Max Planck Institute
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CITA ICAT
Canadian Institute for Theoretical Astrophysics
L'institut canadien d'astrophysique theorique

Faraday rotation in the Milky Way and beyond – A statistical analysis

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with

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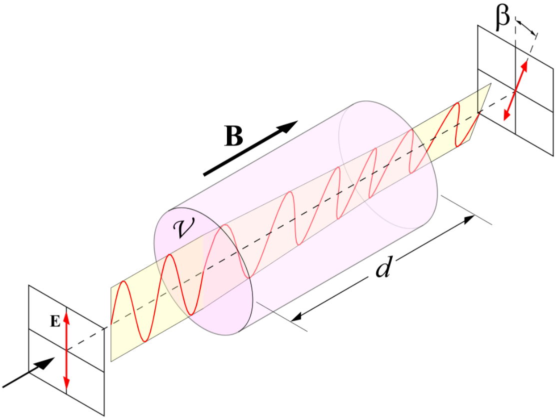
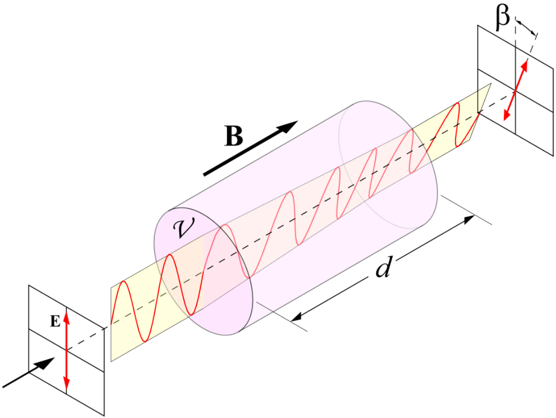
G2000, Toronto, 2013-11-06



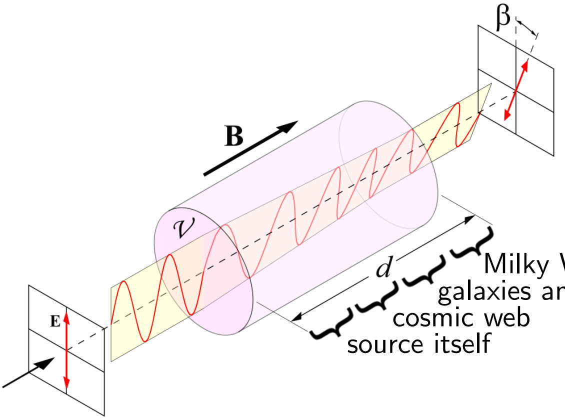
$$\mathrm{d}\beta \propto \lambda^2 n_\mathrm{e}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$

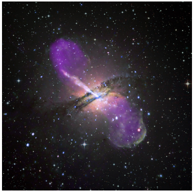
$$\Rightarrow \quad \beta \propto \lambda^2 \int_{r_\mathrm{source}}^{0} n_\mathrm{e}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$



Faraday depth: $\phi \propto \int_{r_{\mathrm{source}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$

$$\beta = \phi \lambda^2$$





Faraday depth: $\phi \propto \int_{r_{\text{source}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$

