The Faraday Sky

Map Making and Helicity Inference

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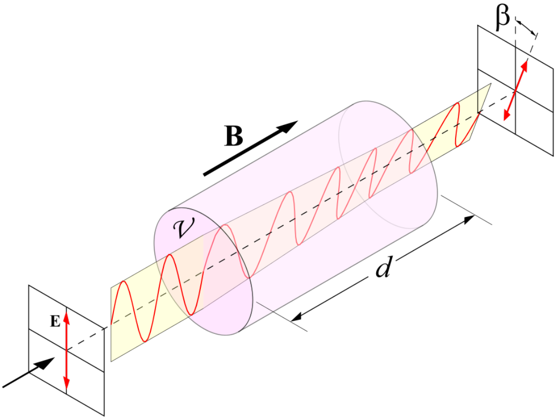
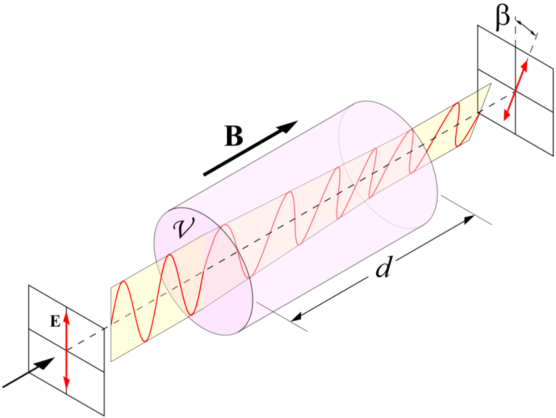


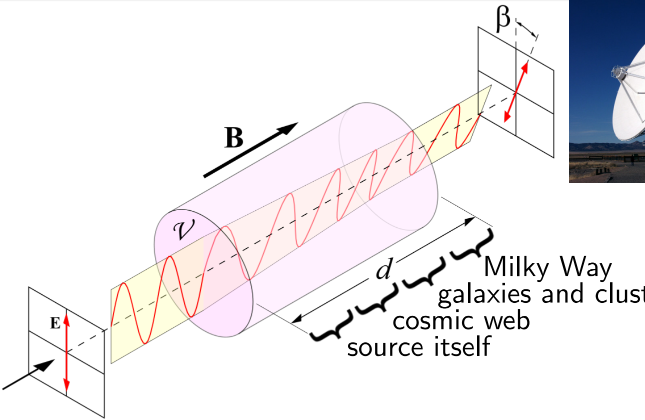
$$\mathrm{d}\beta \propto \lambda^2 n_\mathrm{e}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$

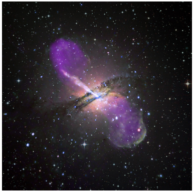
$$\Rightarrow \quad \beta \propto \lambda^2 \int_{r_\mathrm{source}}^{0} n_\mathrm{e}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$



Faraday depth: $\phi \propto \int_{r_{\mathrm{source}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$

$$\beta = \phi \lambda^2$$





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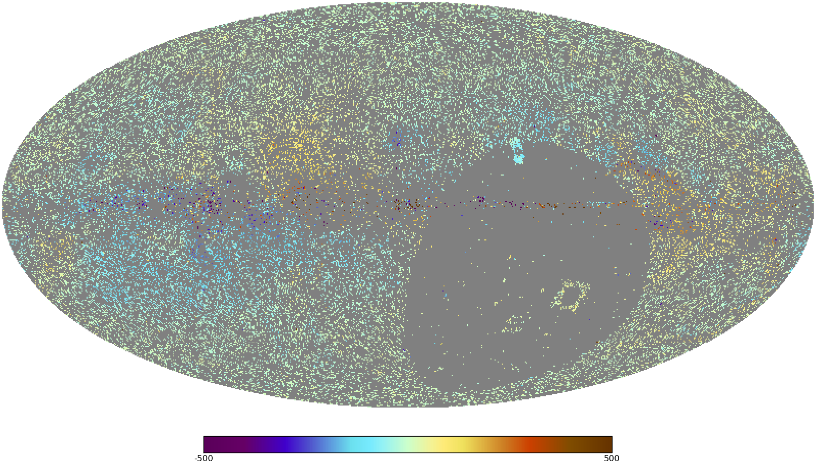
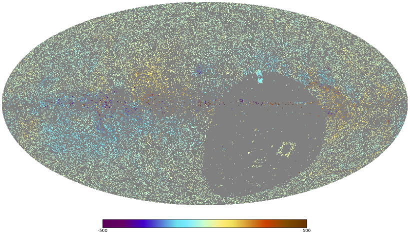


