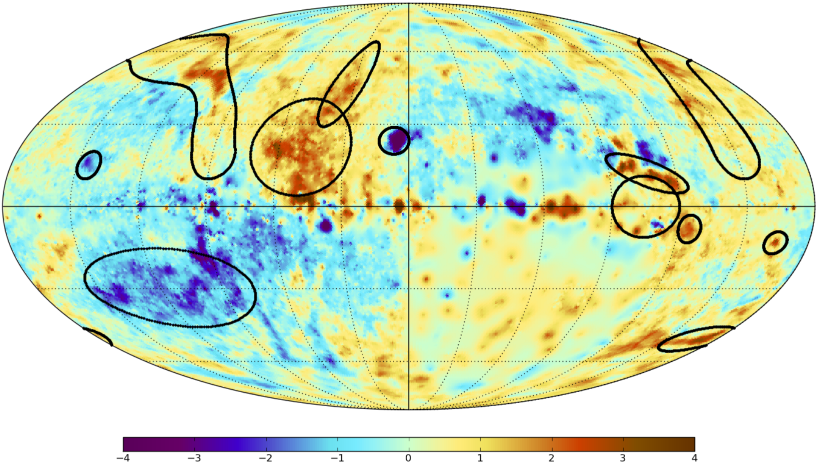
Oppermann et al. (2012/2015)

rescaled Galactic Faraday depth



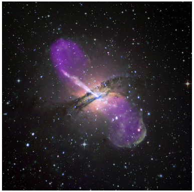
Oppermann et al. (2012/2015)

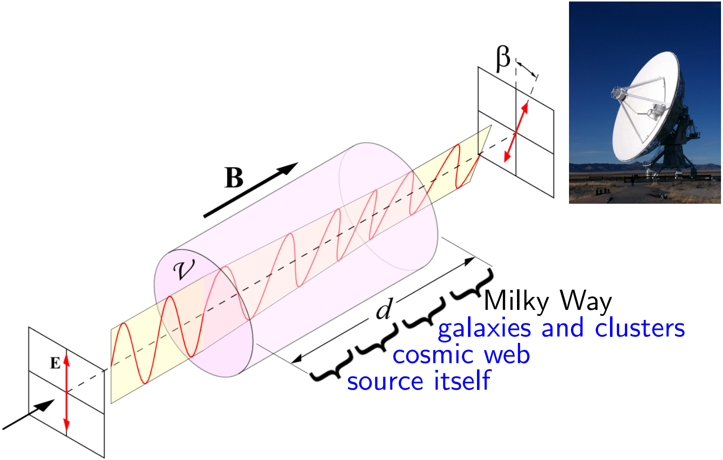
rescaled Galactic Faraday depth



Oppermann et al. (2012/2015)

Extracting the extragalactic contribution





extragalactic Faraday depth:

$$\phi_{\mathrm{e}} \propto \int_{r_{\mathrm{source}}}^{r_{\mathrm{MilkyWay}}} (1+z)^{-2} \; n_{\mathrm{e}} \, B_r \, \mathrm{d}r$$

One slide on statistics

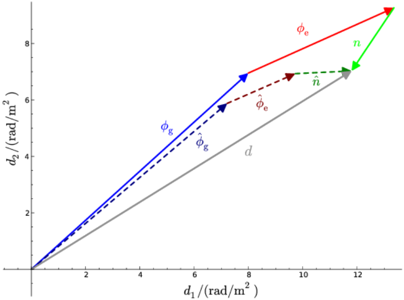
$$d = \phi_{\rm g} + \phi_{\rm e} + n$$

Wiener filter:

$$\hat{\phi}_{\rm g} = G\,(G + E + N)^{-1}\,d$$

Covariance matrices:

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$
$$E_{ij} = \delta_{ij}\,\sigma_{\rm e}^2$$
$$N_{ij} = \delta_{ij}\,\sigma_i^2$$



$$(E + N)_{ij} = \delta_{ij}\,\left(\sigma_{\rm e}^2 + \sigma_i^2\right)\eta_i$$

One slide on statistics

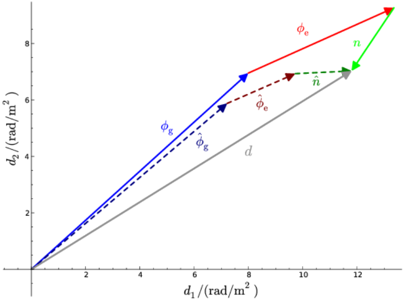
$$d = \phi_{\rm g} + \phi_{\rm e} + n$$

Wiener filter:

$$\hat{\phi}_{\rm e} = E\,(G + E + N)^{-1}\, d$$

Covariance matrices:

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\; C_\ell$$
$$E_{ij} = \delta_{ij}\,\sigma_{\rm e}^2$$
$$N_{ij} = \delta_{ij}\,\sigma_i^2$$



$$(E + N)_{ij} = \delta_{ij}\,\left(\sigma_{\rm e}^2 + \sigma_i^2\right)\,\eta_i$$

One slide on statistics

$$d = \phi_\mathrm{g} + \phi_\mathrm{e} + n$$

Wiener filter:

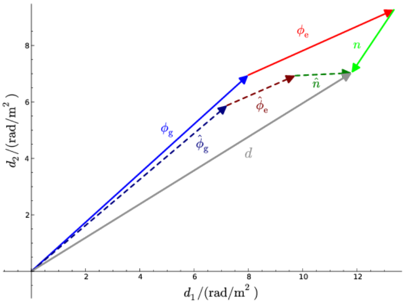
$$\hat{\phi}_\mathrm{e} = E\,(G + E + N)^{-1}\,d$$

Covariance matrices:

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$

$$E_{ij} = \delta_{ij}\,\sigma_\mathrm{e}^2\,\eta_\mathrm{e}$$

$$N_{ij} = \delta_{ij}\,\sigma_i^2\,\eta_i$$



One slide on statistics

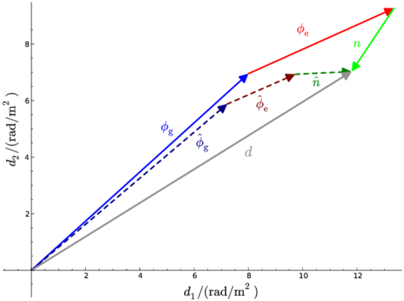
$$d = \phi_\mathrm{g} + \phi_\mathrm{e} + n$$

Wiener filter:

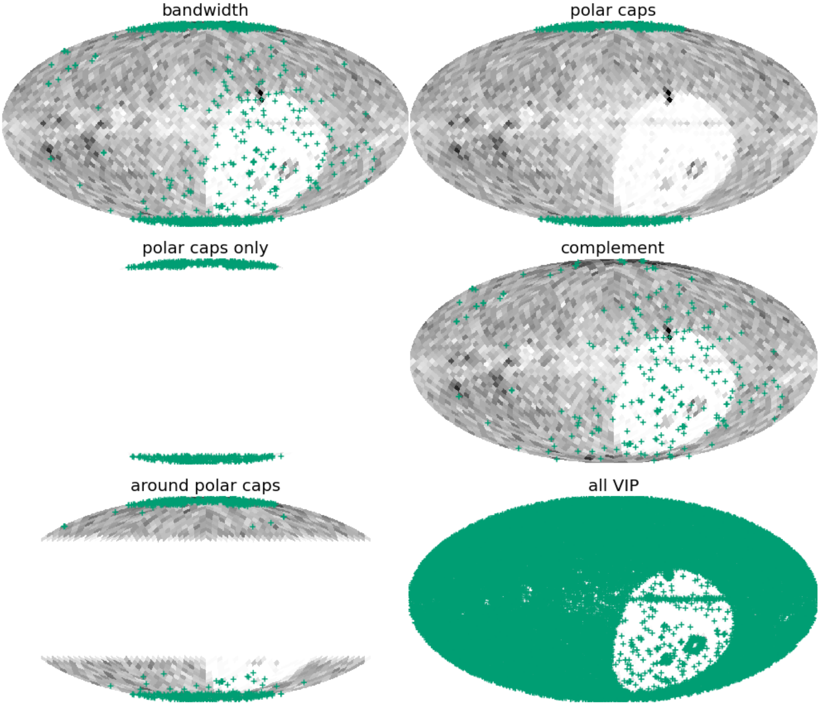
$$\hat{\phi}_\mathrm{e} = E\,(G + E + N)^{-1}\,d$$