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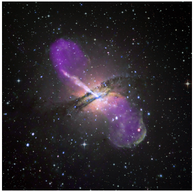
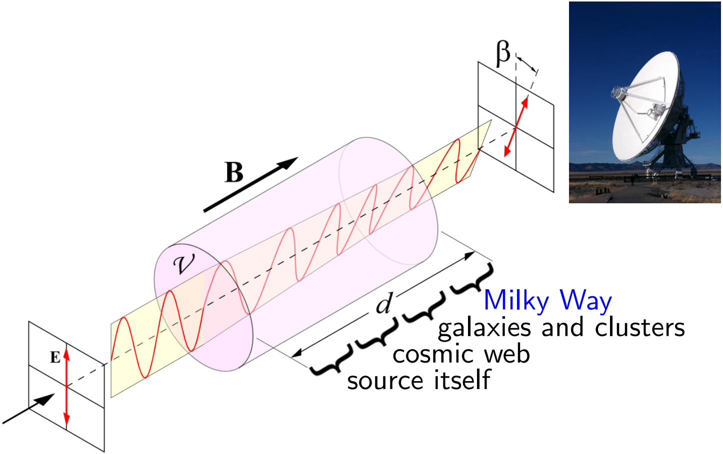
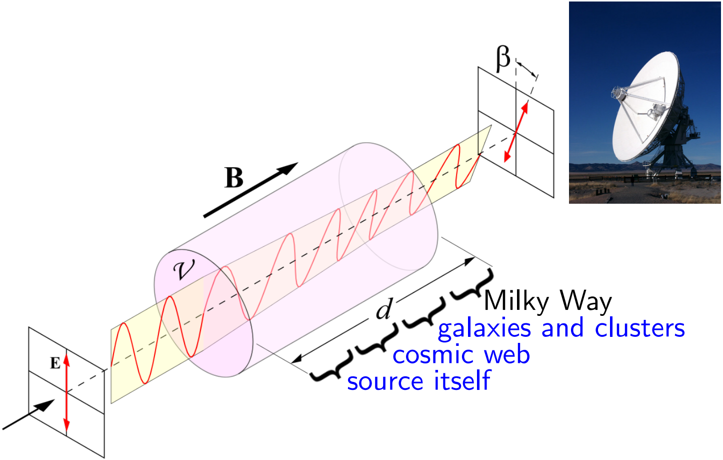
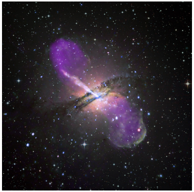




Galactic Faraday depth:

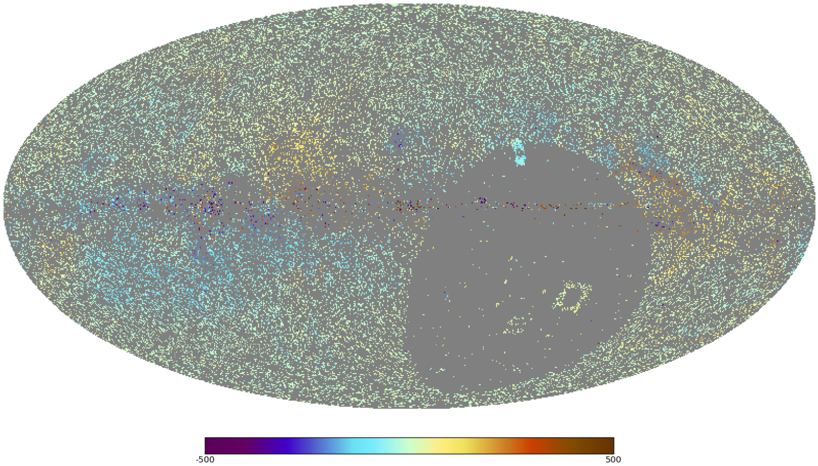
$$\phi_{\mathrm{g}} \propto \int_{r_{\mathrm{Milky\,Way}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$



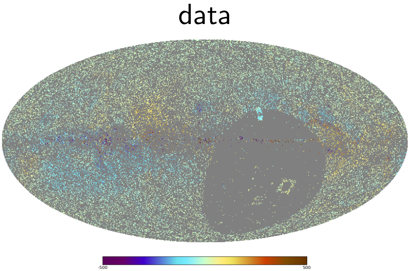


extragalactic Faraday depth:

$$\phi_{\mathrm{e}} \propto \int_{r_{\mathrm{source}}}^{r_{\mathrm{Milky Way}}} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$

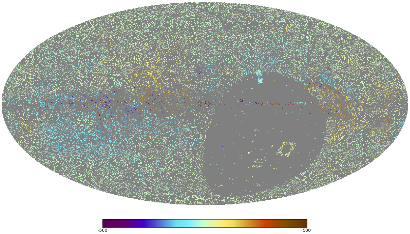


41 330 data points



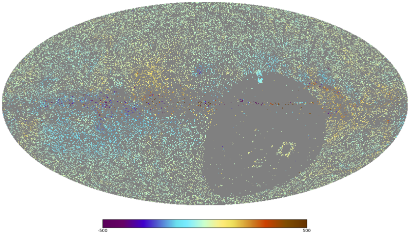
$$d = \quad \phi_{\mathrm{g}} + \phi_{\mathrm{e}} + n$$