41 330 data points



Challenges

- Regions without data
- Uncertain error bars:
  - complicated observations
  - $n\pi$-ambiguity
  - extragalactic contributions unknown

Assumptions

- linear data model $d = Rs + n$
- Gaussian signal field $s \hookleftarrow \mathcal{G}(s, S)$
- Gaussian noise $n \hookleftarrow \mathcal{G}(n, N)$

Wiener Filter

$$m = \int \mathcal{D}s \; s \; \mathcal{P}(s|d)$$

$$m = Dj, \text{ where } \begin{array}{l} j = R^{\dagger} N^{-1} d \\ D = \left(S^{-1} + R^{\dagger} N^{-1} R\right)^{-1} \end{array}$$

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Assumptions

- linear data model $d = Rs + n$
- Gaussian signal field $s \hookleftarrow \mathcal{G}(s, S)$
- s statistically isotropic $\Rightarrow \; S_{(\ell,m)(\ell',m')} = \delta_{\ell\ell'}\delta_{mm'} C_\ell$
- Gaussian noise $n \hookleftarrow \mathcal{G}(n, N)$
- noise uncorrelated $\Rightarrow \; N_{ij} = \delta_{ij}\eta_i\sigma_i^2$

Wiener Filter

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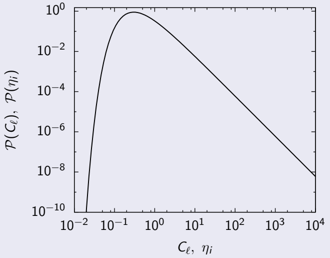
assume priors for parameters

$$\mathcal{P}\left((C_\ell)_\ell\right) = \prod_\ell \frac{1}{q_\ell \Gamma(\alpha_\ell - 1)} \left(\frac{C_\ell}{q_\ell}\right)^{-\alpha_\ell} \exp\left(-\frac{q_\ell}{C_\ell}\right)$$

$$\mathcal{P}\left((\eta_i)_i\right) = \prod_i \frac{1}{q_i \Gamma(\alpha_i - 1)} \left(\frac{\eta_i}{q_i}\right)^{-\alpha_i} \exp\left(-\frac{q_i}{\eta_i}\right)$$

$\Rightarrow$ marginalize over all possible parameters



Wiener Filter

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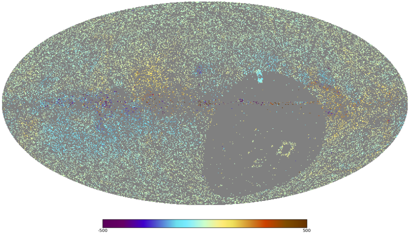
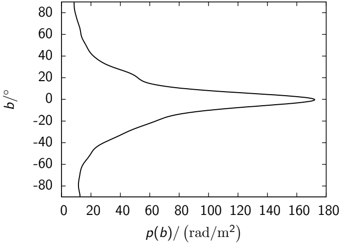
Extended critical filter

$$m = Dj$$

$$C_\ell = \frac{1}{\alpha_\ell + \ell - 1/2}\left[q_\ell + \frac{1}{2}\mathrm{tr}\left(\left(mm^\dagger + D\right) P_\ell\right)\right]$$