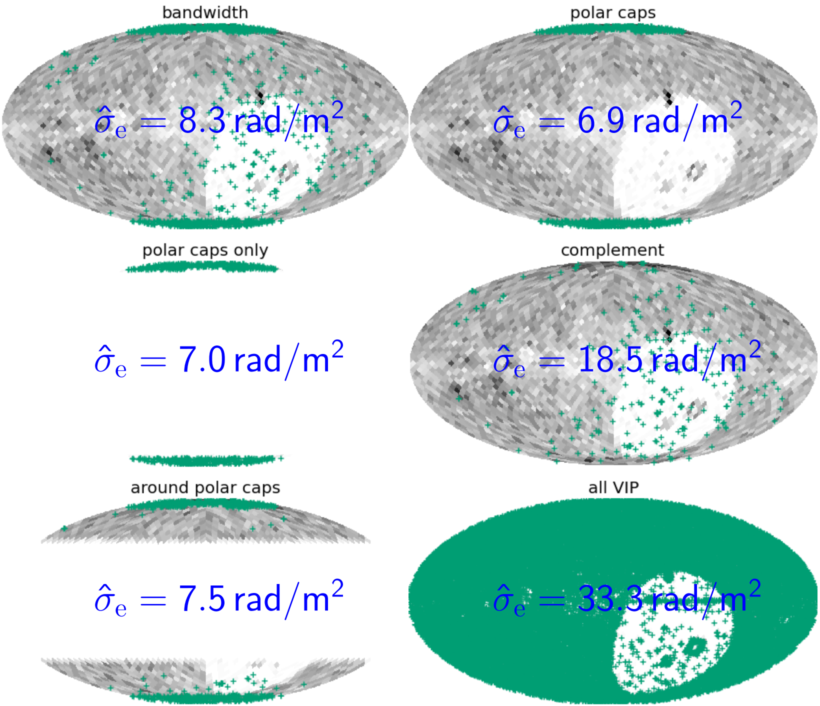
Covariance matrices:

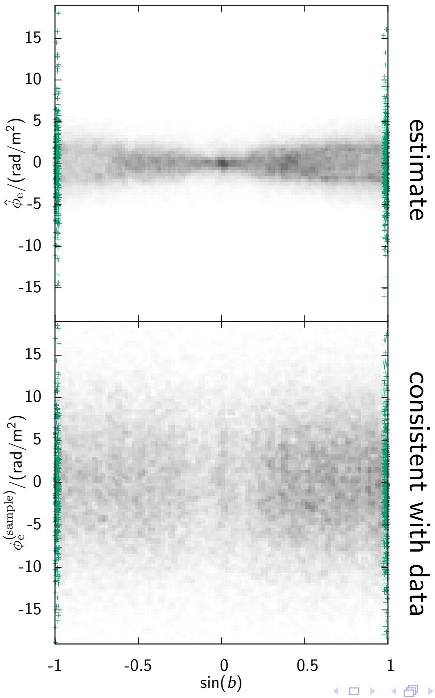
$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$
$$E_{ij} = \delta_{ij}\,\sigma_\mathrm{e}^2\,\eta_\mathrm{e}$$
$$N_{ij} = \delta_{ij}\,\sigma_i^2\,\eta_i$$