data



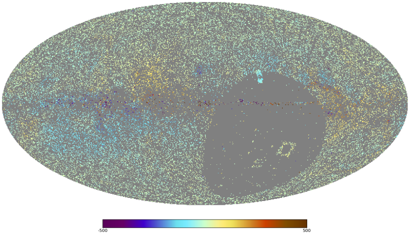
$$d = \phi_{\rm g} + \underbrace{\phi_{\rm e} + n}_{n'}$$

data



$$d = R\phi_{\rm g} + \underbrace{\phi_{\rm e} + n}_{n'}$$

data



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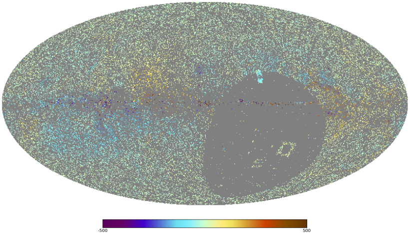
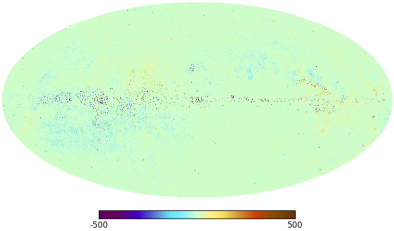
if n' Gaussian

$\Rightarrow$ likelihood $\mathcal{P}(d\,|\phi_{\mathrm{g}})$ Gaussian

data



maximum likelihood solution



$$d = R\phi_{\rm g} + \underbrace{\phi_{\rm e} + n}_{n'}$$

if n' Gaussian
$\Rightarrow$ likelihood $\mathcal{P}(d\,|\phi_{\rm g})$ Gaussian

n' uncorrelated

Bayesian inference

$$\mathcal{P}(\phi_{\rm g}|\, d) = \frac{\mathcal{P}(d|\phi_{\rm g})\,\mathcal{P}(\phi_{\rm g})}{\mathcal{P}(d)}$$

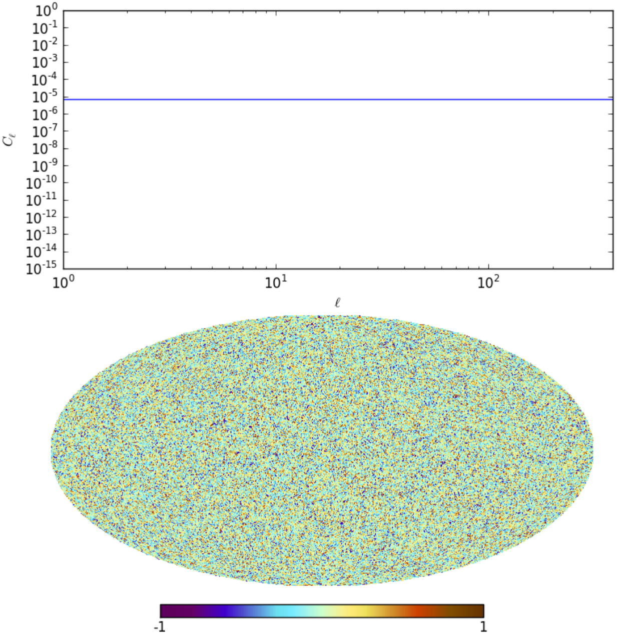
all prior information encoded in $\mathcal{P}(\phi_{\rm g})$

e.g.: $\mathcal{P}(\phi_{\rm g})$ Gaussian $\Rightarrow$ $\mathcal{P}(\phi_{\rm g}|\, d)$ also Gaussian.

Covariance matrices

$$\Phi_{(\ell m),(\ell' m')} = \delta_{\ell\ell'}\,\delta_{mm'}\;C_\ell$$

$\hookrightarrow$ angular power spectrum


C_ℓ
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{0}
10^{1}
10^{2}
ℓ
-1
1

Covariance matrices

$$\Phi_{(\ell m),(\ell' m')} = \delta_{\ell\ell'}\, \delta_{mm'}\, C_\ell$$

$\hookrightarrow$ angular power spectrum

$$N'_{ij} = \delta_{ij}\left(\sigma_i^2 + \sigma_{\mathrm{e}}^2\right)$$

(uncorrelated noise)