$$\eta_i = \frac{1}{2\alpha_i - 1}\left[2q_i + \frac{1}{\sigma_i^2}\left((d - Rm)^2_{ii} + \left(RDR^\dagger\right)_{ii}\right)\right]$$





$$s := \frac{\phi(l,b)}{p(b)} \sim \text{statistically isotropic}$$

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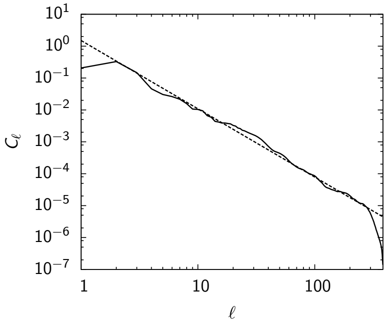
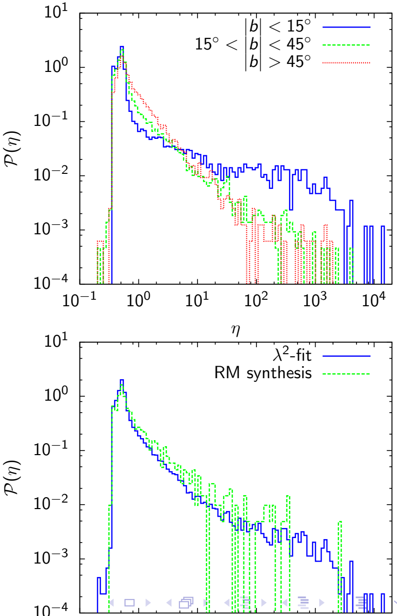
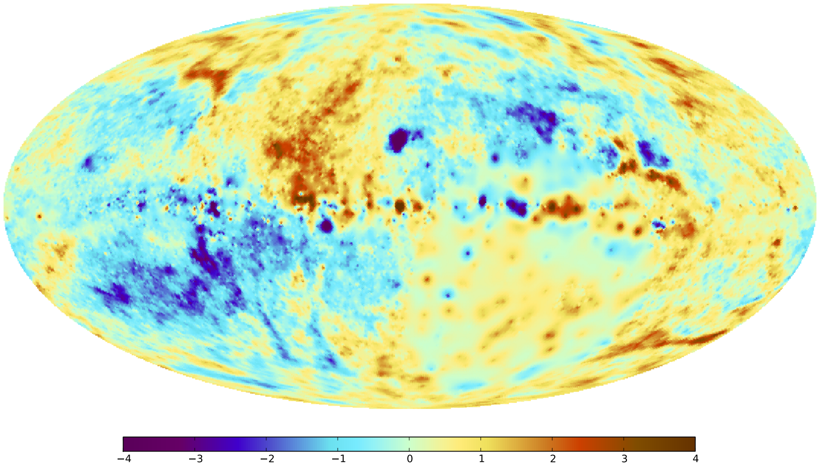
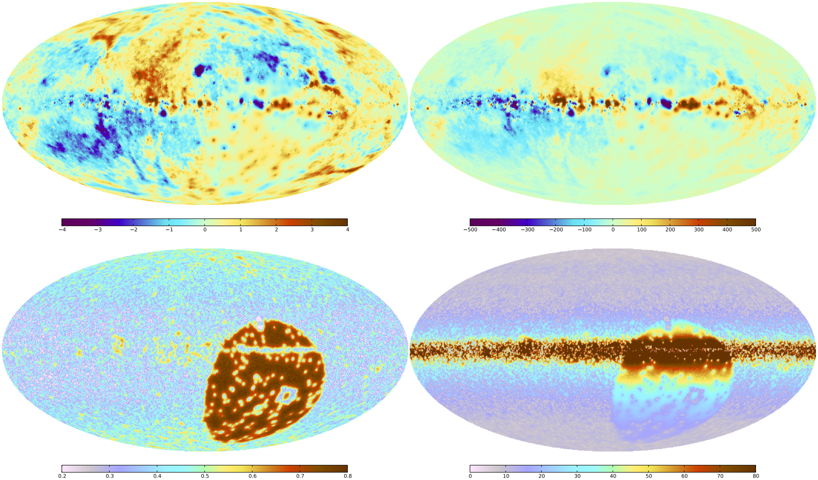