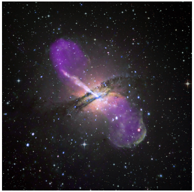
idea: find subset of data for which $\eta_i \equiv 1$

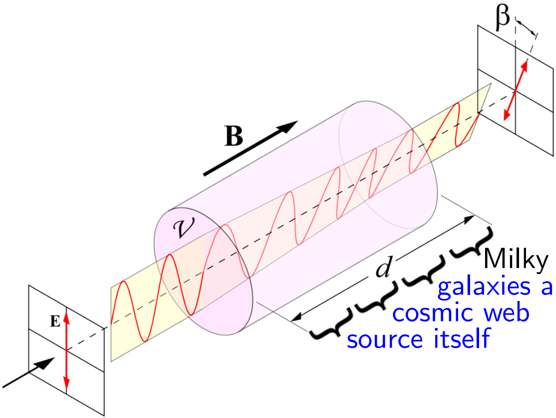

bandwidth
polar caps
polar caps only
complement
around polar caps
all VIP


bandwidth
$\hat{\sigma}_e = 8.3\,\mathrm{rad}/\mathrm{m}^2$
polar caps
$\hat{\sigma}_e = 6.9\,\mathrm{rad}/\mathrm{m}^2$
polar caps only
$\hat{\sigma}_e = 7.0\,\mathrm{rad}/\mathrm{m}^2$
complement
$\hat{\sigma}_e = 18.5\,\mathrm{rad}/\mathrm{m}^2$
around polar caps
$\hat{\sigma}_e = 7.5\,\mathrm{rad}/\mathrm{m}^2$
all VIP
$\hat{\sigma}_e = 33.3\,\mathrm{rad}/\mathrm{m}^2$


estimate
consistent with data
$\hat{\phi}_e/(\mathrm{rad/m^2})$
$\phi_e^{(\mathrm{sample})}/(\mathrm{rad/m^2})$
15
10
5
0
-5
-10
-15
-1
-0.5
0
0.5
1
sin(*b*)

What is the extragalactic contribution?





extragalactic Faraday depth:

$$\phi_{\mathrm{e}} \propto \int_{r_{\mathrm{source}}}^{r_{\mathrm{MilkyWay}}} (1+z)^{-2}\; n_{\mathrm{e}}\, B_r \,\mathrm{d}r$$

One slide on statistics

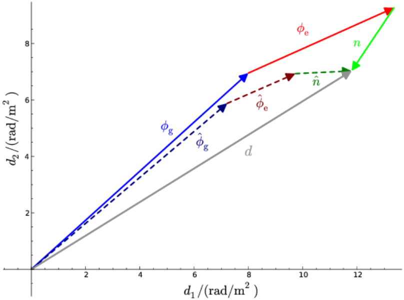
$$d = \phi_{\mathrm{g}} + \phi_{\mathrm{e}} + n$$

Covariance matrices:

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$
$$E_{ij} = \delta_{ij}\,\sigma_{\mathrm{e}}^2\,\eta_{\mathrm{e}}$$
$$N_{ij} = \delta_{ij}\,\sigma_i^2\,\eta_i$$

Wiener filter:

$$\hat{\phi}_{\mathrm{e}} = E\,(G + E + N)^{-1}\,d$$



One slide on statistics

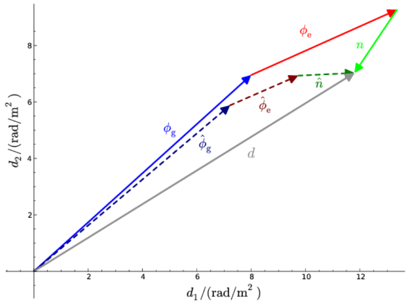
$$d = \phi_{\mathrm{g}} + \phi_{\mathrm{e}} + n$$

Covariance matrices:

Wiener filter:

$$\hat{\phi}_{\mathrm{e}} = E\,(G + E + N)^{-1}\,d$$

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$
$$E_{ij} = \delta_{ij}\,\sigma_{\mathrm{e}}^2\,\eta_{\mathrm{e}}$$
$$N_{ij} = \delta_{ij}\,\sigma_i^2\,\eta_i$$



$$E_{ij} = \delta_{ij}\left(\sigma^{(\mathrm{source})2} + \sigma_i^{(\mathrm{cluster})2} + \sigma_i^{(\mathrm{filament})2} + \sigma_i^{(\mathrm{void})2}\right)$$

One slide on statistics

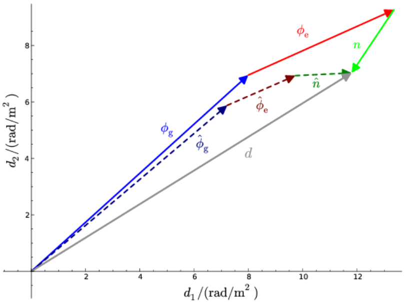
$$d = \phi_\mathrm{g} + \phi_\mathrm{e} + n$$

Covariance matrices:

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$
$$E_{ij} = \delta_{ij}\,\sigma_\mathrm{e}^2\,\eta_\mathrm{e}$$
$$N_{ij} = \delta_{ij}\,\sigma_i^2\,\eta_i$$

Wiener filter:

$$\hat{\phi}_\mathrm{e} = E\,(G + E + N)^{-1}\,d$$

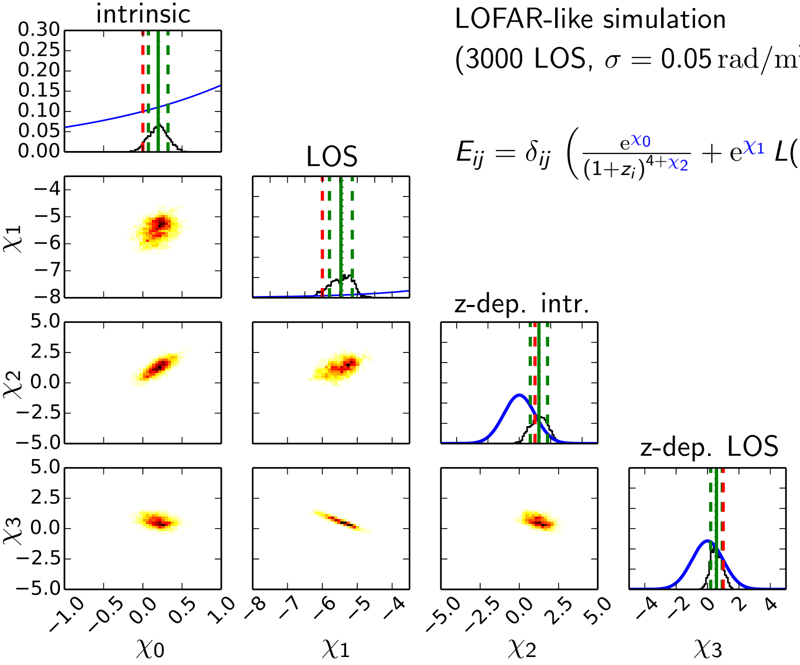


$$E_{ij} = \delta_{ij}\left(\frac{\mathrm{e}^{\chi_0}}{(1+z_i)^{4+\chi_2}} + \mathrm{e}^{\chi_1}\,L(z_i,\chi_3)\right)$$

$$L(z_i,\chi_3) \propto \int_0^{r(z_i)} \frac{\mathrm{d}r}{(1+z(r))^{4+\chi_3}}$$

LOFAR-like simulation
(3000 LOS, $\sigma = 0.05\,\mathrm{rad/m^2}$)

$$E_{ij} = \delta_{ij} \left(\frac{e^{\chi_0}}{(1+z_i)^{4+\chi_2}} + e^{\chi_1}\, L(z_i, \chi_3) \right)$$



plots courtesy of Valentina Vacca

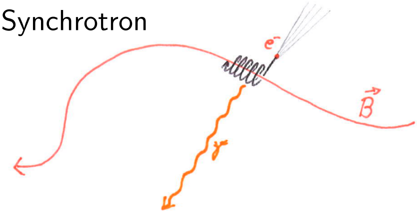
Summary

- Galactic contribution (correlated) can be separated from rest (uncorrelated)
- Rest can be separated statistically into extragalactic and noise
- Uncertainties are large and should not be ignored