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$\hookrightarrow$ angular power spectrum

$$N'_{ij} = \delta_{ij}\left(\sigma_i^2 + \sigma_{\mathrm{e}}^2\right)\eta_i$$

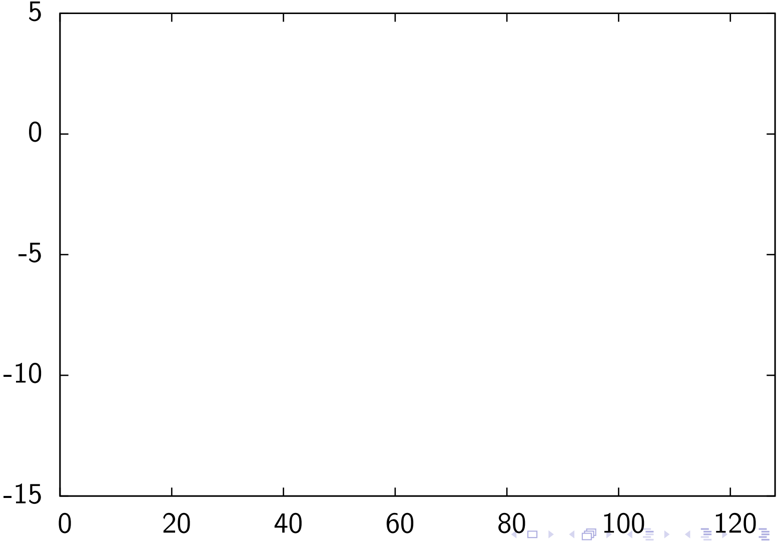
$\hookrightarrow$ error variance correction factors

(uncorrelated noise)

1D example

Assumptions:

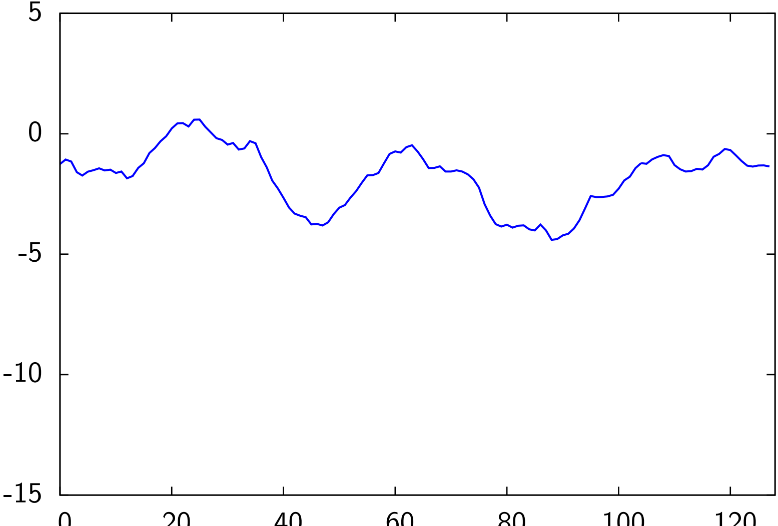
-
-



1D example

Assumptions:

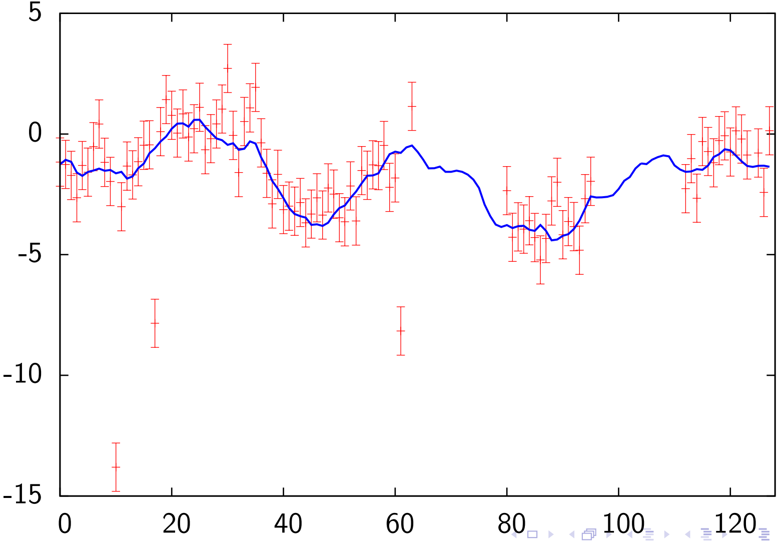
- signal field statistically homogeneous Gaussian random field
-