$$C_\ell \propto \ell^{-2.14}$$



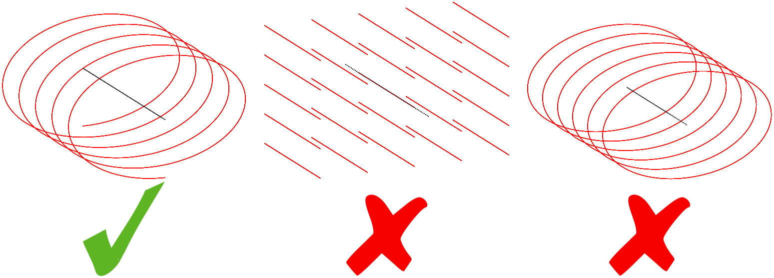

−4
−3
−2
−1
0
1
2
3
4

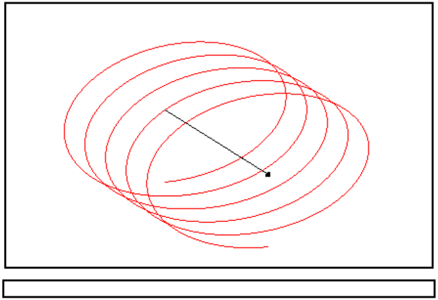

−4 −3 −2 −1 0 1 2 3 4
−500 −400 −300 −200 −100 0 100 200 300 400 500
0.2 0.3 0.4 0.5 0.6 0.7 0.8
0 10 20 30 40 50 60 70 80

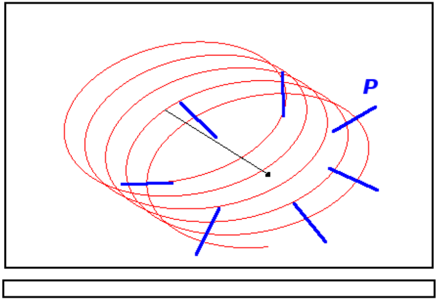
Local
Inference
Test for
Magnetic fields,
which **U**ncovers
helice**S**

Junklewitz & Enßlin (2011)

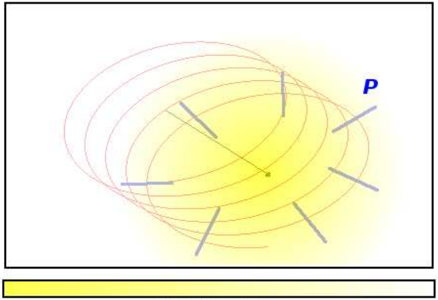
$$H = \int \vec{A} \cdot \vec{B} \, \mathrm{d}V$$







$$P = |P|\, e^{2i\alpha}$$



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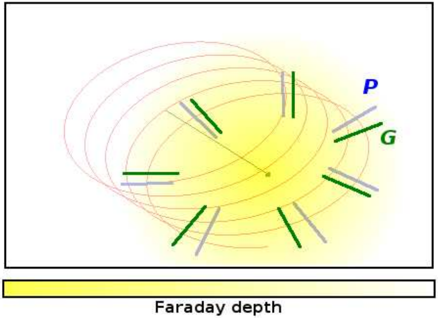
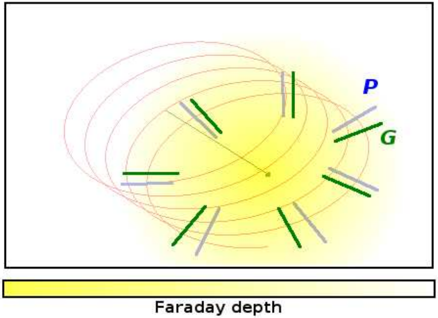