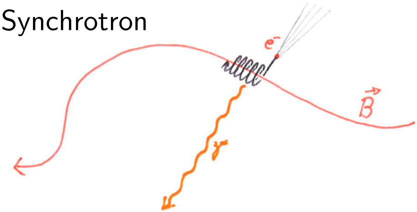
All results at
http://www.mpa-garching.mpg.de/ift/faraday/

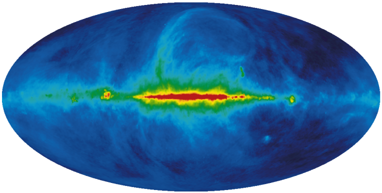
BACKUP

Synchrotron



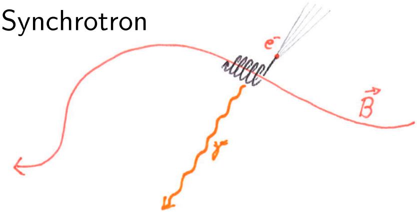
Synchrotron

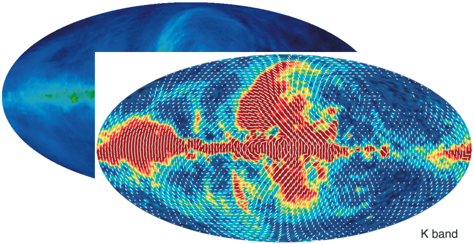




Haslam et al. 1981

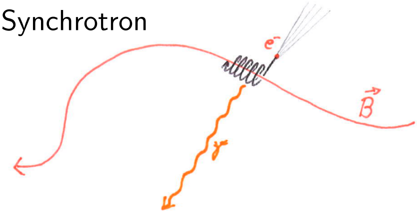
Synchrotron

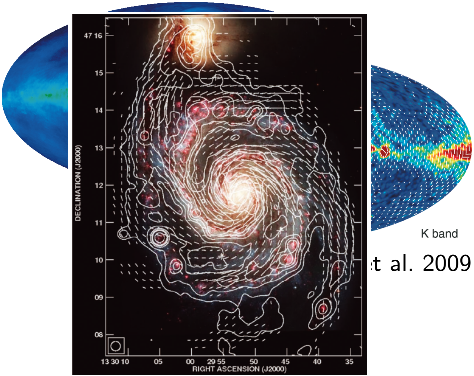




Hinshaw et al. 2009

Synchrotron

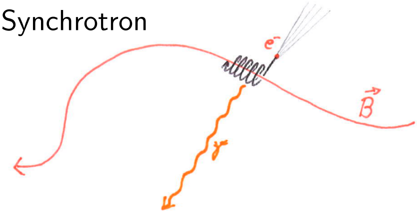




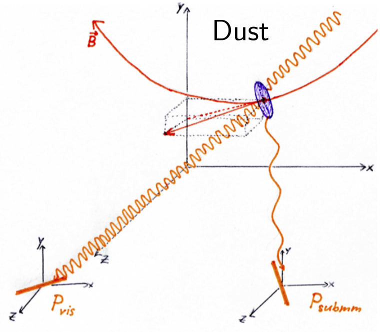
t al. 2009

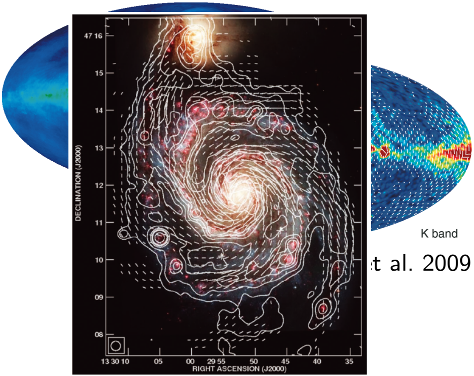