1D example

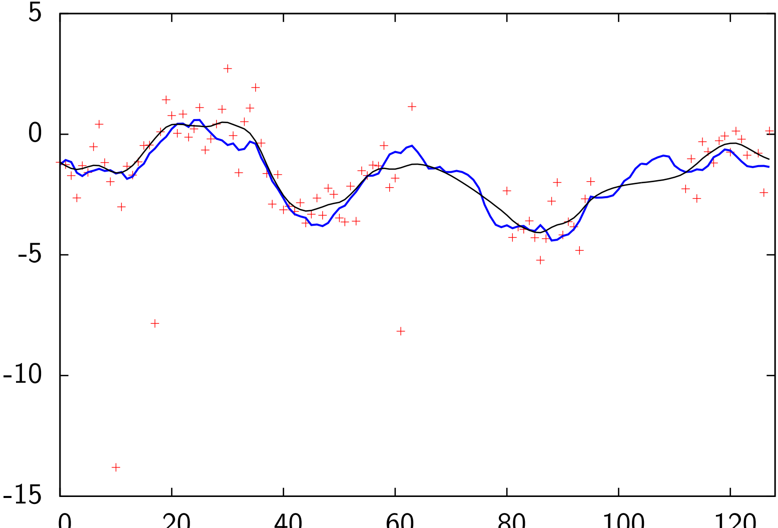
Assumptions:

- signal field statistically homogeneous Gaussian random field
- noise uncorrelated, Gaussian



1D example

- Reconstruct (iteratively):
  signal, power spectrum, noise variance



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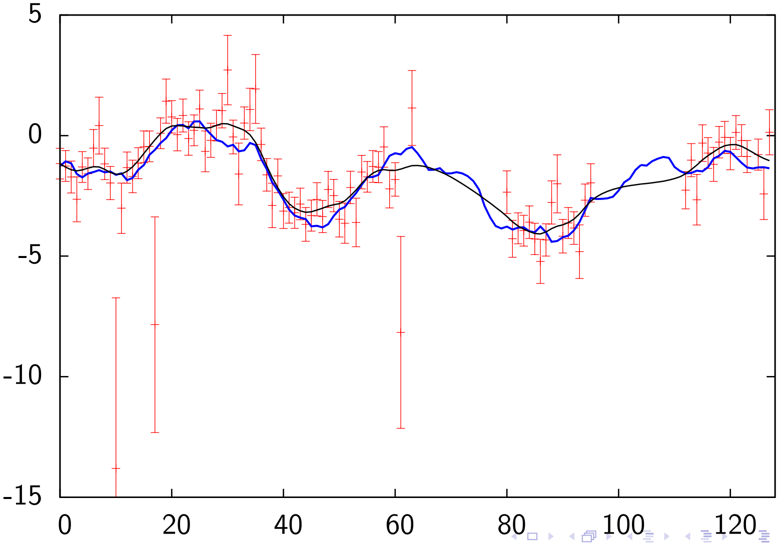