$$P = |P|\, e^{2i\alpha}$$

$$G = T_2\left(\nabla\phi\right) = \left(\partial_x\phi + i\partial_y\phi\right)^2 = |G|\, e^{2i\gamma}$$

$$T_2 : \ \mathbb{R}^2 \to \mathbb{C}$$



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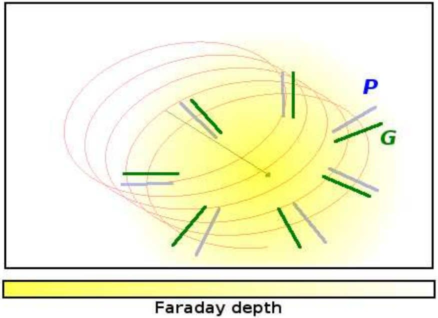
$$T_2 : \ \mathbb{R}^2 \rightarrow \mathbb{C}$$

Helicity

$$\mathrm{Re}\left(GP^*\right) > 0$$



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Helicity

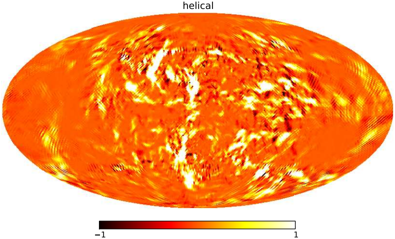
$$\mathrm{Re}\,(GP^*) > 0$$

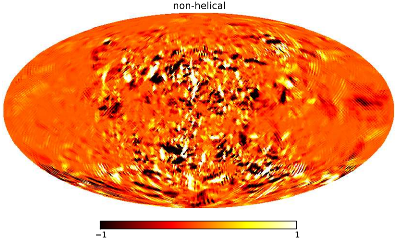
According to theory...

$$\langle P\phi\phi\rangle \propto \epsilon_H^2 \Rightarrow$$

$$\langle GP^*\rangle \propto \left(\int_0^\infty \mathrm{d}k\, \frac{\epsilon_H(k)}{k}\right)^2$$

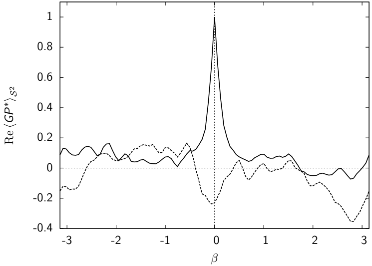
Junklewitz & Enßlin (2011)