Fletcher et al. 2011

Synchrotron



Dust



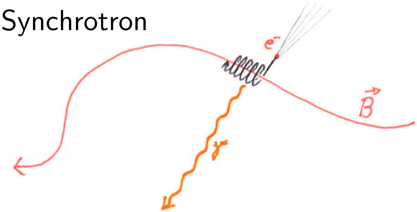


K band

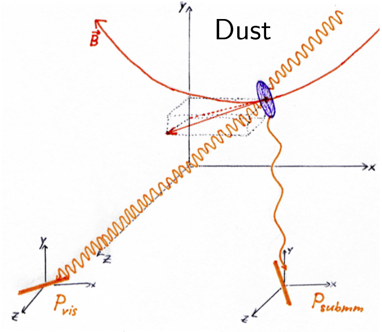
t al. 2009

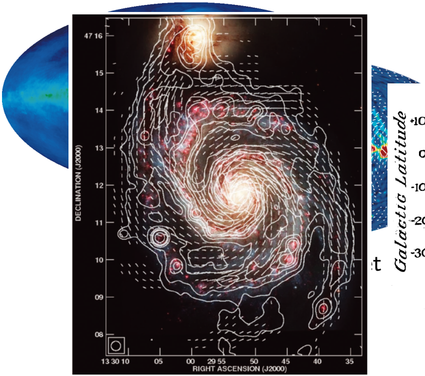
Fletcher et al. 2011

Synchrotron

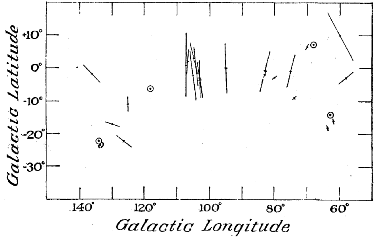


Dust



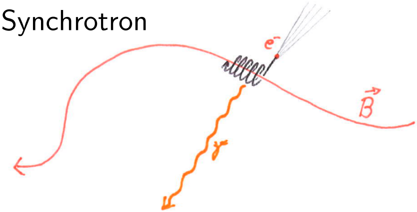


Fletcher et al. 2011

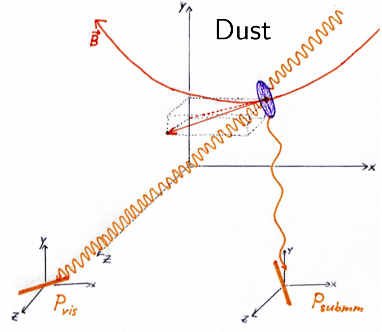


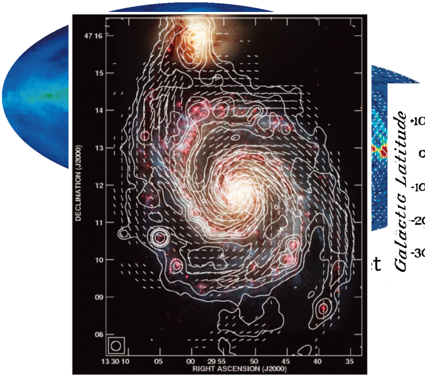
Hall 1949

Synchrotron

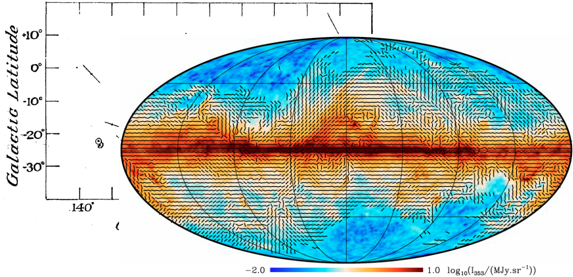


Dust





Fletcher et al. 2011



Planck Collaboration Int. XIX (2014)