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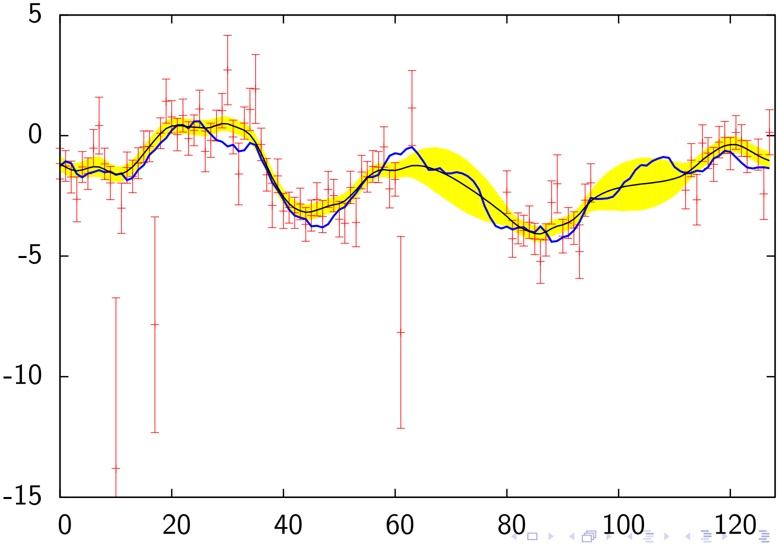
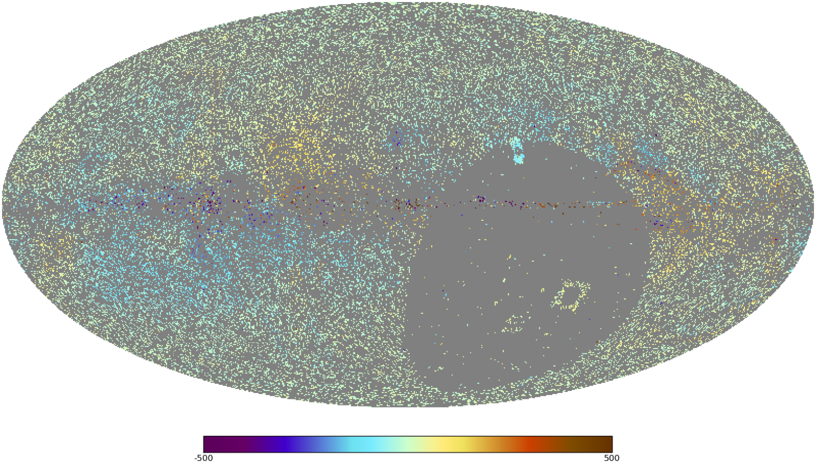
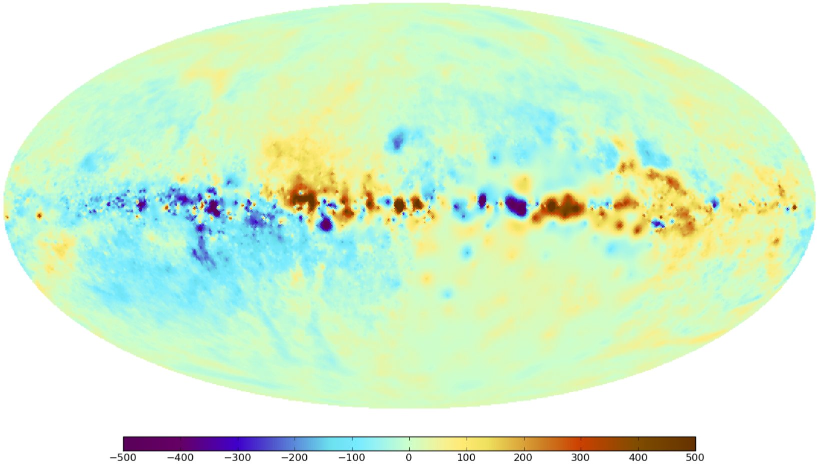


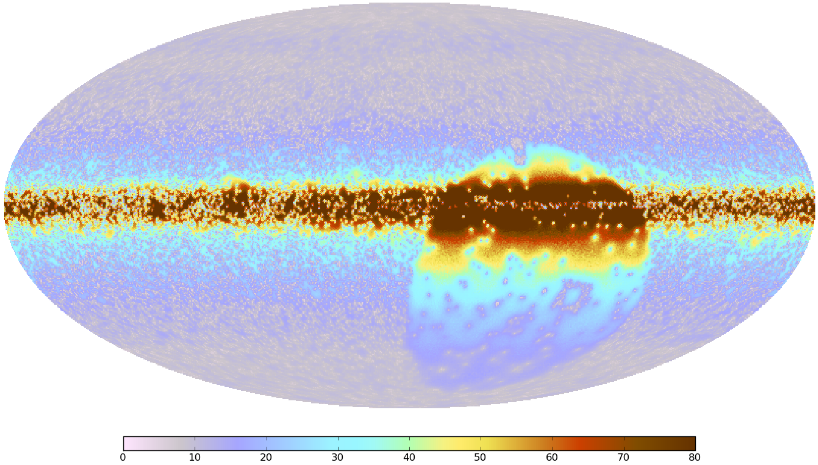
data



posterior mean of the Galactic Faraday depth



uncertainty of the reconstruction



What about extragalactic contributions?

$$d = \phi_{\mathrm{e}} + R\phi_{\mathrm{g}} + n$$

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idea: find subset of data for which $\eta_i \equiv 1$

SUMMARY

- Don't be afraid to use prior information; use posterior to do inference.
- Galactic contribution to Faraday rotation can be separated using its correlation structure
- Extragalactic contribution not so easy, only possible if noise statistics very well understood