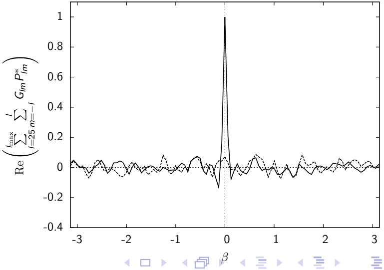


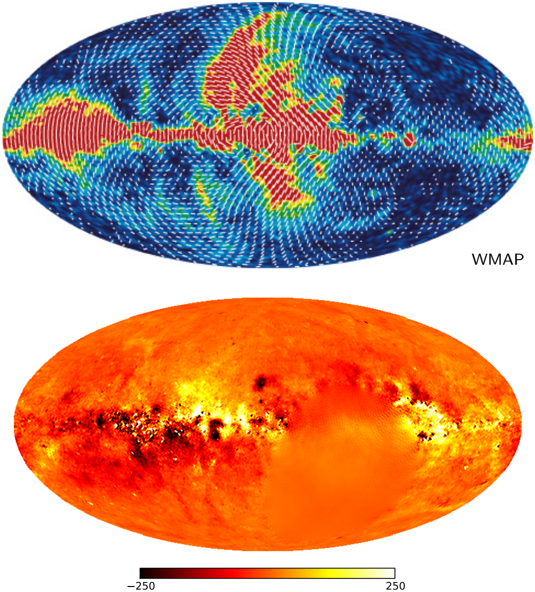


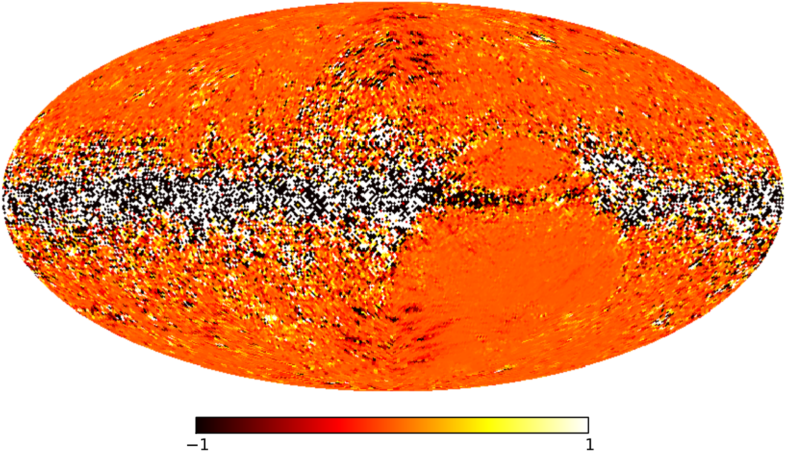
$$\text{with helicity: } \left\langle \mathrm{Re}\left\langle GP^* \right\rangle_{\mathcal{S}^2} \right\rangle_{\mathrm{samples}} = 1.0, \quad \sigma_{\mathrm{Re}\langle GP^* \rangle_{\mathcal{S}^2}} = 0.25$$

$$\text{without helicity: } \left\langle \mathrm{Re}\left\langle GP^* \right\rangle_{\mathcal{S}^2} \right\rangle_{\mathrm{samples}} = -0.27, \quad \sigma_{\mathrm{Re}\langle GP^* \rangle_{\mathcal{S}^2}} = 0.23$$

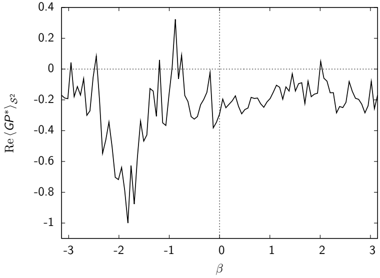




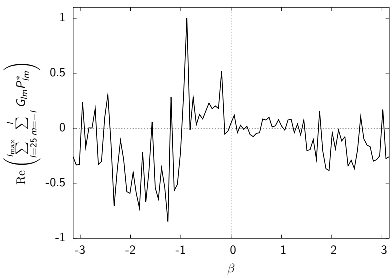

WMAP
−250
250


−1
1

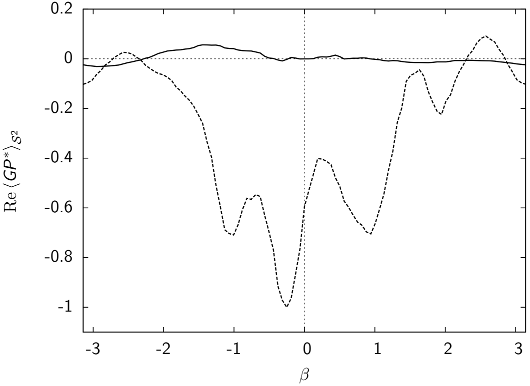
contributions of all scales



small-scale contributions



Test case with non-trivial electron densities:



Summary

- *Extended critical filter* produces excellent map with
  - angular power spectrum
  - robustness against outliers
- *LITMUS* test works, provided the electron densities don't vary too much.
- $\Rightarrow$ *LITMUS* test has problems if electron density varies on scales of helicity.

Outlook

- better maps are available:
  - Faraday depth from Oppermann et al.
  - synchrotron polarization from Planck
  - thermal dust polarization from Planck